Estimation of Monetary Policy Preferences in a Forward-Looking Model: A Bayesian Approach*

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In this paper we adopt a Bayesian approach toward the estimation of monetary policy preference parameters in a general equilibrium framework for the euro area. We assume that monetary policy authorities optimize an intertemporal quadratic loss function under commitment and study two alternative specifications of the loss function. The first specification includes inflation, output gap, and the interest rate differential as targets. The second loss function includes an additional wage-inflation target. The weights assigned to target variables—i.e., monetary policy preferences—are estimated jointly with the structural model parameters. The results imply that inflation variability remains the main concern of optimal monetary policy. Interest rate smoothing and the output gap appear to be, to a lesser extent, important target variables as well. Due to the time-inconsistency problem under commitment, we propose to initialize the estimates by a presample period of forty quarters. This allows us to approach, empirically, the timeless-perspective framework.

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1. Introduction

Correct knowledge of the variables that are of main concern for monetary policy is an important asset, since knowing the alternative monetary policy targets will have an effect on the formation of private-sector expectations. Given the role these expectations play in stabilizing the economy, it is desirable for monetary policymakers to provide the private agents with sufficient information concerning the relative importance of each target variable. The value attached to a particular target—for example, the inflation target—can be described by the relative weight assigned to this target in the loss function that the central bank aims to minimize over the infinite horizon. The relative weights therefore generally reflect the preferences of monetary policymakers with respect to the corresponding target variables.

In order to infer the monetary policy preferences, one could analyze empirical monetary policy reaction functions and study the behavior of monetary policymakers. This kind of approach, however, has often been criticized (Svensson 2002b, 2003, and Dennis 2000, 2002, 2004a, 2006, and 2007). While an estimated reaction function gives a good description of monetary policy behavior, the intertemporal loss function is a more appropriate measure of (changes in) monetary policy objectives. The variables entering the reaction function mainly provide monetary policy with information needed to achieve the policy objectives. These variables are therefore not necessarily equal to the target variables in the loss function and cannot be attributed directly to the monetary policy objectives. A more theoretical justification for assuming a representative monetary policymaker that systematically optimizes an intertemporal loss function is that this approach will bring monetary policy behavior in line with the behavior of private agents (Svensson 1999 and Woodford 2003). In this paper, we adopt a general equilibrium framework with rational and optimizing agents, where all structural equations result from optimal decisions made by private agents as well as monetary policymakers.

An extensive number of studies in the literature have recently focused attention on the estimation or calibration of preferences of optimizing monetary policy authorities, which is analogous to estimating the weights assigned to the target variables in the loss
function of the central bank. Many of these estimation exercises consider the case of discretionary monetary policy, as in Dennis (2000, 2004a), Söderström, Söderlind, and Vredin (2005), and Castelnuovo (2006) for the U.S. economy and Lippi and Neri (2007) for the euro-area economy. The case of full commitment as in Söderlind (1999) and Ilbas (forthcoming) or commitment to a simple rule of the kind adopted by Salemi (2006) and Givens and Salemi (2008) for the U.S. economy has, to our knowledge, not been applied to the euro-area economy. The aim of this paper is to study the case of monetary policy that systematically minimizes an intertemporal quadratic loss function under full commitment in a forward-looking model for the euro area. A commitment strategy, if credible, enables the central bank to control the expectations of private agents and provides it with an additional stabilization tool. We consider the Smets and Wouters (henceforth SW) (2003) model for the euro area as the benchmark model, where we replace the estimated Taylor rule with optimal monetary policy under commitment. This enables us to estimate the preference parameters of the monetary policy objective function jointly with the structural parameters of the model economy. We further show that the time-inconsistency problem under commitment can be dealt with empirically by considering an initialization period that is long enough to reduce the effect of the initial values on the estimation results. This enables us to implement the timeless-perspective framework of Woodford (2003) in empirical exercises.

This paper is organized as follows. In the next section, we outline the theoretical framework. We start from the SW (2003) model and describe the assumed structural behavior of the private agents in the economy, followed by the introduction of optimal monetary policy, which leads to a set of Euler equations that can be estimated accordingly. In introducing optimal monetary policy, we consider two types of loss functions. The first loss function includes inflation, the model-consistent output gap, and the interest rate differential as target variables, whereas the second loss function considers an additional wage-inflation target. The third section explains the data

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set used and the methodology adopted in the Bayesian estimation procedure, followed by a discussion of the results. We compare the results for the structural parameters obtained from the modified model with optimal monetary policy with the results of the original SW (2003) model. In section 4, we compare the alternative models based on their marginal-likelihood values and discuss the impulse responses obtained under the best-performing model that is characterized by optimal policy with respect to the benchmark impulse responses of SW (2003). Finally, section 5 concludes, followed by the appendices.

2. Theoretical Framework

The structural behavior of the euro-area economy is described by the model developed in Smets and Wouters (2003). However, instead of capturing the behavior of the monetary policy authorities by an empirical Taylor rule as is done in the original setup of the SW (2003) model, we will assume that monetary policy is performed optimally under commitment. In the following, we present a brief summary of the linearized SW (2003) model and introduce the optimizing monetary policy authorities.

2.1 The Smets and Wouters Model for the Euro Area

The dynamic stochastic general equilibrium (DSGE) framework presented in SW (2003) for the euro area consists of a household sector that supplies a differentiated type of labor, leading to nominal rigidities in the labor markets. The goods markets are characterized by intermediate and final goods producers. The former type of agents are monopolistically competitive and produce a differentiated type of intermediate goods, leading to nominal rigidities in the goods markets. The latter type of agents operate in a perfectly competitive market and produce one final good.

The model also features real rigidities like habit formation, costs of adjustment in capital accumulation, and variable capital utilization. There are many similarities with the model presented in Christiano, Eichenbaum, and Evans (2005). However, the SW (2003) model includes an additional number of structural shocks, includes
partial indexation to past inflation in the labor and the goods markets, and is estimated using (Bayesian) estimation methods. The linearized rational-expectations equations that result from private-sector optimizing behavior are summarized next, where the same notation as in SW (2003) is adopted and where all variables are expressed as log-deviations from their steady-state levels denoted by $\hat{\cdot}$, i.e. $\hat{x} = \log(\frac{x}{x^*})$.

Each household supplies a differentiated type of labor and maximizes an intertemporal utility function of the following type:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^\tau,$$

where $E$ is the rational-expectations operator and $\beta$ is the discount factor. The utility function $U_t^\tau$ is separable in consumption, $C_t^\tau$, and labor supply, $l_t^\tau$, defined as follows (see linearized equations below for a description of the parameters):

$$U_t^\tau = \varepsilon^b_t \left( \frac{1}{1 - \sigma_c} (C_t^\tau - H_t)^{1 - \sigma_c} - \frac{\varepsilon^L_t}{1 + \sigma_c} (l_t^\tau)^{1 + \sigma_c} \right).$$

This setup leads to the following consumption Euler equation, which includes an external habit variable that leads to a backward-looking component to capture habit persistence:

$$\hat{C}_t = \frac{h}{1 + h} \hat{C}_{t-1} + \frac{1}{1 + h} E_t \hat{C}_{t+1} - \frac{1 - h}{1 + (1 + h)\sigma_c} (\hat{R}_t - E_t \hat{\pi}_{t+1})$$

$$+ \frac{1 - h}{(1 + h)\sigma_c} (\hat{\varepsilon}^b_t - E_t \hat{\varepsilon}_{t+1}),$$

where $h$ is the degree of habit persistence in the external habit variable $H_t = hC_{t-1}$. The symbol $\sigma_c$ represents the inverse of the intertemporal elasticity of substitution between consumption at different dates. $\hat{R}_t$ is the nominal interest rate, $\hat{\pi}_t$ is the inflation rate, and $\hat{\varepsilon}^b_t$ is an AR(1) preference shock to the discount rate with an

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2For a detailed description of the optimizing behavior of the agents that led to the linearized version of the model, we refer to the original SW (2003) paper.
i.i.d. normal error term: \( \varepsilon^b_t = \rho_b \varepsilon^b_{t-1} + \eta^b_t \). Nominal wages are set by households according to a Calvo (1983) type of scheme. Households that cannot reoptimize will adjust their nominal wages partially to past inflation with a degree \( 0 \leq \gamma_w \leq 1 \), leading to the following real-wage \((w_t)\) equation:

\[
\hat{w}_t = \frac{\beta}{1 + \beta} E_t \hat{w}_{t+1} + \frac{1}{1 + \beta} \hat{w}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{\pi}_{t+1} - \frac{1 + \beta \gamma_w \hat{\pi}_t}{1 + \beta} \\
+ \frac{\gamma_w}{1 + \beta} \hat{\pi}_{t-1} - \frac{1}{1 + \beta} \frac{(1 - \beta \xi_w)(1 - \xi_w)}{(\lambda_w + (1 + \lambda_w)\sigma_L)} \xi_w \\
\times \left[ \hat{w}_t - \sigma_c \hat{L}_t - \frac{\sigma_c}{1 - h} (\hat{C}_t - h \hat{C}_{t-1}) - \hat{\varepsilon}_L - \hat{\eta}_L \right], \tag{4}
\]

where \( \xi_w \) is the probability that the wage will not be reoptimized in the next period. \( \lambda_w \) is a constant in \( \lambda_{w,t} = \lambda_w + \eta^w_t \), with \( \lambda_{w,t} \) a shock to the wage markup assumed to be i.i.d. normal around the constant term. \( \sigma_L \) is the inverse of the elasticity of work with respect to the real wage, \( \hat{L}_t \) is the aggregate labor, and \( \hat{\varepsilon}_L \) is a shock to labor supply assumed to follow an AR(1) process with an i.i.d. normal error term: \( \varepsilon^L_t = \rho_L \varepsilon^L_{t-1} + \eta^L_t \). Households also make the investment and capital accumulation decision; they are owners of the capital stock and rent it out to the producers of intermediate goods at the rental rate of capital \( r^k_t \). The supply of rental services can be adjusted by increasing investments \( I_t \) or by increasing the utilization rate \( z_t \) of installed capital. The linearized investment equation, which is characterized by adjustment costs depending on the size of investments, is given by the following equation:

\[
\hat{I}_t = \frac{1}{1 + \beta} \hat{I}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{I}_{t+1} + \frac{\varphi}{1 + \beta} \hat{Q}_t + \frac{\beta E_t \hat{\varepsilon}_{t+1} - \hat{\varepsilon}_t}{1 + \beta}, \tag{5}
\]

with \( \varphi \) the inverse of the investment adjustment costs \((S'')\), which captures the investment dynamics, and \( \hat{Q}_t \) the real value of capital expressed in units of consumption goods. \( \hat{\varepsilon}_t \) is an AR(1) shock to investment costs with an i.i.d. normal error term: \( \varepsilon^I_t = \rho_I \varepsilon^I_{t-1} + \eta^I_t \). The real value of capital is represented by
\[ Q_t = -(R_t - \pi_{t+1}) + \frac{1 - \tau}{1 - \tau + \bar{r}^k} E_t Q_{t+1} + \frac{\bar{r}^k}{1 - \tau + \bar{r}^k} E_t \hat{r}_{t+1} + \eta_t^Q, \]

(6)

with \( \tau \) the depreciation rate, \( \bar{r}^k \) the steady-state real rental rate of capital, and \( \eta_t^Q \) an i.i.d. normal shock that captures changes in the external finance premium due to informational frictions. However, this shock is not related to the structure of the economy, and it is further assumed that distortions caused by informational frictions are zero in the steady state. By definition, \( \beta = 1/(1 - \tau + \bar{r}^k) \). The capital-accumulation equation fulfills

\[ \hat{K}_t = (1 - \tau)\hat{K}_{t-1} + \tau \hat{I}_t, \]

(7)

where \( K_t \) stands for capital. There is a final-goods sector and an intermediate-goods sector. The final-goods sector is perfectly competitive and produces one final good used for consumption and investment by the households. The intermediate-goods sector is characterized by monopolistic competition where each firm is the only producer of the intermediate good \( j \). In the intermediate-goods sector each monopolist sets its price, given the total demand function for its differentiated good in line with Calvo (1983). As in the case of wage setting by households, each firm can only reoptimize its price after receiving a price-change signal with a constant probability \( 1 - \xi_p \). Firms that cannot reoptimize will adjust their price partially to past inflation with a degree \( 0 \leq \gamma_p \leq 1 \). This leads to the following New Keynesian Phillips curve:

\[ \hat{\pi}_t = \frac{\beta}{1 + \beta \gamma_p} E_t \hat{\pi}_{t+1} + \frac{\gamma_p}{1 + \beta \gamma_p} \hat{\pi}_{t-1} \]

\[ + \frac{1}{1 + \beta \gamma_p} \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} \left[ \alpha \hat{r}^k_t + (1 - \alpha) \hat{w}_t - \hat{\varepsilon}_t + \eta_t^p \right], \]

(8)

with \( \alpha \) the share of capital in total output in the Cobb-Douglas production function in the intermediate-goods sector with constant

\[ \text{Note that we correct for a typo in SW (2003) by replacing lagged investment with its current value. In addition, we include in equation (10) the cost-of-capital-adjustment component (last term) in the first line, which is omitted in SW (2003).} \]
returns to scale. \( \varepsilon_{t}^{a} \) is an AR(1) productivity shock with an i.i.d. normal error term: \( \varepsilon_{t}^{a} = \rho a \varepsilon_{t-1}^{a} + \eta_{t}^{a} \). The final term in square brackets represents the real marginal costs augmented with an i.i.d. (cost-push) shock \( \eta_{t}^{p} \) to the markup in the goods market: \( \lambda_{p,t} = \lambda_{p} + \eta_{t}^{p} \).

The labor-demand equation for a given capital stock is given by

\[
\hat{L}_{t} = -\hat{w}_{t} + (1 + \Psi)\hat{r}_{t}^{k} + \hat{K}_{t-1},
\]

where \( \Psi \) is the inverse of the elasticity of the capital-utilization cost function, which comes from the condition \( \hat{r}_{t}^{k} = \frac{1}{\Psi} \hat{z}_{t} \) and determines the dynamics. Finally, the goods-market equilibrium condition is represented by the following expression, where total output, \( \hat{Y}_{t} \), equals total demand for output by households and the government (first line) and total supply of output by the firms (second line):

\[
\hat{Y}_{t} = c_{y}C_{t} + g_{y}\varepsilon_{t}^{G} + \tau k_{y}\hat{I}_{t} + \tau k_{y}\Psi \hat{r}_{t}^{k} = \phi(\hat{\varepsilon}_{t}^{a} + \alpha \hat{K}_{t} + \alpha \Psi \hat{r}_{t}^{k} + (1 - \alpha)\hat{L}_{t}),
\]

where \( \varepsilon_{t}^{G} \) is an AR(1) government spending shock with an i.i.d. normal error term: \( \varepsilon_{t}^{G} = \rho G \varepsilon_{t-1}^{G} + \eta_{t}^{G} \). \( c_{y}, g_{y}, \) and \( k_{y} \) stand for the steady-state ratio of consumption to output, government spending to output, and capital to output, respectively. \( \phi \) is equal to \( 1 + \frac{\Phi}{Y} \), where the second term is the ratio of fixed production cost (\( \Phi \)) to output in the steady state (\( \overline{Y} \)).

In order to complete the model, we introduce optimizing monetary policy authorities, discussed in the next section, which distinguishes our model from the original SW (2003) specification where monetary policy is described by a generalized empirical Taylor rule of the following type:\footnote{Note that we correct a typo in SW (2003) in the first line of the rule where the lag of the output gap appears instead of the current output gap.}

\[
\hat{R}_{t} = \rho \hat{R}_{t-1} + (1 - \rho)\{ \hat{\pi}_{t} + r_{\pi}\frac{\hat{\pi}_{t-1} - \hat{\pi}_{t}}{\hat{\pi}_{t}} + r_{y}(\hat{Y}_{t-1} - \hat{Y}_{t-1}) \} \\
+ \tau_{\pi}\frac{\hat{\pi}_{t} - \hat{\pi}_{t-1}}{\hat{\pi}_{t}} + r_{\Delta y}(\hat{Y}_{t} - \hat{Y}_{t}) + r_{\Delta y}(\hat{Y}_{t-1} - \hat{Y}_{t-1}) + \eta_{R}^{R}.
\]
The interest rate $\hat{R}_t$ is the monetary policy instrument; $\pi_t$ is the inflation target that follows an AR(1) process with i.i.d. normal error term and is normalized to zero: $\pi_t = \rho \pi_{t-1} + \eta_t^\pi$. $\rho$ is the monetary authorities’ smoothing parameter, $\eta_t^R$ is an i.i.d. normal monetary policy shock, and $(\hat{Y}_t - \hat{Y}_t^p)$ is the output gap defined as the difference between the actual output level and the level of potential output. The latter is assumed to be the output level under the absence of nominal rigidities (i.e., full price and wage flexibility) and the three i.i.d. cost-push shocks ($\eta_t^Q, \eta_t^p, \eta_t^w$). In addition to the one-period difference in inflation and the output gap as measured by $r_{\Delta \pi}$ and $r_{\Delta y}$, respectively, a gradual response to lagged inflation ($r_\pi$) and lagged output gap ($r_y$) is assumed.

2.2 Optimal Monetary Policy

Monetary policy authorities are assumed to minimize a discounted intertemporal quadratic loss function of the following type:

$$E_t \sum_{i=0}^{\infty} \delta^i [y_{t+i} W y_{t+i}], \quad 0 < \delta < 1,$$

with $\delta$ the monetary policy discount factor, which in the estimations will be fixed to the value chosen for the households’ discount factor: $\delta = \beta = 0.99$. $E_t$ is the expectations operator conditional on information available at time $t$. This type of quadratic loss function is commonly adopted in the literature, as in Rudebusch and Svensson (1999), Söderlind (1999), Giannoni and Woodford (2003), Svensson (2003), and Dennis (2007). The vector $y_t = [x_t' u_t]'$ contains the $n \times 1$ endogenous variables and AR(1) exogenous variables in the model included in $x_t$ and the $p \times 1$ vector of control variables included in $u_t$. Since we assume only one control variable—i.e., the interest rate—$u_t$ is a scalar with $u_t = \hat{R}_t$. $W$ is a time-invariant symmetric, positive semi-definite matrix of policy weights which reflect the monetary policy preferences of the central bank over the target variables. An alternative and theoretically more justifiable approach toward monetary policy is to derive the approximated welfare-based loss function. Levin et al. (2006) derive a welfare-based loss function in a DSGE model similar to the one of SW (2003) and consider the
effects of uncertainty on the conduct of monetary policy. The authors show that presence of uncertainty about model specification and the parameter estimates has an effect on the specification of the welfare-based loss function. However, adopting a welfare-based loss function would make our results rely more heavily on the specific assumptions about the underlying model of the economy than would be the case under an ad hoc loss function. Angeloni, Coenen, and Smets (2003) claim that the welfare loss from not using a welfare-based loss function is small enough to justify the use of an ad hoc loss function in the SW (2003) model. As shown by Woodford (2003), in more simple models this type of loss function can be obtained from the microfoundations.\textsuperscript{5} Recent developments in the literature, however, have made it possible to consider linear-quadratic approximate problems that yield a solution representing a correct local linear approximation to the actual non-linear optimal policy. Benigno and Woodford (2008), among others, show that in a large number of cases, under the linear-quadratic approximation to the exact optimal policy problem, it is possible to correctly rank alternative simple rules as long as the size of the shocks is small enough. Altissimo, Cúrdia, and Rodriguez Palenzuela (2005) provide a standard algorithm that makes it possible to apply the theoretical framework of Benigno and Woodford (2008) to various type of models, including the SW (2003) model. Debortoli and Nunes (2006) in addition prove that the methodology of Benigno and Woodford (2006) can be generalized, and that in the case of a distorted steady state the timeless-perspective assumption is crucial in obtaining a linear-quadratic approximation to the welfare measure.

We follow the literature\textsuperscript{6} in assuming that the one-period ad hoc loss function for (12) includes inflation, a measure of the output gap, and a smoothing component for the policy instrument:

\begin{equation}
\text{Loss}_t = \pi_t^2 + q_y (\hat{Y}_t - \hat{Y}_t^p)^2 + q_r (\hat{R}_t - \hat{R}_{t-1})^2.
\end{equation}

\textsuperscript{5}See also Rotemberg and Woodford (1999) and Giannoni and Woodford (2003), among others, for the same exercise in DSGE models.

As explained in the next section, the data set used in the estimation procedure contains series on (detrended) inflation considered in deviation from its sample mean. Since we normalize the inflation target in (13) to zero, this implies that monetary policy aims to stabilize inflation around the sample mean, which is a known constant.\(^7\) The inclusion of the term \(\hat{R}_t - \hat{R}_{t-1}\) in the loss function can be justified by concerns about financial stability or in order to take into account the observed inertial behavior in the policy instrument, which suggests a gradualist monetary policy approach. The output gap considered here is the one implied by the model—i.e., the deviation of output from its natural level.\(^8\) As is done in most applications in the literature, the weight assigned to inflation in the above loss function is normalized to one.\(^9\) Hence, the weights corresponding to the output gap and the interest rate smoothing component—i.e., \(q_y\) and \(q_r\), respectively—are relative weights with respect to the inflation target variable. We will estimate the preferences of monetary policy reflected by these parameters along with the structural parameters of the economy.

Since the structural model of the economy outlined above is characterized by nominal wage rigidities, we investigate also the case where monetary policy is concerned about stabilizing nominal wage inflation in addition to the target variables in (13). A case for a nominal wage-inflation target is provided by Erceg, Henderson, and Levin (1998, 2000) and Woodford (2003). In a dynamic general equilibrium model featuring wage and price inertia, monetary policy faces a trade-off between stabilizing wage and price inflation (in addition to the output gap) since rigidities in the two markets at the same

\(^7\)One could criticize this approach and alternatively consider the inflation target as an additional parameter to be estimated. This approach is considered in Ilbas (forthcoming). Conditions under which the inflation target can be identified and estimated are provided in Dennis (2004a, 2004b).

\(^8\)Although it is more common in empirical applications to use deviations of output from a linear trend as an approximation to the output gap, we prefer to adopt the theoretical concept of the output gap. To support this choice, we experiment with alternative definitions of the output gap in the next section which yield less favorable results in terms of the marginal likelihood.

\(^9\)Examples can be found in Rotemberg and Woodford (1999), Rudebusch and Svensson (1999), Woodford (2003), and Dennis (2004a).
time make the Pareto optimal equilibrium unattainable for monetary policy. Moreover, Erceg, Henderson, and Levin (2000) derive a social welfare function under the circumstances of nominal wage and price rigidities and show that there are high welfare costs attached to targeting price inflation only and ignoring wage-inflation stabilization. Therefore, we analyze the following alternative specification of the loss function:

\[
Loss_t = \pi_t^2 + q_y(Y_t - \hat{Y}_t)^2 + q_r(R_t - \hat{R}_t)^2 + q_w(W_t - \hat{W}_{t-1} + \pi_t)^2,
\]

where \(q_w\) is the weight attached to the wage-inflation target. The specification of the one-period loss function in (13) can be considered as a special case of (14) where \(q_w\) is set to zero. We will estimate the model under both specifications of the loss function, by treating the case under each one of the two specifications as a separate model. Therefore, we will refer to the case where \(q_w = 0\)—i.e., period loss function (13)—as model M1 and consider the case where nominal wage inflation is an additional target variable as model M2. We then use the corresponding marginal-likelihood values in order to assess to what extent monetary policy has been concerned about nominal wage-inflation stabilization over the sample period.\(^\text{10}\)

The central bank minimizes the intertemporal loss function (12), the one-period loss function of which is given by either (13) or (14), under commitment subject to the structural equations of the economy (3)–(10) augmented by their flexible-price versions, written and represented by the following second-order form:

\[
Ax_t = BE_{t}x_{t+1} + Fx_{t-1} + Gu_t + Dz_t, \quad z_t \sim iid[0, \Sigma_{zz}],
\]

with \(z_t\) an \(n \times 1\) vector of stochastic innovations to the variables in \(x_t\), having mean zero and variance-covariance matrix \(\Sigma_{zz}\), and the

\(^{10}\text{We admit that in this study we make the strong assumption that the monetary policy regime has been unchanged in the euro area throughout the sample period. This might be doubtful given the separate monetary policy strategies adopted in the individual countries before the introduction of the euro. However, the transition period toward the euro and the restrictions imposed by the Maastricht Treaty justify our assumption that the policy regimes were more likely to have been in line.}\)
matrices $A$, $B$, $F$, $G$, and $D$ containing the structural parameters. We follow the optimization routine for commitment suggested by Dennis (2007). For details concerning this procedure, we therefore refer to Dennis (2007). The Euler equations of the monetary policy optimization problem are represented in the following second-order form:

$$A1^*\Upsilon_t = B1^*E_t\Upsilon_{t+1} + C1^*\Upsilon_{t-1} + D1^*z_t,$$

(16)

with $\Upsilon_t = \begin{bmatrix} x_t \\ u_t \\ \rho_t \end{bmatrix} = \begin{bmatrix} y_t \\ \rho_t \end{bmatrix}$ and the final term in $\Upsilon_t$, $\rho_t$, the vector of Lagrange multipliers. The matrices $A1^*$, $B1^*$, and $C1^*$ have dimension $(2n + p) \times (2n + p)$. In the next section, we will estimate the Euler equations (16) resulting from the optimization procedure.

3. Estimation

In this section we discuss the data set used and the methodology followed in estimating the system (16), which yields estimates of the structural parameters resulting from optimizing private agents and policy preferences of optimizing monetary policy authorities. Next, we present the results under $M1$ and $M2$ and compare them with each other and with the estimates obtained from the benchmark SW (2003) model.

3.1 Data

We use the updated version of the euro-area data set used in SW (2003), i.e., constructed by Fagan, Henry, and Mestre (2005). The data set contains observed series on real GDP, real consumption, real investment, GDP deflator, real wages, employment, and the nominal interest rate. The series range over the period 1980:Q2–2005:Q4, preceded by an initialization period of forty quarters. In appendix 2, we show that this pre-sample method in addition makes it possible to overcome the time-inconsistency problem in the estimation procedure by yielding non-zero initial values for the Lagrange multipliers of forward-looking variables, which is in line with the timeless-perspective approach of Woodford (2003).
As in SW (2003), all variables are considered in deviations from their sample means. Inflation and nominal interest rates are detrended by the inflation trend, whereas the remaining variables in the data set are detrended separately by a linear trend. Following SW (2003), we introduce an additional equation for employment to correct for the use of data on employment instead of data on aggregate hours worked in the euro area, since the latter is not available for the euro area:

\[
\hat{E}_t - E_{t-1} = \hat{E}_{t+1} - E_t + \frac{(1 - \beta \xi_e)(1 - \xi_e)}{\xi_e} (L_t - \hat{E}_t).
\] (17)

This equation reflects the fact that employment moves more slowly in response to shocks than aggregate hours worked. \(\hat{E}_t\) is the amount of people employed and \(\xi_e\) is the fraction of firms that is able to respond to shocks in a given period by adjusting the level of employment. The labor demand in a given period is satisfied by adjusting the number of hours worked per employer, which is flexible and not observed. In the line of Hansen and Sargent (1980), we further assume that the econometrician does not necessarily have access to the same (i.e., more complete) data set possessed, and responded to, by the monetary authorities. In order to capture the orthogonal information possessed by the monetary authorities, which we are not able to observe, we therefore introduce an i.i.d. error \(\epsilon_t^R\) to account for this discrepancy in the monetary policy instrument, i.e., the interest rate. Hence, the relation between the observed series of the nominal interest rate (\(\hat{R}_t^{obs}\)) and the non-observed policy instrument rate (\(\hat{R}_t^{nobs}\)) is expressed as follows:

\[
\hat{R}_t^{obs} = \hat{R}_t^{nobs} + \epsilon_t^R.
\] (18)

As opposed to SW (2003), the two monetary policy shocks that appear in the generalized Taylor rule—i.e., a shock to the inflation objective and an interest rate shock—are absent from the models \(M1\) and \(M2\). Given that there are seven observables only, we need a sufficient amount of restrictions to enable identification. As in SW (2003), identification is obtained through the assumption that all
shocks are uncorrelated, that the three cost-push shocks together with the interest rate error term follow a white noise process, and that the remaining shocks related to preferences and technology are AR(1). We also follow SW (2003) in fixing parameters that are related to the steady state of the state variables and cannot be estimated because the data set is demeaned. The discount factors $\beta$ and $\delta$ are set equal to 0.99, implying an annual real interest rate of 4 percent in the steady state. The annual depreciation rate on capital is assumed to be 10 percent, i.e., $\tau = 0.025$. The income share of labor in total output is assumed to be 0.7 in the steady state, i.e., $\alpha = 0.3$. The share of consumption and investment in total output is 0.6 and 0.22 in the steady state, respectively. Finally, there is one parameter that needs to be fixed since it is not identified: the constant wage markup parameter $\lambda_w$ is calibrated to be 0.5.

3.2 Methodology

We apply Bayesian methods in order to estimate the parameters of the alternative models.\textsuperscript{11} The posterior density distribution is derived by combining the prior distribution with the likelihood function. After maximizing the posterior mode, we use the Metropolis-Hastings algorithm to generate draws from the posterior distribution in order to approximate the moments of the distribution, calculate the modified harmonic mean, and construct the Bayesian impulse responses.\textsuperscript{12} In discussing the estimation results, we will focus on the maximized posterior mode and the Hessian-based standard errors.

Panel A in table 1 shows the details of the prior distributions for the shock processes, i.e., the standard errors $\sigma$ of all nine shocks and the AR(1) coefficients $\rho$ of the five preference shocks. The type of the prior distributions, the prior means, and the prior standard errors are identical to the assumptions made in SW (2003) and are kept constant throughout the estimation processes for both models $M_1$ and $M_2$. All variances of the shocks are assumed to have an

\textsuperscript{11}For a detailed discussion on Bayesian estimation of DSGE models, we refer to SW (2003, 2005), Schorfheide (2006), and An and Schorfheide (2007).

\textsuperscript{12}All estimations are performed using Michel Juillard’s software Dynare, which can be downloaded from the web site www.dsge.net.
Table 1. Prior Specifications of the Shocks and the Structural Parameters

<table>
<thead>
<tr>
<th>A. Parameters of Shock Processes</th>
<th>Prior Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distribution Type</td>
</tr>
<tr>
<td>σ Productivity Shock</td>
<td>Inverse Gamma</td>
</tr>
<tr>
<td>σ Preference Shock</td>
<td>Inverse Gamma</td>
</tr>
<tr>
<td>σ Government Spending Shock</td>
<td>Inverse Gamma</td>
</tr>
<tr>
<td>σ Labor Supply Shock</td>
<td>Inverse Gamma</td>
</tr>
<tr>
<td>σ Investment Shock</td>
<td>Inverse Gamma</td>
</tr>
<tr>
<td>σ Equity Premium Shock</td>
<td>Inverse Gamma</td>
</tr>
<tr>
<td>σ Price Markup Shock</td>
<td>Inverse Gamma</td>
</tr>
<tr>
<td>σ Wage Markup Shock</td>
<td>Inverse Gamma</td>
</tr>
<tr>
<td>σ Inflation Objective Shock (Only SW (2003))</td>
<td>Inverse Gamma</td>
</tr>
<tr>
<td>σ Monetary Policy Shock (Only SW (2003))</td>
<td>Inverse Gamma</td>
</tr>
<tr>
<td>σ Interest Rate Error (Only M1 and M2)</td>
<td>Gamma</td>
</tr>
<tr>
<td>ρ Inflation Objective Shock (Only SW (2003))</td>
<td>Beta</td>
</tr>
<tr>
<td>ρ Productivity Shock</td>
<td>Beta</td>
</tr>
<tr>
<td>ρ Preference Shock</td>
<td>Beta</td>
</tr>
<tr>
<td>ρ Government Spending Shock</td>
<td>Beta</td>
</tr>
<tr>
<td>ρ Labor Supply Shock</td>
<td>Beta</td>
</tr>
<tr>
<td>ρ Investment Shock</td>
<td>Beta</td>
</tr>
</tbody>
</table>

(continued)
## Table 1. (Continued)

<table>
<thead>
<tr>
<th>B. Structural Parameters</th>
<th>Prior Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distribution Type</strong></td>
<td><strong>Prior Mean</strong></td>
</tr>
<tr>
<td>$S''(.)$ Investment Adjustment Cost</td>
<td>Normal</td>
</tr>
<tr>
<td>$\sigma_c$ Consumption Utility</td>
<td>Normal</td>
</tr>
<tr>
<td>$h$ Consumption Habit</td>
<td>Beta</td>
</tr>
<tr>
<td>$\xi_w$ Calvo Wages</td>
<td>Beta</td>
</tr>
<tr>
<td>$\sigma_L$ Labor Utility</td>
<td>Normal</td>
</tr>
<tr>
<td>$\xi_p$ Calvo Prices</td>
<td>Beta</td>
</tr>
<tr>
<td>$\xi_e$ Calvo Employment</td>
<td>Beta</td>
</tr>
<tr>
<td>$\gamma_w$ Indexation Wages</td>
<td>Beta</td>
</tr>
<tr>
<td>$\gamma_p$ Indexation Prices</td>
<td>Beta</td>
</tr>
<tr>
<td>Capital-Utilization Adjustment Cost</td>
<td>Normal</td>
</tr>
<tr>
<td>$\rho$ Fixed Cost</td>
<td>Normal</td>
</tr>
<tr>
<td>$\rho$ Smoothing Parameter Empirical Taylor Rule</td>
<td>Beta</td>
</tr>
<tr>
<td>$r_\pi$ Lagged Inflation Parameter Empirical Taylor Rule</td>
<td>Normal</td>
</tr>
<tr>
<td>$r_y$ Lagged Output-Gap Parameter Empirical Taylor Rule</td>
<td>Normal</td>
</tr>
<tr>
<td>$r_{\Delta\pi}$ Inflation Differential Empirical Taylor Rule</td>
<td>Normal</td>
</tr>
<tr>
<td>$r_{\Delta y}$ Output-Gap Differential Empirical Taylor Rule</td>
<td>Normal</td>
</tr>
<tr>
<td>$q_r$ Interest Smoothing Preference</td>
<td>Gamma</td>
</tr>
<tr>
<td>$q_y$ Output-Gap Preference</td>
<td>Gamma</td>
</tr>
<tr>
<td>$q_w$ Wage-Inflation Preference</td>
<td>Gamma</td>
</tr>
</tbody>
</table>

**Notes:** Panel A summarizes the assumed prior distributions, the prior means, and the prior standard errors in the estimation of the standard errors of the shocks ($\sigma$), followed by the AR(1) coefficients of the persistent shocks ($\rho$). Panel B in the table shows the assumed prior distributions, the prior means, and the prior standard errors in the estimation of the structural parameters and the policy preference parameters.
inverted gamma distribution with two degrees of freedom, except for the interest rate error term, which we assume to be gamma distributed with a prior mean of 0.05 and standard error of 0.025. The five AR(1) coefficients are assumed to have a beta distribution with a prior mean of 0.85 and a strict prior standard error of 0.1 in order to distinguish the persistent shocks clearly from the i.i.d. shocks.

The prior specifications of the structural and the monetary policy preference parameters—i.e., the weights assigned to the target variables $q_y$, $q_r$, and $q_w$—are reported in panel B of table 1. The monetary policy preference parameters are assumed to be gamma distributed, in order to rule out negative values, with 0.5 prior mean. We further assign a loose prior standard error of 0.4 given that our prior knowledge about these parameters is rather restricted.

3.3 Results

In this section, we report and discuss the estimation results obtained for $M1$ and $M2$.

3.3.1 Structural Shocks and Private-Sector Parameters

The results obtained from the posterior maximization—i.e., the posterior mode and the (Hessian-based) standard errors—for the models $M1$, $M2$, and the benchmark SW (2003) model are reported in table 2.

In panel A of the table, the results for the shock processes are shown. The symbol $\sigma$ stands for the standard error of each shock and $\rho$ for the AR(1) parameter of the persistent shocks. The estimated

\[\sigma \text{ and } \rho\]

\[\text{In our experiments we consider alternative loss functions where we replace the interest rate smoothing term by the interest rate level (as in Giannoni and Woodford 2002a, 2002b; Woodford 2003; and Onatski and Williams 2004), or the output gap by the differences in output. We also studied the case where we used simply output deviations from a linear trend instead of the model-consistent output gap. We examined loss functions including a difference in the output gap, a difference in the inflation rate, or of the welfare-approximated type presented by Onatski and Williams (2004) as well. None of these cases, however, could yield better outcomes in terms of their corresponding marginal-likelihood values.} \]
Table 2. Estimates of the Shock Processes and the Structural Parameters for the Period 1980:Q2–2005:Q4

<table>
<thead>
<tr>
<th>A. Parameters of Shock Processes</th>
<th>Results from Posterior Maximization $M1(q_w = 0)$</th>
<th>Results from Posterior Maximization $M2(q_w \neq 0)$</th>
<th>Results SW (2003) (Reestimated)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode</td>
<td>se (Hessian)</td>
<td>Mode</td>
</tr>
<tr>
<td>$\sigma$ Productivity Shock</td>
<td>0.5767</td>
<td>0.0754</td>
<td>0.5769</td>
</tr>
<tr>
<td>$\sigma$ Preference Shock</td>
<td>0.1799</td>
<td>0.0497</td>
<td>0.1749</td>
</tr>
<tr>
<td>$\sigma$ Government Spending Shock</td>
<td>0.2961</td>
<td>0.0205</td>
<td>0.2961</td>
</tr>
<tr>
<td>$\sigma$ Labor Supply Shock</td>
<td>2.0571</td>
<td>0.4004</td>
<td>2.0676</td>
</tr>
<tr>
<td>$\sigma$ Investment Shock</td>
<td>0.0392</td>
<td>0.0120</td>
<td>0.0393</td>
</tr>
<tr>
<td>$\sigma$ Equity Premium Shock</td>
<td>0.5735</td>
<td>0.0611</td>
<td>0.5745</td>
</tr>
<tr>
<td>$\sigma$ Price Markup Shock</td>
<td>0.1601</td>
<td>0.0134</td>
<td>0.1597</td>
</tr>
<tr>
<td>$\sigma$ Wage Markup Shock</td>
<td>0.2854</td>
<td>0.0211</td>
<td>0.2858</td>
</tr>
<tr>
<td>$\sigma$ Inflation Objective Shock (Only SW (2003))</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\sigma$ Monetary Policy Shock (Only SW (2003))</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\sigma$ Interest Rate Error (Only M1 and M2)</td>
<td>0.0439</td>
<td>0.0069</td>
<td>0.0444</td>
</tr>
<tr>
<td>$\rho$ Inflation Objective Shock (Only SW (2003))</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\rho$ Productivity Shock</td>
<td>0.9867</td>
<td>0.0096</td>
<td>0.9867</td>
</tr>
<tr>
<td>$\rho$ Preference Shock</td>
<td>0.8752</td>
<td>0.0322</td>
<td>0.8772</td>
</tr>
<tr>
<td>$\rho$ Government Spending Shock</td>
<td>0.9342</td>
<td>0.0216</td>
<td>0.9342</td>
</tr>
<tr>
<td>$\rho$ Labor Supply Shock</td>
<td>0.9892</td>
<td>0.0071</td>
<td>0.9898</td>
</tr>
<tr>
<td>$\rho$ Investment Shock</td>
<td>0.9262</td>
<td>0.0520</td>
<td>0.9251</td>
</tr>
</tbody>
</table>

(continued)
Table 2. (Continued)

<table>
<thead>
<tr>
<th>B. Structural Parameters</th>
<th>Results from Posterior Maximization M1($q_w = 0$)</th>
<th>Results from Posterior Maximization M2($q_w \neq 0$)</th>
<th>Results SW (2003) (Reestimated)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode</td>
<td>se (Hessian)</td>
<td>Mode</td>
</tr>
<tr>
<td>$S''(.)$</td>
<td>4.4435</td>
<td>1.0543</td>
<td>4.4222</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>1.5026</td>
<td>0.3015</td>
<td>1.5080</td>
</tr>
<tr>
<td>$h$</td>
<td>0.5655</td>
<td>0.0642</td>
<td>0.5651</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>0.8930</td>
<td>0.0161</td>
<td>0.8953</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>1.4510</td>
<td>0.4848</td>
<td>1.4605</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>0.9038</td>
<td>0.0097</td>
<td>0.9040</td>
</tr>
<tr>
<td>$\xi_r$</td>
<td>0.7438</td>
<td>0.0289</td>
<td>0.7438</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>0.7704</td>
<td>0.1265</td>
<td>0.7684</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>0.4379</td>
<td>0.0726</td>
<td>0.4476</td>
</tr>
<tr>
<td>Capital Utilization</td>
<td>0.2153</td>
<td>0.0730</td>
<td>0.2146</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.3437</td>
<td>0.1169</td>
<td>1.3454</td>
</tr>
<tr>
<td>$\rho$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_\pi$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_y$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{\Delta \pi}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{\Delta \pi}^p$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_r$</td>
<td>0.9267</td>
<td>0.4128</td>
<td>0.9258</td>
</tr>
<tr>
<td>$q_y$</td>
<td>0.0410</td>
<td>0.0191</td>
<td>0.0406</td>
</tr>
<tr>
<td>$q_w$</td>
<td></td>
<td></td>
<td>0.1092</td>
</tr>
</tbody>
</table>

Notes: Panel A shows the posterior mode and the corresponding Hessian-based standard errors for the shock processes (with standard errors $\sigma$ and AR(1) coefficients of the persistent shocks $\rho$). Panel B in the table shows the estimated posterior mode of the structural parameters and the policy preference parameters (in bold), followed by the Hessian-based standard errors for each estimate in $M_1$ and $M_2$, and in the SW (2003) specification of the model for the sample period 1980:Q2–2005:Q4. A pre-sample period of forty quarters is used. $M_1$ corresponds to the loss function specification that does not include the wage-inflation target, i.e., Loss$_{M_1} = \pi_t^2 + q_y(Y_t - \hat{Y}_t)^2 + q_r(\hat{R}_t - \hat{R}_{t-1})^2$. $M_2$ is the case where an additional wage-inflation target is considered in the estimations, i.e., Loss$_{M_2} = \pi_t^2 + q_y(\hat{Y}_t - \hat{Y}_t)^2 + q_r(\hat{R}_t - \hat{R}_{t-1})^2 + q_w(\hat{W}_t - \hat{W}_{t-1} + \pi_t)^2$. Both of these specifications are compared with the results obtained for the benchmark SW (2003) specification of the model where monetary policy is described by the empirical Taylor rule $\hat{R}_t = \hat{\rho}\hat{R}_{t-1} + \hat{\pi} + (1 - \hat{\rho})(\pi_t + r_\pi(\pi_{t-1} - \pi_t) + r_y(\hat{Y}_{t-1} - \hat{Y}_{t-1})) + r_{\Delta \pi}(\pi_t - \pi_{t-1}) + r_{\Delta \pi}^p(\hat{Y}_t - \hat{Y}_{t-1} - (\hat{Y}_{t-1} - \hat{Y}_{t-1})) + \eta_t^R$. 

Capital Utilization: 2153.0730 - 2146.0731 = 0.0089
parameters and their corresponding standard errors under our specification $M_1$ of the model are similar to those under $M_2$. The preference, labor supply, and investment shocks are estimated to be lower under the models $M_1$ and $M_2$ compared with SW (2003). Furthermore, a more persistent labor supply shock is estimated under $M_1$ and $M_2$ than under the SW (2003) specification.

Comparing the SW (2003) estimates of the structural parameters with those obtained under $M_1$ and $M_2$, which are reported in panel B of table 2, yields the following conclusions. The investment adjustment cost parameter is estimated to be lower than the baseline estimated value. This suggests a higher elasticity of investment with respect to an increase in the current price of installed capital of 1 percent under $M_1$ and $M_2$. A strikingly higher value is obtained under $M_1$ and $M_2$ for the Calvo wage parameter $\xi_w$ than in the baseline case. Exactly the same conclusion holds for the wage-indexation parameter $\gamma_w$. Hence, a significantly higher wage stickiness and wage indexation is present whenever monetary policy is assumed to behave optimally, suggesting an average duration of wage contracts of slightly more than two years. The reverse conclusion can be drawn from estimates for the Calvo price $\xi_p$ and the price-indexation parameter $\gamma_p$. These parameters are lower under the models characterized by optimal monetary policy. This implies a lower degree of price stickiness in the goods markets under $M_1$ and $M_2$ compared with SW (2003), with an average duration of price contracts of more than two years, which is very similar to the average duration of the wage contracts. The estimates of $\gamma_p$ imply that price indexation is lower under $M_1$ and $M_2$, and is in line with the findings of Galí, Gertler, and López-Salido (2001) for the euro area, where a low degree of backward-looking behavior in the goods market is estimated. Salemi (2006) likewise finds support for forward-looking behavior in a New Keynesian model when monetary policy is assumed to be committed to a simple rule; in particular, the coefficient on the forward-looking term in the Phillips curve turns out to be significant, which is not the case when the optimality assumption is not imposed. Hence, the degree of forward-lookingness is more precisely estimated under the assumption of monetary policy optimality in Salemi (2006). This finding is supported by our estimation results for the inflation equation. Finally, the estimate of the
labor-utility parameter $\sigma_L$ is considerably lower under the models $M1$ and $M2$ than under SW (2003).\footnote{Note that, as was the case in the original SW (2003) paper, our estimates of this parameter did not appear to be robust across specifications.}

3.3.2 Monetary Policy Preference Parameters

Table 2 also shows the parameter estimates of our main interest, i.e., the monetary policy preferences in the two models $M1$ and $M2$.\footnote{Since inflation and the interest rates, both target variables, are measured on a quarterly basis and the literature occasionally considers target variables on a yearly frequency, the weights obtained from the estimates have to be adjusted in order to make the results comparable to those in the literature that base their results on yearly data. Taking this into account boils down to multiplying $q_y$ by a factor of 16 and converting the inflation and the interest rate in the model to a yearly frequency.} The preference for interest rate smoothing is estimated to be higher than the preference for output-gap stabilization, while overall the main concern is still the inflation target. However, although the importance of the output gap as a target variable is estimated to be small, the posterior probability interval for this parameter $q_y$ ranges above zero, as can be deduced from the posterior distribution for this parameter in model $M1$ displayed in figure 5 in appendix 1. The 90 percent posterior lower and upper bound for this parameter is $[0.0209 - 0.1249]$ in model $M1$. This implies that although the weight on the output gap is estimated to be small, it is significantly different from zero.\footnote{It should also be noted that in order to evaluate the relative importance of the components in the loss function, their corresponding weights should be combined with the relative volatility of the related variables. Hence the weights per se are not sufficient to evaluate the importance of the alternative target variables, due to the fact that the output-gap concept we use in this study is a theoretical one.} The estimated weight assigned to both the output gap $q_y$ and the interest rate smoothing preference parameter $q_r$ hardly changes over varying specifications of the loss function, with values around 0.04 and 0.92, respectively. The high weight on interest rate smoothing suggests that although inflation remains the main target, monetary policy in the euro area has been prudent due to the initial concerns of establishing a credible single monetary policy regime in the euro area. In addition, financial-stability concerns
might have played an important role in attaching a high weight on smoothing the interest rate.

In general, estimates of a small role for the output gap seem to find support in the literature. Lippi and Neri (2007) estimate a very low value for $q_y$ for the euro area, although they use a different output-gap concept\(^{17}\) and assume that monetary policy is conducted under discretion, which requires some caution in comparing the results. Dennis (2004a) likewise finds an ignorable weight for the output gap for the United States under discretionary monetary policy.\(^{18}\) Salemi (2006) and Givens and Salemi (2008), who consider commitment to a simple rule, estimate a close-to-zero and insignificant weight on the output gap in the context of a forward-looking model for the United States. Söderlind (1999) and Ilbas (forthcoming), considering the case of full commitment, however, estimate a relatively high value for $q_y$ in the framework of a standard loss function similar to (13). However, it is important to keep in mind that, as in Ilbas (forthcoming), our output-gap concept differs from the one used in the other studies, which makes comparison of the results a bit troublesome. While Svensson (1999, 2002a) argues for a case of gradual monetary policy where some weight should be given to stabilizing the output gap, findings in the literature mentioned above for $q_y$ generally do not support this concept of flexible inflation targeting. On the other hand, the computed posterior interval for $q_y$ in this paper suggests that monetary policy in the euro area has considered the output gap as an important target variable.

Dennis (2004a) and Lippi and Neri (2007) estimate a weight on the interest rate smoothing component that is higher than the weight on the inflation target.\(^{19}\) This is not the case in our study. Although our estimates of $q_r$ show that interest rate smoothing is a relatively important target, inflation remains the main policy goal. From an

\(^{17}\) Lippi and Neri (2007) describe the output gap as the deviation of output from a linear trend. On the contrary, we assume that the output gap is the deviation of output from the natural output level in the absence of nominal rigidities and the three i.i.d. cost-push shocks.

\(^{18}\) Söderström, Söderlind, and Vredin (2005) show in their calibration exercise under discretion analogously a low concern for output-gap stabilization based on U.S. data. See also Favero and Rovelli (2003).

\(^{19}\) Söderström, Söderlind, and Vredin (2005) show in their calibration exercise under discretion also a high importance for interest rate smoothing based on U.S. data.
economic point of view, this finding is plausible and in line with the statements that inflation should be the main target variable in monetary policy’s objective function. Castelnuovo (2006) finds, through a calibration exercise in the framework of discretionary monetary policy, a value for the interest rate smoothing weight close to ours whenever forward-looking agents are added to the model. When agents are assumed to be backward looking only, this weight increases considerably up to a point where interest rate smoothing becomes twice as important as the inflation target. This leads Castelnuovo (2006) to conclude that finding an economically difficult justifiable high value for \( q_r \) is probably due to model misspecification by the omission of factors like forward-looking behavior. Salemi (2006) confirms the empirical importance of forward-looking behavior by estimating a relatively low weight on interest rate stability in the loss function, suggesting inflation to be the main target, as in Givens and Salemi (2008) and Ilbas (forthcoming), while finding a high coefficient on the lagged interest rate in monetary policy’s reaction function. Since the model we consider includes forward-looking agents, our values for \( q_r \) are therefore in line with these results.\(^{20}\)

An intuitive explanation for this moderating effect of the presence of forward-looking agents on the estimated weight assigned to the interest rate smoothing component in the loss function can be given as follows.\(^{21}\) Whenever (rational) agents are forward looking, their expectations will play a key role in the stabilization process of monetary policy and the law of motion of the target variables. If the economy is hit by a shock in the current period, requiring a change in the policy instrument rate in order to stabilize the target variables, expectations will adjust accordingly, and since agents are rational, they will take into account the fact that interest rate smoothing is a

\(^{20}\)In order to assess this positive link between the degree of backward-lookingness and the estimates of the preference for interest rate smoothing in our model, we look at the correlation between the series on the inflation-indexation parameter \( \gamma_p \) and the interest rate smoothing preference parameter \( q_r \) obtained from the Markov chain Monte Carlo (MCMC) draws. Based on these draws, we detect a positive correlation of around 0.3, which is in line with Castelnuovo (2006).

\(^{21}\)Castelnuovo (2006) provides a detailed explanation on this issue and quantifies the role played by forward-looking agents in lowering the calibrated values of the weights on the interest rate smoothing component. See also Sack and Wieland (2000) as a key reference on this issue.
target variable. Therefore, a slow and persistent move in the interest rates is anticipated. Hence, expectations will have a stabilizing effect on current inflation and output gap, which in turn results in a slow and inertial behavior in the interest rates. If agents were assumed to be backward looking, like in the case of Ozlale (2003) and Dennis (2006), this inertial behavior in the interest rates could only be taken into account by the assumption that smoothing receives a high weight in the loss function of the central bank. On the other hand, if agents are forward looking, interest rate inertia is attributed to the stabilizing effect of expectations, which results in lower concern for the interest rate smoothing target. Also note that the commitment framework assumed in this paper enforces this history dependence more than would be the case under discretion like in most studies previously mentioned. This suggests that if we would perform a similar exercise under discretionary monetary policy, the estimated values for $q_r$ would probably be higher. Finally, our estimates of $q_r$ do not seem to support the argument of Svensson (2002a, 2003) that an interest rate stabilization or smoothing component should not enter the loss function, since the values obtained for the smoothing target are significantly higher than zero.

When nominal wage inflation is introduced in the loss function of monetary policy—i.e., the case of $M_2$—this additional target receives a weight of 0.1092. However, on the basis of comparison of marginal-likelihood values in table 3, the $M_1$ specification of the model where $q_w = 0$ is preferred over $M_2$. Hence, we can conclude that inflation, interest rate smoothing, and output-gap stabilization are the only targets of monetary policy.

Figures 5–7 in appendix 1 plot the prior and the posterior distributions of the parameters for model $M_1$. We apply the Metropolis-Hastings sampling algorithm based on 100,000 draws in order to derive the posterior distributions.  

\[22\] Given that the $M_2$ specification of the model includes an additional parameter to $M_1$, the worse performance of $M_2$ is mainly driven by the prior. We experimented with different prior assumptions for the monetary policy preference parameters but found that this observation does not change with the alternative prior assumptions.

\[23\] Convergence is assessed graphically by the Brooks and Gelman (1998) MCMC univariate diagnostics for each individual parameter and the MCMC multivariate diagnostics for all parameters simultaneously. These graphs are available upon request.
Table 3. Model Comparison Based on the Marginal Likelihood for the Period 1980:Q2–2005:Q4

<table>
<thead>
<tr>
<th></th>
<th>$M1 (q_w = 0)$</th>
<th>$M2 (q_w \neq 0)$</th>
<th>SW (2003)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laplace Approximation</td>
<td>–275.35</td>
<td>–276.26</td>
<td>–245.21</td>
</tr>
<tr>
<td>$\log \frac{p(Q_T</td>
<td>M_i)}{p(Q_T</td>
<td>SW(2003))}$ $i = 1, 2$</td>
<td>–30.14</td>
</tr>
<tr>
<td>Modified Harmonic Mean</td>
<td>–274.65</td>
<td>–275.18</td>
<td>–244.65</td>
</tr>
</tbody>
</table>

Notes: All estimations are performed with the same pre-sample period 1970:Q2–1980:Q1. The table shows the comparison for the period 1980:Q2–2005:Q4. The first row reports the Laplace approximation to approximate the marginal likelihood through the evaluation at the posterior mode. The table also reports the modified harmonic mean for each model (third row), obtained through the Markov chain Monte Carlo simulations, which does not differ much from the Laplace approximation. The second row in the table computes the performance of the models $M1$ and $M2$ with respect to the benchmark SW (2003) model. The fourth row computes the marginal likelihood by correcting for the effect of common priors, since same priors are used over the alternative model specifications, which can bias the model selection process.

Finally, given that the model-consistent output-gap framework adopted in this paper differs from the more commonly used output-gap definition, we plot the natural output level implied by the estimated model $M1$ against the historical path of the actual output level. Figure 1 illustrates a positive output-gap pattern until the late seventies, i.e., the period during which inflation was high. From the early eighties until the end of the nineties, the output gap is negative but turns slightly positive again around 2003.

4. Model Comparison

In this section we compare the models characterized by optimal monetary policy—i.e., the models $M1$ and $M2$—and the benchmark SW (2003) model, based on their marginal-likelihood values\textsuperscript{24} reported in table 3. In a next step, we compare Bayesian impulse responses for

\textsuperscript{24}See Geweke (1999) and Schorfheide (2006) for a detailed discussion on the marginal-likelihood function in Bayesian estimation.
Figure 1. Actual vs. Natural Output Levels in Model $M_1$

![Graph showing actual vs. natural output levels]

Notes: The dark line in the figure describes the historical path of the actual output level; the shaded line represents the estimate of the natural output level in model $M_1$. The latter is the output level in the absence of nominal rigidities (i.e., full price and wage flexibility) and the three i.i.d. cost-push shocks ($\eta^Q_t, \eta^p_t, \eta^w_t$).

selected shocks under optimizing monetary policy authorities ($M_1$) with those under SW (2003).

4.1 Marginal-Likelihood Comparison

The marginal likelihood of a model can be represented as follows:

$$p(Q_T \mid M_i) = \int_{\omega} p(Q_T \mid \omega, M_i)p(\omega \mid M_i)d\omega,$$

(19)

where $Q_T$ contains the observable data series, $\omega$ the vector of parameters, and $M_i$ the model under consideration—in our case of three models, $i = 1, 2$, or SW (2003). The likelihood function $p(Q_T \mid \omega, M_i)$ of the data series is conditional on the parameter vector $\omega$ and the model $M_i$. $p(\omega \mid M_i)$ is the prior density of the parameters conditional on the model. As in Schorfheide (2000), we use the Laplace approximation to approximate the marginal likelihood through the evaluation at the posterior mode. Table 3 also reports the modified harmonic mean for each model, obtained through the
Markov chain Monte Carlo simulations, which does not differ much from the Laplace approximation. As pointed out by Sims (2003) and Del Negro and Schorfheide (2008), using the same priors for alternative specifications of a model can bias our choice toward one type of specification. Given this potential pitfall in model comparison, we correct for the effect of common priors by estimating and evaluating the models over the training sample 1970:Q2–1980:Q1 as well, and subtract accordingly the corresponding marginal likelihood from the one obtained by estimation over the whole sample period 1970:Q2–2005:Q4. The conclusions based on this comparison, however, are in line with the comparisons reported in the first three rows in table 3.

Although from the table we can conclude that model $M_1$, where $q_{w} = 0$, fits the data better than model $M_2$, both models perform relatively worse than the benchmark SW (2003) specification of the model. This result indicates that monetary policy was not optimal (under commitment) during the sample period, which is not very surprising due to the restrictive assumption of optimizing monetary policy in the models $M_1$ and $M_2$. A similar comparison exercise is performed by Salemi (2006), who constructs a likelihood-ratio statistic to test the restrictions imposed by the optimal policy hypothesis under which the parameters of the policy rule are over-identified. Based on this formal test, Salemi (2006) shows that the estimates based on the structural estimation strategy with an optimal reaction function (i.e., the optimal monetary policy restriction) provide a worse fit to the data than in the case where the optimal monetary policy restriction is not imposed, which leads to the rejection of the hypothesis that monetary policy was optimal in the United States over the period 1965–2001. Moreover, Salemi (2006) and Givens and Salemi (2008) show that imposing the restrictions associated with the optimal monetary policy hypothesis leads to an alteration and sharpening of the estimates of some of the structural parameters, which is, as mentioned in the previous section, in line with our findings for the structural parameters.

Also note that in the SW (2003) description of monetary policy behavior, the Taylor rule includes five parameters, while there are only two monetary policy preference parameters to be estimated in $M_1$. Hence, it is not very surprising that the SW (2003) model with more free parameters to be estimated performs better than $M_1$. We therefore test whether this conclusion would change if the Taylor
rule (11) in the benchmark SW (2003) is replaced by two alternative specifications that contain fewer free parameters, and compare accordingly their corresponding marginal-likelihood values with $M_1$. In the first alternative case, the SW (2003) model is reestimated with a Taylor rule that responds to only lagged inflation and lagged output gap:  

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1 - \rho) \{ \hat{\pi}_t + r_\pi (\hat{\pi}_{t-1} - \bar{\pi}_t) + r_y (\hat{Y}_{t-1} - \bar{Y}_{t-1}) \} + \eta_t^R. \quad (20)$$

The corresponding value of the marginal likelihood is $-292.23$, which worsens the fit of the SW (2003) benchmark against $M_1$ characterized by optimal policy. In the second alternative case, the SW (2003) model is reestimated with a forward-looking version of the rule (20), where monetary policy reacts to a four-quarter change in the inflation rate and a four-quarter change in the output gap:

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1 - \rho) \{ \hat{\pi}_t + r_\pi (E_t \hat{\pi}_{t+4} - \bar{\pi}_t)$$

$$+ r_y E_t (\hat{Y}_{t+4} - \bar{Y}_{t+4}) \} + \eta_t^R. \quad (21)$$

This version of the Taylor rule increases the value of the marginal likelihood to $-244.69$, i.e., improving the fit of the SW (2003) benchmark against the $M_1$ specification of the model.  

Therefore, the performance of the model when monetary policy is assumed to be optimal against the benchmark where policy is simply described by a Taylor rule largely depends on how the latter is defined.

4.2 Bayesian Impulse-Response Analysis

In this section, we visualize the consequences of assuming optimizing monetary policy authorities on the impact and the dynamics of the variables in the case of a supply shock (productivity shock), a demand shock (equity premium shock), and a cost-push shock (price markup shock) over a period of twenty quarters in figures 2–4. We

\[ \text{Note that this rule is a special case of the original Taylor rule (11) where} \]

\[ r_{\Delta \pi} = r_{\Delta y} = 0. \]

\[ \text{Note that when we assume a four-quarter change in the inflation rate only, i.e.,} \]

\[ \hat{R}_t = \rho \hat{R}_{t-1} + (1 - \rho) \{ \hat{\pi}_t + r_\pi (E_t \hat{\pi}_{t+4} - \bar{\pi}_t) + r_y (\hat{Y}_t - \bar{Y}_t) \} + \eta_t^R, \text{ the marginal likelihood worsens to } -357.06. \]
Notes: The figure shows the impulse responses of selected variables to a productivity shock in the SW (2003) model (shaded lines) and in the model $M_1$ (dark lines) estimated for the period 1980:Q2–2005:Q4. The solid lines are the mean impulse responses, whereas the dotted lines are the 10 percent and the 90 percent posterior intervals. The horizon of twenty quarters is given on the horizontal axis; the vertical axes show the percentage deviations of the variables from the steady state.

take the SW (2003) model, which includes an estimated policy reaction function as the benchmark case (shaded lines), and assess to which extent the reactions of the variables differ when monetary policy minimizes an intertemporal loss function with one-period loss as specified under model $M_1$ (dark lines).$^{27}$

Figure 2 shows the responses of the variables to a productivity shock. The interest rate shows a slightly lower impact and becomes more negative (accommodative) around the third quarter under $M_1$

$^{27}$We focus only on the impulse responses obtained under $M_1$. However, the impulse responses under $M_2$ are, with the exception of responses to the equity premium shock, very similar to those under $M_1$. 
Figure 3. Equity Premium Shock

Notes: The figure shows the impulse responses of selected variables to an equity premium shock in the SW (2003) model (shaded lines) and in the model $M1$ (dark lines) estimated for the period 1980:Q2–2005:Q4. The solid lines are the mean impulse responses, whereas the dotted lines are the 10 percent and the 90 percent posterior intervals. The horizon of twenty quarters is given on the horizontal axis; the vertical axes show the percentage deviations of the variables from the steady state.

as opposed to the benchmark SW (2003) case. Hence, consumption and investment both increase to a greater extent, resulting in a higher increase in output and lower decrease in employment. The output gap becomes only slightly negative, in contrast to the SW (2003) benchmark case, since monetary policy accommodates the productivity shock more strongly. While the impact on wages is similar in both models, the initial decline in the rental rate of capital is slightly smaller under $M1$. Finally, the initial effect on inflation is slightly more negative.

Figure 3 shows the impulse responses to an equity premium shock. The interest rate responds more strongly to the equity premium shock and gets more positive (more activist policy) around
Figure 4. Price Markup Shock

Notes: The figure shows the impulse responses of selected variables to a price markup shock in the SW (2003) model (shaded lines) and in the model M1 (dark lines) estimated for the period 1980:Q2–2005:Q4. The solid lines are the mean impulse responses, whereas the dotted lines are the 10 percent and the 90 percent posterior intervals. The horizon of twenty quarters is given on the horizontal axis; the vertical axes show the percentage deviations of the variables from the steady state.

the third quarter, in contrast to the baseline model, which explains the stronger initial decline in consumption and the weaker response of investment and hence employment, output, and the output gap. The impact on both wages and rental rate of capital is much weaker and even turns slightly negative under M1. Therefore, inflation responds negatively; however, the effect is very small.

Comparing the impulse responses to a price markup shock in figure 4 with the benchmark case yields the following conclusions. The impact and dynamics of the interest rate are similar to the benchmark case. The initial decrease in investment is lower than the benchmark model, followed by weak dynamics that are close to zero. In contrast to what we observe for SW (2003), the impact of
the shock on consumption is slightly positive under $M_1$, followed by smaller dynamics also in this case. Hence, it is not surprising that output and employment are affected very weakly in the initial period. The former reaches its lowest value around the tenth quarter, which is still higher than in SW (2003). Interestingly, and in line with the findings of Salemi (2006), inflation shows very similar patterns in both models.

5. Conclusion

In this paper, we estimate the preferences of optimizing monetary policy authorities in a DSGE framework. Taking the Smets and Wouters (2003) model for the euro area as the benchmark case, we look at the effects of the assumption that monetary policy is conducted optimally under commitment. Accordingly, we estimate the preferences in the monetary policy’s objective function, which is assumed to take a standard quadratic form, using Bayesian estimation techniques. The optimizing monetary authorities are modeled in two alternative ways. The first model adopts a one-period loss function that includes inflation, the model-consistent output gap, and the one-period difference in the policy instrument. The second model includes an additional wage-inflation component. The estimation results for the structural parameters under these two model specifications are compared with the results of the benchmark Smets and Wouters (2003) model.

The estimate of the wage-inflation preference parameter in the monetary policy’s loss function in our second alternative model does not turn out to be significant. This model performs relatively worse by yielding a lower marginal likelihood than our first alternative model with three target variables defining the loss function. The results for the first model suggest a significant value assigned to both the output gap and interest rate smoothing, in addition to the main inflation objective, in monetary policy’s loss function.

We show that by using an initialization sample in the estimations, we are able to approach the framework of optimal monetary policy under the timeless perspective of Woodford (2003) very closely.

There are a number of points on which we would like to improve and extend this work in the future. The results presented in this paper are conditional on the assumption that monetary policy performs optimally under commitment. Therefore, these results may
change and lead to different conclusions if one considers the alternative case of discretion. A simplifying and perhaps naive assumption throughout our estimations is that of a constant loss function over the sample period, implying stable preferences over the target variables. Moreover, we have restricted our analysis to a case where the inflation target is assumed to be a known constant. In Ilbas (forthcoming), these assumptions are relaxed and the out-of-sample forecasting performance of the model under the optimal monetary policy assumption is examined for the U.S. economy.

Appendix 1. Prior and Posterior Distributions of Parameters in Model $M_1$

Figure 5. Posterior Distributions of the Parameters of $M_1$

Notes: Prior distributions are indicated by shaded lines, and estimated posterior distributions are indicated by dark lines. The posterior mode is indicated by the vertical lines. The Metropolis-Hastings sampling algorithm is based on 100,000 draws using the sample 1980:Q2–2005:Q4.
Figure 6. Posterior Distributions of the Parameters of M1

Notes: Prior distributions are indicated by shaded lines, and estimated posterior distributions are indicated by dark lines. The posterior mode is indicated by the vertical lines. The Metropolis-Hastings sampling algorithm is based on 100,000 draws using the sample 1980:Q2–2005:Q4.
Figure 7. Posterior Distributions of the Parameters of $M_1$

Notes: Prior distributions are indicated by shaded lines, and estimated posterior distributions are indicated by dark lines. The posterior mode is indicated by the vertical lines. The Metropolis-Hastings sampling algorithm is based on 100,000 draws using the sample 1980:Q2–2005:Q4.

Appendix 2. Use of a Pre-Sample and the Timeless-Perspective Approach

Due to our assumption of optimal monetary policy under commitment, we are dealing with a policy that is not time consistent. Woodford (2003) addresses this potential time-inconsistency problem by considering the concept of optimal policy from a timeless perspective (see also Svensson and Woodford 1999). In this paper, we perform the estimations by adopting a pre-sample period; i.e., the first forty...
quarters (1970:Q2–1980:Q1) of our sample are treated as the initialization period preceding the estimations. This implies that although the Lagrange multipliers are set equal to zero at the start of the presample (the initial period), at the start of the estimation the Lagrange multipliers will be derived from their past values. Hence, the use of a pre-sample has implications for the time consistency of monetary policy since it determines the way the lagged Lagrange multipliers are treated and the values they are assigned at the start of the estimation.

In order to test the effect of the initial values chosen for the Lagrange multipliers on the estimation results, we experiment with different initial values at the start of the initialization period (i.e., at time 1970:Q2) for the Lagrange multipliers of the forward-looking variables. The estimation results are almost unaffected by changes in initial values thanks to a relatively long initialization period. Therefore, we check for sensitivity of the estimation results to the length of the initialization period. For this purpose, we decrease the size of the pre-sample gradually below forty and reestimate the model $M_1$ twice, i.e., once by setting all initial values equal to zero and once by setting the Lagrange multipliers of forward-looking variables equal to non-zero, for a given pre-sample size. While decreasing the sample size, the estimation results start to show higher differences once the pre-sample reaches the size of around twenty quarters and below. Therefore, given our pre-sample choice of forty quarters, our estimation results appear to be robust to the initial values of the Lagrange multipliers of forward-looking variables and the pre-sample length. Hence, our results hold in the context of optimal policy approached from a timeless perspective.

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28 Unfortunately, we do not know the exact initial values of the Lagrange multipliers of the forward-looking variables because these are not observable. Therefore, we experiment with arbitrary values.

29 We perform this experiment for model $M_1$ only. The reasoning that the optimal time-inconsistent policy approaches closely the timeless-perspective policy after a sufficiently long period following the initial period of the optimization is similar to the one followed by Juillard and Pelgrin (2007).

30 A detailed report of this sensitivity exercise is available on request.

31 There is an alternative empirical application to the timeless-perspective solution through the pre-sample approach proposed in this paper. Juillard and Pelgrin (2007) show a two-step method to estimate policy from a timeless perspective in a Bayesian framework.
References


