

# Discussion of “Risky Mortgages in a DSGE Model”

Tomoyuki Nakajima  
Kyoto University

This is an interesting paper on a timely topic. As the authors argue, the recent global financial crisis has its roots in increased mortgage delinquencies caused by the bursting of the housing bubble in the United States. By what mechanism does such an increase in mortgage delinquencies affect the rest of the economy? And how should policy react to it? To address these issues, the authors develop a dynamic stochastic general equilibrium model with endogenous defaults on mortgage loans.

Specifically, they apply Bernanke and Gertler’s (1989) model of financial constraint to housing investment.<sup>1</sup> The economy produces two kinds of goods: non-durable consumption goods and durable investment goods (“houses”). The value of the housing stock held by each individual is subject to idiosyncratic risk. Mortgage loans take the form of a debt contract, where borrowers who experience a sufficiently large decline in the value of their housing stock declare defaults. In the case of a default, the lender incurs a monitoring cost and seizes the collateral, i.e., the housing stock held by the defaulting borrower. Furthermore, prices of non-durable goods are sticky (prices of housing stock are flexible). Monetary policy is given by an interest rate rule of the Taylor type.

The aggregate shock they focus on is the shock to the volatility of the idiosyncratic shock to the value of housing stock. Its fluctuations

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<sup>1</sup>The model of Bernanke and Gertler (1989) is based on the costly-state-verification model of Townsend (1979), and has been elaborated further by Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999), among others. Aoki, Proudman, and Vlieghe (2004) is an earlier application of the Bernanke-Gertler model to housing investment.

are interpreted as exogenous changes in the mortgage risk. Based on such a framework, they analyze impulse responses to an exogenous increase in the standard deviation of the idiosyncratic housing risk. An increase in the volatility of the idiosyncratic risk increases the default rate, reduces the supply of loans, and leads to a recession (i.e., both output and consumption fall). These features are roughly consistent with the evidence. The authors argue, however, that there are two counterfactual implications. First, the relative price of housing stock recovers too fast after the shock to the mortgage risk. Second, the decline in the level of output is too small. In addition, they compare interest rate rules with different degrees of interest rate inertia. They find that an increase in the mortgage risk results in a larger recession under a more inertial interest rate rule. Based on this, they argue that interest rate flexibility is important when responding to a shock to the mortgage risk.

The key feature of the model of this paper is to apply the financial friction model of Bernanke and Gertler (1989) to the model of housing investment. An alternative approach would be to apply the collateral constraint model of Kiyotaki and Moore (1997), just as Iacoviello (2005) has done. I'd like to start this discussion by comparing these two approaches.

In this paper, there are two types of households: "Borrowers" and "Savers." Borrowers are less patient than Savers so that, in equilibrium, Borrowers borrow from Savers. Each household consists of a continuum of individual members. Each individual manages a part of the housing stock owned by the household he/she belongs to. The value of the housing stock managed by an individual is subject to an idiosyncratic shock: if he/she manages  $h_t$  of housing stock in period  $t$ , it becomes  $\omega_t h_t$  during the period, where  $\omega_t$  is i.i.d. across individuals.

By assumption, individuals in Borrowers households obtain loans from lenders (Savers). Consider such an individual who manages housing stock  $H_t$  with mortgage loan  $L_t$ . It is assumed that a mortgage loan is a debt contract of the following form. Let  $R_{Z,t}$  denote the contractual rate of the loan—that is, the rate that applies as long as the borrower is solvent. The Borrower is solvent if the value of the housing stock which he/she manages is greater than the amount that he/she is supposed to repay. Thus, the Borrower is solvent if and only if  $\omega_t \geq \bar{\omega}_t$ , where  $\bar{\omega}_t$  is the threshold value defined by

$$\bar{\omega}_t(1 - \delta)P_{H,t}H_t = (1 + R_{Z,t})L_t,$$

where  $P_{H,t}$  is the price of housing stock and  $\delta$  is its depreciation rate. If the Borrower experiences a bad enough shock  $\omega_t < \bar{\omega}_t$ , he/she declares default and the lender seizes the collateral. Thus what the Borrower repays to the lender depends on the idiosyncratic shock  $\omega_t$ , and is given as follows:

$$\begin{cases} (1 + R_{Z,t})L_t, & \text{if } \omega_t \geq \bar{\omega}_t, \\ \omega_t(1 - \delta)P_{H,t}H_t, & \text{if } \omega_t < \bar{\omega}_t. \end{cases}$$

In addition, in the case of default, the lender must incur a monitoring cost which is proportional to the value of the collateral he/she is seizing:  $\mu\omega_t(1 - \delta)P_{H,t}H_t$ .

Lenders hold a completely diversified portfolio of loans. Consider a representative Saver who has made loans of total amount  $L_t$  in period  $t - 1$ . Let  $R_{L,t-1}$  denote the rate of return on his/her portfolio of loans, net of the monitoring cost:<sup>2</sup>

$$\begin{aligned} (1 + R_{L,t-1})L_t &= \int_0^{\bar{\omega}_t} \omega_t(1 - \mu)(1 - \delta)P_{H,t}H_t f(\omega_t) d\omega_t \\ &+ \int_{\bar{\omega}_t}^{\infty} (1 + R_{Z,t})L_t f(\omega_t) d\omega_t, \end{aligned} \tag{1}$$

where  $f(\omega)$  is the probability density function. This equation is referred to as the participation constraint of lenders. Note that, gross of the monitoring costs, the amount that the lender receives from Borrowers is  $(1 + R_{L,t-1})L_t + \mu G(\bar{\omega}_t)P_{H,t}H_t$ , where  $G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega f(\omega) d\omega$ .

Now consider a representative Borrowers household that owns housing stock  $H_t$  and has mortgage loans  $L_t$  at the beginning of period  $t$ . Given that the gross amount that the Borrowers household pays to lenders is  $(1 + R_{L,t-1})L_t + \mu G(\bar{\omega}_t)P_{H,t}H_t$ , its flow budget constraint in period  $t$  is expressed as

$$\begin{aligned} P_{C,t}C_t + P_{H,t}H_{t+1} - L_{t+1} &= W_t N_t + (1 - \delta)P_{H,t}H_t \\ &- \{(1 + R_{L,t-1})L_t + \mu G(\bar{\omega}_t)P_{H,t}H_t\}, \end{aligned} \tag{2}$$

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<sup>2</sup>It is assumed that this rate is predetermined in period  $t - 1$ .

where  $C_t$  is consumption of the non-durable good,  $P_{C,t}$  is its price,  $N_t$  is labor supply, and  $W_t$  is the wage rate. The lender's participation constraint (1) can be rewritten as

$$(1 + R_{L,t})L_{t+1} = \Phi(\bar{\omega}_{t+1})(1 - \delta)P_{H,t+1}H_{t+1}, \quad (3)$$

where

$$\Phi(\bar{\omega}) \equiv \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega + (1 - \mu)G(\bar{\omega}).$$

Note that this constraint may be interpreted as the collateral constraint: the amount that the Borrowers household can borrow (LHS) is limited by the value of its collateral (RHS). The Borrowers household maximizes its lifetime utility subject to the flow budget constraint (2) and the collateral constraint (3).

Now, how different is this model from the Kiyotaki-Moore-Iacoviello (KMI) model? In the KMI model, the Borrowers household's flow budget constraint and collateral constraint would be given respectively as

$$P_{C,t}C_t + P_{H,t}H_{t+1} - L_{t+1} = W_tN_t + (1 - \delta)P_{H,t}H_t - (1 + R_{L,t-1})L_t \quad (4)$$

and

$$(1 + R_{L,t})L_{t+1} = \Phi(1 - \delta)E_tP_{H,t+1}H_{t+1}, \quad (5)$$

where  $\Phi$  is a constant parameter determining the loan-to-value ratio.

Comparing equations (2)–(3) and (4)–(5), we notice two differences: First, in the model of this paper, the loan-to-value ratio,  $\Phi(\bar{\omega}_t)$ , is endogenously determined and fluctuates over time, while it is exogenously given in the KMI model. In particular, in this paper's model, the loan-to-value ratio has a close relationship with the default rate,  $F(\bar{\omega})$ , where  $F(\omega)$  is the cumulative distribution function. I find that this is a very attractive feature of the model. The second difference is that  $\bar{\omega}$  also directly affects the flow budget constraint (2), because it affects the amount of monitoring costs lenders must incur. These are the differences from the KMI model, which can make the model here more interesting.

Using this framework, the authors try to analyze the effect of an increase in the mortgage risk on the aggregate economy. For this purpose, they assume that the idiosyncratic shock to the value of housing stock,  $\omega_t$ , has time-varying standard deviation,  $\sigma_{\omega,t}$ . Specifically, it is assumed to follow an AR(1) process:

$$\ln(\sigma_{\omega,t}) = (1 - \rho_\sigma) \ln(\sigma_\omega) + \rho_\sigma \ln(\sigma_{\omega,t-1}) + \epsilon_{\sigma_{\omega,t}}.$$

Then the authors examine impulse responses of various variables to an innovation to the standard deviation of the idiosyncratic risk,  $\epsilon_{\sigma_{\omega,t}}$ .

This is a very interesting exercise, which helps to enhance our understanding on how disturbances to the housing market are propagated to the rest of the economy. In the rest of the discussion, I'd like to make some suggestions for further work.

First, land is abstracted from the analysis of this paper. However, evidence suggests that residential land is a very important part of the value of housing stock. It is, for instance, illustrated by Davis and Heathcote (2007), who decompose the aggregate value of the U.S. housing stock into structures and land components over the period 1975–2006. There they find not only that residential land is an important component of the value of the U.S. housing stock, but also that both trend growth and cyclical fluctuations in the price of housing stock are primarily attributable to changes in the price of residential land, rather than changes in the price of structures. Given such evidence, I think it is worth trying to add land to the model of this paper.

Second, in this paper the authors focus on the effects of an increase in the volatility of the idiosyncratic risk to the value of housing. I wonder if this is really what happened at the onset of the recent global financial crisis. Since it plays a central role in the analysis of this paper, I think it would be beneficial to conduct an empirical analysis to see if the volatility of the idiosyncratic risk to the value of housing rose enough to cause the crisis. For instance, in the impulse response analysis of this paper, it is assumed that the standard deviation of the idiosyncratic risk increases by 40 percent.

Third, related to the second comment, many economists have argued that what caused the recent financial crisis was the bursting of the housing bubble. Indeed, the authors themselves provide

such a view in the introduction of this paper. Then an interesting question would be how the bursting of the housing bubble affects the economy in the model of this paper. For this exercise we do not need to assume a change in the volatility of the idiosyncratic risk. For instance, let us consider the following kind of a news-shock experiment.<sup>3</sup> Suppose that at a point in time people receive a signal which suggests that the productivity of housing stock increases in some near future; they believe that signal for a while, but at some point they realize that the signal is wrong. This type of news shock will generate a boom and bust of housing prices. It seems to me an interesting exercise to conduct.

Fourth, exploring welfare implications of different monetary policy rules would be an interesting direction of future research. The authors do consider how differently the aggregate variables are affected by the volatility shock under alternative monetary policy rules. However, they do not discuss welfare in this paper. In the model of this paper, there are two distinct types of households: Borrowers and Savers. They presumably have different preferences over monetary policy rules. I think it is interesting to see what kind of conflict of interest they have. In the literature on optimal monetary policy, it is typical to consider a representative household, and thus there is no conflict of interest among households. Such a simplification may be okay in some contexts, but not in others. To me, it appears that explicitly considering such a conflict would become increasingly important.

## References

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<sup>3</sup>Kobayashi, Nakajima, and Inaba (2010) study the effects of a news shock in a related model.

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