Fixed- and Variable-Rate Tenders in the Management of Liquidity by the Eurosystem: Implications of the Recent Credit Crisis*

Margarida Catalão-Lopes
CEG-IST, Instituto Superior Técnico, Technical University of Lisbon

Most liquidity-providing operations of the European Central Bank (ECB) have been conducted through variable-rate tenders. However, fixed rates were first employed in the main refinancing operations (MROs) and are still used in other liquidity management operations. In October 2008, the ECB decided to carry MROs again at a fixed rate. In a simple three-stage game in which banks can obtain liquidity through the open-market operations of the ECB, through interbank transactions, or through "standing facilities," this paper revisits the dilemma between fixed- and variable-rate procedures, with an emphasis on the scenarios that are particularly relevant under the recent credit crisis, namely collateral shortage, rationing in the interbank market, and non-acute estimation by the ECB of the system's liquidity needs.

JEL Codes: D44, E52, G21.

1. Introduction

The primary objective of the monetary policy of the Eurosystem is to maintain price stability. The Governing Council's decisions on the level of the reference interest rates serve this objective, and so do the various instruments and procedures at disposal used to keep money market interest rates in line with the key policy rates. Open-market operations are a crucial instrument of the monetary policy of the European Central Bank (ECB). Among them, main

*Author contact: Instituto Superior Técnico–DEG, Universidade Técnica de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal. Tel: +351 218418177; Fax: +351 218417979; E-mail: mcatalao@ist.utl.pt.

refinancing operations (MROs) are the most important in terms of liquidity provision to banks but, during the financial market turmoil that began in the summer of 2007 and intensified in 2008, longer-term refinancing operations (LTROs) have also played an important role to improve the liquidity position of banks and provide broad access to funding. Indeed, supplementary LTROs were carried out with a high frequency, in addition to the Eurosystem's regular tender operations. Also, to alleviate pressures on the market for U.S. dollars (USD), several liquidity-providing operations in USD have been launched, at a fixed rate (equal to the marginal rate of the simultaneous Federal Reserve System tender). In October 13, 2008, the ECB announced that tenders of USD funding would be conducted regularly at least until January 2009, every Wednesday, at a fixed rate with full allotment. The use of fine-tuning operations also intensified during the credit crisis, with an enlargement in the range of institutions that are eligible to participate (October 3, 2008). Most of these fine-tuning operations took place to absorb excess liquidity, at a fixed rate (equal to the minimum bid rate of MROs).

Since the beginning of Stage Three of the Economic and Monetary Union (EMU) in January 1999, and until June 2000, the Eurosystem conducted its MROs through fixed-rate tenders (without full allotment). This procedure, aimed at clearly signaling the stance of monetary policy, involved some distortions in the bidding behavior of banks (overbidding) and led to very small allotment ratios. The change to a variable-rate procedure was introduced in the MRO of June 28, 2000, and was in practice until the operation settled on October 8, 2008. According to the European Central Bank, "The switch to variable rate tenders was a response to the severe overbidding which had developed in the context of the fixed rate tender procedure. . . . In the last two main refinancing operations executed prior to the switch to the variable rate tender, the allotment ratio was below 1%" (European Central Bank 2000, p. 37). The return to the fixed-rate format, however, had never been excluded by the ECB, which retained the option of reverting to it, if and when this is deemed appropriate. The two changes introduced in March 2004 in the operational framework of the Eurosystem—namely, the reduction in the maturity of the MROs and the commitment that rate changes would not occur during the maintenance period—were recognized by the ECB as improving the efficiency of the fixed-rate procedure, because they would prevent the occurrence of excessive overbidding associated with expectations of rising interest rates (European Central Bank 2003). On October 8, 2008, the Governing Council decided to perform the next MROs at a fixed rate, starting with the one settled on October 15 and at least until the end of the first maintenance period of 2009. These operations would take place with full allotment at the interest rate that resulted from the simultaneous ease in monetary conditions (3.75 percent).

LTROs, in turn, have usually been executed in the form of variable-rate tenders, but the possibility of using fixed-rate procedures, "under exceptional circumstances," is not excluded (European Central Bank 2008a, p. 15).

Bearing in mind all these aspects, the present paper revisits the debate between fixed- and variable-rate procedures, with a special focus on the scenarios most relevant under the recent credit crisis, namely (i) rationing in the interbank market—due to high uncertainty as to the solvency of borrowers and as to the evolution of own liquidity requirements; (ii) binding collateral (to address this problem, the ECB expanded the collateral acceptable in lending operations, in its biennial review of September 4, 2008); and (iii) non-acute estimation by the ECB of the true liquidity needs of the system, which are difficult to predict under the prevailing tensions.

Banks can obtain/place liquidity in three markets: (i) the primary market, through the open-market operations of the ECB;[1] (ii) the secondary market or interbank market, where liquidity is traded among banks; and (iii) the "standing facilities"—lending or deposit—of the ECB, at penalty rates. All operations with the ECB must be collateralized, either in the primary or in the tertiary market. In contrast, the exchange of liquidity in the secondary market is assumed to be uncollateralized, which reinforces the risk involved.

During the periods of fixed-rate MROs, the ECB sets the interest rate at which the operations are conducted (the main refinancing rate) and also the marginal lending and deposit interest rates. Only the interbank market rate (a short-term rate) is determined by demand and supply forces. In a variable-rate tender procedure, the prevailing primary market rate fluctuates according to market

[1] For a detailed description of the auction environment, see, for instance, Scalia, Ordine, and Bruno (2005).

conditions in a given interval (the minimum bid rate is established by the Governing Council in its monetary policy meetings), and the ECB uses the amount of liquidity allotted to steer its behavior.[2] The marginal lending and deposit rates are used as policy instruments too, delimiting the "corridor" for interbank rates. In the management of the recent crisis, this "corridor" was shortened from 200 to 100 basis points.

Taking into account the variety of instruments currently in use, this paper compares, in a simple three-stage game, the two tender formats with respect to induced bidding behavior and allocation of liquidity in the primary market, functioning of the secondary market, and resorting to "standing facilities" (tertiary market). The conclusions as to these aspects reinforce previous results in the literature. Specifically, the overbidding phenomenon is indeed shown to be an equilibrium outcome of fixed-rate tenders (without full allotment), a problem which may be avoided by variable-rate tenders. Furthermore, variable-rate tenders allow keeping some informational content of quantity bids, as opposed to fixed-rate tenders. Announcing that there will be full allotment in fixed-rate tenders (October 8, 2008) helps prevent these problems, but in reality there is no auctioning taking place, because the Central Bank has a perfectly elastic supply and fully satisfies the liquidity needs at the prevailing rate.[3] Although simple, the present model further allows us to address the possible implications of some facts deriving from the recent credit crisis, such as collateral shortage, credit rationing in the interbank market, and wrong estimation by the ECB of the true system's liquidity needs. In the 2008 ECB annual report on EU banking structures, and reflecting the prevalent high degree of uncertainty, banks in the European Union have, for the first time, identified financial markets as the major source of risk for the coming year (see European Central Bank 2008b).

[2]For example, to contain pressures on euro overnight rates, the ECB has accommodated counterparties' wishes to fulfill reserve requirements early in the maintenance period, therefore alloting excess liquidity in the MROs, which is later absorbed at the end of the period.

[3]Prior to this announcement, some MROs of the Eurosystem had already been conducted with full allotment, allocating liquidity in excess of the benchmark amount.

To my knowledge, among other studies addressing the open-market operations of the ECB, the closest to this are Ayuso and Repullo (2003), Bindseil (2002), and Nautz and Oechssler (2003), in the sense that they also compare fixed- and variable-rate procedures. However, apart from Bindseil (2002), who takes care of situations in which the central bank tends to provide surplus liquidity or is systematically tight, none of them addresses the other relevant scenarios for the recent credit crisis: binding collateral and interbank rationing. Ayuso and Repullo (2003) assume that the ECB wants to minimize deviations of the interbank rate from the target rate that signals the stance of monetary policy. They show that when the penalty is higher for rates below the target, pre-announcing the amount of the liquidity injection eliminates overbidding in a variable-rate tender, but the equilibrium in a fixed-rate tender is still characterized by extreme overbidding. Bindseil (2002) compares the two types of tenders, alternatively in the absence and in the presence of interest rate change expectations. He concludes that the fixed-rate tender works well when interest rates are stable (a result that would support the change in the operational framework of the ECB; see European Central Bank 2003), but that the variable-rate tender is better suited in an environment of strong interest rate change expectations. Nautz and Oechssler (2003) conclude for bidding behavior indeterminacy in fixed-rate tenders, and, using a dynamic version of their model, they also conclude that overbidding is an increasing phenomenon in these auctions. The authors also develop an empirical analysis, confirming the weakness of fixed-rate tenders and showing that the move to variable-rate tenders significantly improved liquidity management by the ECB.

There have actually been many empirical studies on the performance of the monetary policy of the ECB. Among them, Ayuso and Repullo (2001) test whether the overbidding problem during the fixed-rate tenders period was due to expectations of a future tightening of monetary policy or to the liquidity allotment decisions of the ECB that resulted in a positive spread between the interbank rate and the interest rate of the MROs. The authors conclude in favor of the latter hypothesis. Scalia, Ordine, and Bruno (2005) address the degree of integration and the bidding efficiency in the primary market during a sample period for variable-rate tenders, using panel information with country effects, bank size effects, and bank group effects. They conclude that bidders in the Eurosystem auctions behave efficiently and that, although the effects mentioned may be significant to some extent, market integration is promoted by liquidity supply through MROs and by the bidding behavior of banks. Nautz and Oechssler (2006) investigate the subsistence of overbidding in fixed-rate tenders if one controls for interest rate expectations (which the ECB proposes in its recent modifications to the operational framework), for the magnitude of the spread between the main refinancing rate and the interbank rate, and for the possibility of banks being squeezed. Their results suggest that none of these three hypotheses is sufficient to eliminate the overbidding problem.

The remainder of the paper is organized as follows. Section 2 presents the notation common to both models (fixed and variable rate). In section 3 the equilibrium bidding behaviors in the fixed-rate auction are derived under several scenarios, namely lack of collateral, credit rationing in the secondary market, and wrong estimation by the ECB of banks' liquidity needs. The corresponding analysis for the variable-rate auction is performed in section 4.[4] Section 5 compares and summarizes results for both types of tender and section 6 concludes.

2. Notation

Consider two representative banks, one which expects not to be a liquidity supplier in the secondary market (i), and one which has the opposite expectation (j). In equilibrium these expectations are confirmed. The equilibrium behaviors of banks i and j replicate the bids of banks that expect to have these positions in the interbank market.

Non-zero banks' liquidity needs are denoted by $\bar{l}_i$ and $\bar{l}_j$. Liquidity obtained in the primary market is l_i and l_j, so bank i still needs $\bar{l}_i - l_i \geq 0$, while bank j has $l_j - \bar{l}_j \geq 0$ in excess.

The interest paid by the central bank's deposit facility is d, and the interest charged on the marginal lending facility is denoted by m. In a fixed-rate tender there is another administered interest rate,

[4] Full details are available in Catalão-Lopes (2001).

denoted by r, with $r \in]d, m[$, and which is the rate paid for the liquidity obtained in the MRO. In a variable-rate tender there may exist a minimum interest rate for bids, also chosen by the ECB. The amount of liquidity that bank i (j) proposes to buy in the MRO is b_i (b_j). In a variable-rate procedure the bank also makes a bid, denoted by r_i (r_j), for the price to pay for this liquidity.

The unsecured interbank rate is denoted by o (overnight rate). Bank j is not willing to sell liquidity at a price below d, so o is bounded from below by d; however, bank i may be willing to buy at an interest rate higher than m, in case collateral is insufficient to satisfy the remaining liquidity needs through the lending facility (remember that, contrary to operations with the ECB, transactions in the interbank market are uncollateralized).

Banks wish to minimize the expenditure they must incur in order to comply with reserve requirements. Both games—fixed- and variable-rate auctions—have three stages: MRO, interbank market, and "standing facilities."[5] The problems are solved backwards.

The optimum bidding behavior in the first market arises from the first-order conditions of the expenditure minimization problem (assuring second-order conditions for a minimum are verified). In a fixed-rate tender there is only one decision variable, the quantity bid, whereas in a variable-rate tender there is an additional one, the interest rate that the bank proposes to pay for the quantity bid it makes. Banks and the monetary authority are assumed to be risk neutral.

3. Fixed-Rate Tenders

In a fixed-rate tender without full allotment, each bank is allocated an amount proportional to its bid (under the implicit assumption that total bids exceed the allotted amount). The allotment ratio is defined as $\frac{\bar{l}_i+\bar{l}_j+v}{b_i+b_j}$, where $\bar{l}_i + \bar{l}_j + v$ is the total amount of liquidity that the ECB decides to allocate. A parameter $v > 0$ ($v < 0$) means an overestimation (underestimation) by the ECB of the system's liquidity needs (see subsection 3.4).

[5]Instead of MROs, one could also consider, for instance, LTROs.

Denote by α_i the proportion of bank i's bid in terms of total bids: $\alpha_i = \frac{b_i}{b_i+b_j}$. In the primary market bank i is allocated with

$$l_i = \frac{\bar{l}_i + \bar{l}_j + \upsilon}{b_i + b_j} b_i = \alpha_i(\bar{l}_i + \bar{l}_j + \upsilon).$$

If this is insufficient to cover the liquidity needs, it still has to acquire

$$\bar{l}_i - l_i = \alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i \upsilon.$$

Bank j, in turn, buys

$$l_j = \frac{\bar{l}_i + \bar{l}_j + \upsilon}{b_i + b_j} b_j = \alpha_j(\bar{l}_i + \bar{l}_j + \upsilon)$$

in the primary market, and, if this amount exceeds its liquidity needs, it has an excess equal to

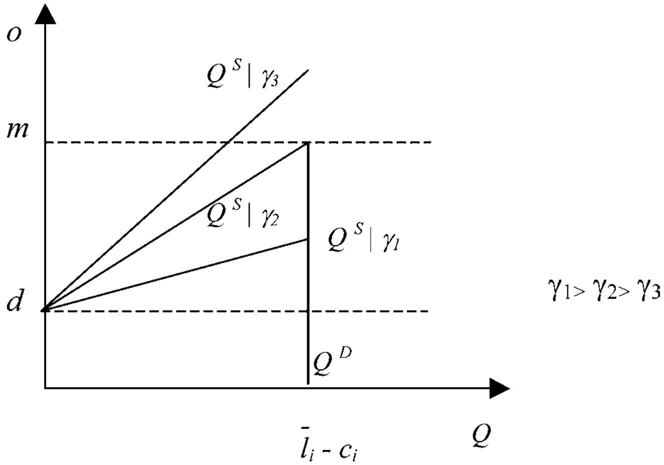
$$l_j - \bar{l}_j = \alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j \upsilon.$$

If collateral is not restrictive, bank i is not willing to buy liquidity in the secondary market at a price above m, the marginal lending rate. For prices below m it wants to fulfill all its needs, so the demand curve is given by

$$Q^D = \begin{cases} \alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i \upsilon \Leftarrow o \leq m \\ 0 \Leftarrow o > m. \end{cases}$$

In turn, banks of the j-type are not willing to sell their excess liquidity at a price below the deposit rate with the ECB. Their supply curve is assumed to be linear. I consider that, for precautionary motives related to uncertainty concerning the future evolution of primary rates, these banks may choose to retain some liquidity. This is modeled by an exogenous parameter $\gamma > 0$. In other words, γ stands for the "willingness" to lend. The inclusion of this parameter allows us to capture the possibility that i-banks are not able to acquire all the liquidity needed in the secondary market and have to make use of the lending facility. As figure 1 shows, this happens

Figure 1. Demand and Supply in the Interbank Market



for low-enough γ.[6] It should be stressed, however, that the main conclusions of the model as to the relative performance of fixed- and variable-rate tenders do not depend on the value of γ. The supply curve can be written as

$$Q^S = \begin{cases} 0 \Leftarrow o < d \\ \gamma(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j v) \frac{o-d}{m-d} \Leftarrow o \geq d. \end{cases}$$

Equating demand and supply, the equilibrium overnight rate arises:

$$o^* = \begin{cases} d + \frac{m-d}{\gamma} \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j} \Leftarrow \gamma > \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j} \\ m \Leftarrow \gamma \leq \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j}. \end{cases}$$

[6]This happens, for example, in periods of high financial markets turbulence. In the limit, when γ tends to zero, the money market disappears and all liquidity must be obtained from the Central Bank. To set money markets to work again during the recent credit crisis, authorities announced that all interbank operations would be guaranteed by each State, up to a certain amount.

The quantity traded in the secondary market is

$$Q^* = \begin{cases} \alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i \upsilon \Leftarrow \gamma > \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j} \\ \gamma(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j \upsilon) \Leftarrow \gamma \leq \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j}. \end{cases}$$

In the tertiary market there is no interaction between banks. Liquidity in short is obtained at m, and liquidity in excess gives rise to a deposit remunerated at d. Liquidity in short (for bank i) is

$$\bar{l}_i - l_i - Q^* = \begin{cases} 0 \Leftarrow \gamma > \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j} \\ (1-\gamma)(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j) - (\alpha_i + \gamma \alpha_j)\upsilon \Leftarrow \gamma \leq \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j} \end{cases}$$

and liquidity in excess (for bank j) is

$$l_j - \bar{l}_j - Q^* = \begin{cases} (\alpha_i + \alpha_j)\upsilon \Leftarrow \gamma > \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j} \\ (1-\gamma)(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j \upsilon) \Leftarrow \gamma \leq \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j}. \end{cases}$$

Recall that $\alpha_i = \frac{b_i}{b_i + b_j}$. Then the problem of bank i in the primary market can be stated as

$$min_{b_i} \quad r\alpha_i(\bar{l}_i + \bar{l}_j + \upsilon) + \left(d + \frac{m-d}{\gamma} \frac{\alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i \upsilon}{\alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j \upsilon} \right) \times (\alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i \upsilon) \Leftarrow \gamma > \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j}$$

$$min_{b_i} \quad r\alpha_i(\bar{l}_i + \bar{l}_j + \upsilon) + m\gamma(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j \upsilon) \Leftarrow \gamma \leq \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j}.$$

The problem of bank j is

$$min_{b_j} \quad r\alpha_j(\bar{l}_i + \bar{l}_j + \upsilon) - \left(d + \frac{m-d}{\gamma} \frac{\alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i \upsilon}{\alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j \upsilon} \right) \times (\alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i \upsilon) - d(\alpha_i + \alpha_j)\upsilon \Leftarrow \gamma > \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j}$$

$$min_{bj} \quad r\alpha_j(\bar{l}_i + \bar{l}_j + \upsilon) - (\gamma m + (1-\gamma)d) \times (\alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j \upsilon) \Leftarrow \gamma \leq \frac{\bar{l}_i - l_i}{l_j - \bar{l}_j}.$$

To begin with, let us consider the simple case with $\upsilon = 0$ (the estimation of liquidity needs by the ECB is acute). This scenario is further elaborated by considering the possibilities of collateral shortage and credit rationing. I then present the changes in results when the ECB fails to correctly estimate the system's needs ($\upsilon \neq 0$).[7]

3.1 Absence of Estimation Errors

When the ECB correctly estimates the system's liquidity needs ($\upsilon = 0$), liquidity in excess matches liquidity in short for the interbank market. The equilibrium overnight rate is then

$$o^* = \begin{cases} d + \frac{m-d}{\gamma} \Leftarrow \gamma > 1 \\ m \Leftarrow \gamma \leq 1. \end{cases}$$

If $\gamma > 1$ (supply is sufficiently elastic so that all remaining liquidity needs of bank i may be satisfied in the secondary market), the problem of bank i resumes to

$$min_{b_i} \quad r\alpha_i(\bar{l}_i + \bar{l}_j) + \left(d + \frac{m-d}{\gamma}\right)(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j)$$

and the problem of bank j to

$$min_{b_j} \quad r\alpha_j(\bar{l}_i + \bar{l}_j) - \left(d + \frac{m-d}{\gamma}\right)(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j).$$

To be consistent with the assumption that i will not be a liquidity supplier and that j will not be a liquidity demander in the interbank market, their bids have to satisfy the following condition (arising from $Q^D \geq 0$ and $Q^S \geq 0$, with $\upsilon = 0$): $b_j \bar{l}_i - b_i \bar{l}_j \geq 0 \Leftrightarrow$

[7] Note that bidders have rational expectations, in the sense that they fully anticipate the Central Bank's allotment strategy.

$$b_i \leq \frac{\bar{l}_i}{\bar{l}_j} b_j$$
$$b_j \geq \frac{\bar{l}_j}{\bar{l}_i} b_i.$$

So, $b_i \in (0, \frac{\bar{l}_i}{\bar{l}_j} b_j]$, and $b_j \in [\frac{\bar{l}_j}{\bar{l}_i} b_i, +\infty)$. From this, the following is clear:

LEMMA 1. *In a fixed-rate tender without full allotment, bids can grow infinitely high, no matter what the expected position of the bank in the secondary market, which is at the origin of potentially low allotment ratios.*

This result confirms previous findings in the literature and is consistent with evidence from the first months of Stage Three of the EMU. It is not specific to the scenario under analysis ($\upsilon = 0$) and is a direct consequence of the fact that bids are not limited by available collateral—only the liquidity received is. As noted in the Introduction, a fixed-rate procedure with announced full allotment avoids this problem as well as the non-informativeness of bids as regards true funding requirements: banks just have to bid equal to their needs and the allotment ratio is simply equal to one.[8]

Optimizing the objective functions of i and j yields

$$(1 - \alpha_i)(\bar{l}_i + \bar{l}_j)(r - o)$$
$$(1 - \alpha_j)(\bar{l}_i + \bar{l}_j)(r - o).$$

If $o > r$ (which corresponds to $1 < \gamma < \frac{m-d}{r-d}$), the higher b_i, the better for bank i, and the higher b_j, the better for bank j, so bids will tend to reach very high levels. However, if γ is sufficiently high so that the equilibrium overnight rate falls below the MRO interest rate, both banks have an incentive to bid very low quantities in the MRO.

[8] Of course, in periods of high financial markets uncertainty, banks may be uncertain about their true liquidity needs and bid higher for precautionary motives.

The equilibrium locus is

$$b_i = \frac{\bar{l}_i}{\bar{l}_j} b_j.$$

The two reaction curves are upward sloping and fully coincide. There are multiple equilibria. The allotment ratio can thus turn out to be very low, if the equilibrium bids locate at high levels, which corresponds to the outcome of most MROs conducted through fixed-rate auctions at the beginning of Stage Three. This multiplicity of equilibria reduces (or even extinguishes) the informational content of bids as regards liquidity needs.

Bids of banks i and j are strategic complements: the higher the expectation of the bid of the rival, the higher the optimal bid of each bank. As expected, given the bid of the opponent, a bank's optimal bid is increasing in its own liquidity needs.

If $\gamma < 1$ ($\bar{l}_i - l_i$ is not entirely solved in the secondary market) or $\gamma = 1$, the problem of bank i can be written as

$$min_{b_i} \quad r\alpha_i(\bar{l}_i + \bar{l}_j) + m(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j)$$

and the problem of bank j as

$$min_{b_j} \quad r\alpha_j(\bar{l}_i + \bar{l}_j) - (\gamma m + (1-\gamma)d)(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j).$$

Differentiating with respect to own bid,

$$(1-\alpha_i)(\bar{l}_i + \bar{l}_j)(r - m) < 0$$
$$(1-\alpha_j)(\bar{l}_i + \bar{l}_j)(r - d - \gamma(m - d)).$$

For bank i, the higher b_i, the better, which is quite intuitive from the fact that the overnight rate will reach m. If $\frac{r-d}{m-d} < \gamma < 1$, bank j also wants to bid high in the first market, but for low-enough γ this result may be reversed, as j will be left with much liquidity to deposit with the ECB. The equilibrium locus is again given by $b_i = \frac{\bar{l}_i}{\bar{l}_j} b_j$, and the allotment ratio may reach very low levels.

Note that in periods of high turbulence γ will tend to zero, corresponding to the vanishing of liquidity supply in the money market. In these circumstances i-banks will bid very high in the primary market, but not j-banks.

3.2 Collateral Shortage

Suppose now that bank i is limited by its amount c_i of collateral, such that $c_i < \bar{l}_i$. Bank i will try to acquire an amount equal to c_i (the maximum allowed) in the primary market.[9] In the secondary market the supply curve does not change and the demand curve is perfectly inelastic, with

$$Q^D = \bar{l}_i - c_i.$$

Because of collateral shortage, bank i has to fully satisfy Q^D. The equilibrium overnight rate may thus be higher than m and will be given by

$$o^* = d + \frac{(m-d)}{\gamma} \frac{\bar{l}_i - c_i}{\alpha_j \bar{l}_i - \alpha_i \bar{l}_j}.$$

A lower c_i increases the likelihood of o^* rising above m.

In the tertiary market, bank j deposits

$$\alpha_j(\bar{l}_i + \bar{l}_j) - \bar{l}_j - Q^S = c_i - \alpha_i(\bar{l}_i + \bar{l}_j),$$

which varies positively with c_i: shortage of collateral (lower c_i) intensifies trading in the interbank market, and so less liquidity remains to be placed at d.

In order for i to guarantee itself an amount of liquidity equal to c_i in the primary market, its minimum bid is $b_i^{min} = \frac{c_i}{\bar{l}_i + \bar{l}_j - c_i} b_j$, arising from the condition $\alpha_i(\bar{l}_i + \bar{l}_j) \geq c_i$. This is actually the optimal bid, since the objective function appears to be increasing in b_i. It does not benefit bank i to make a bid above this one, since that would not assure any additional liquidity in the primary market and would increase the prevailing interest rate in the secondary market (because bank j would be allocated with less liquidity in the MRO).

So, bidding equilibria are now given by the relationship $b_i = \frac{c_i}{\bar{l}_i + \bar{l}_j - c_i} b_j$. In equilibrium the shortage of funds for i, equal to $\bar{l}_i - c_i$, is solved in the secondary market, at an interest rate equal to $d + \frac{m-d}{\gamma}$.

[9] There is no point in saving collateral for the third market, because the price of liquidity is higher than in the MRO.

LEMMA 2. *Lack of enough collateral to cover the allotted amount in the primary market raises price and quantity traded in the interbank market, whereas the use of the deposit facility decreases.*

To fulfill its liquidity needs, bank i spends $rc_i+(d+\frac{m-d}{\gamma})(\bar{l}_i-c_i)$, higher than in the case in which collateral is not restrictive if γ is low enough ($\gamma < \frac{m-d}{r-d}$), the most likely scenario under a credit crisis. For this range of values of γ, bank j is able to take advantage of i's shortage of collateral, and its expenses decline. The ECB's revenues, in turn, remain unchanged as compared with the case of unbinding collateral.[10] The total amount spent by i and j together is not altered (so there is no loss of efficiency), but the distribution may change in favor of the banks that are not restricted by collateral.

3.3 Credit Rationing in the Interbank Market

Let us admit that, because of information problems related with the repayment capacity of the borrower, banks of type j are not willing to lend at an overnight rate higher than t, with $d < t < m$ (recall that transactions in the interbank market are uncollateralized). Banks of type i are thus subject to credit rationing in the secondary market. Rationing may also occur because of bank j's uncertainty about future own liquidity needs. I elaborate on the case with collateral restrictions, so in the equilibrium i-banks will use all the collateral in the primary market. This is the most relevant scenario under financial market turmoil.

In this case equilibrium bids are again given by $b_i = \frac{c_i}{\bar{l}_i+\bar{l}_j-c_i}b_j$. Quantity traded in the interbank market, at the interest rate t, is equal to $\frac{\gamma(t-d)(\bar{l}_i-c_i)}{m-d}$. As expected, it varies positively with t: the more rationing there is (the lower t), the lower Q. Credit rationing is active if and only if $t < d + \frac{m-d}{\gamma}$, the equilibrium overnight rate obtained before.

Bank i is unable to satisfy reserve requirements by an amount equal to what bank j deposits with the ECB. This amount varies positively with the level of rationing and negatively with γ, as expected.

[10]The reference to the ECB's revenues is just a remark. The Central Bank has mainly policy concerns.

LEMMA 3. *Due to the existence of rationing in the interbank market, banks that are restricted by collateral may be unable to satisfy their liquidity needs. In turn, the other banks have excess liquidity.*

This excess liquidity may be applied through the deposit facility or it may also be absorbed through the ECB's fine-tuning operations, which occurred frequently since the beginning of the recent financial market turmoil.

The credit-rationing situation could also be addressed, with identical conclusions, by looking at the parameter γ. In case credit institutions effectively suspend interbank transactions (γ tends to zero), an extreme rationing situation that characterizes markets during periods of high turbulence, an expansion in the set of eligible assets for collateral purposes may be necessary for banks of type i to be able to meet their liquidity needs, through the Central Bank's providing operations (recall the biennial review of September 4, 2008). Notice that as γ approaches zero, the expenses of a bank with binding collateral grow significantly (see last subsection).

3.4 Estimation Errors

In this subsection the hypothesis that $\upsilon = 0$ is abandoned. The ECB may underestimate ($\upsilon < 0$) or overestimate ($\upsilon > 0$) the system's liquidity needs, which is quite plausible in a period of financial turmoil and sharp liquidity tensions. This case (with no collateral restrictions) corresponds to the general problem formulated in the beginning of the section. The wrong estimation derives from an incorrect forecast of the "autonomous factors" of liquidity injection/absorption and affects banks of both types (i and j).[11]

Suppose that $\upsilon < 0$. Then it can be seen that bank i is allocated a quantity equal to $\bar{l}_i + \upsilon$ (below its needs), while bank j receives $\bar{l}_j$. So, i is the one that bears all the burden of the ECB's estimation

[11] The autonomous liquidity factors of the euro area are the Eurosystem's balance-sheet items whose amount is independent of the monetary policy operations of the Central Bank and comprise, namely, banknotes in circulation, government deposits, and net foreign assets. Note that under a fixed-rate tender with full allotment format, there is no need to estimate the system's needs, because the banks' bids truly reveal them, so $\upsilon = 0$.

error. As a consequence, it will have to acquire $|v|$ through the lending facility; total expenses are higher than without estimation errors (if i is restricted by collateral, it may be unable to comply with reserve requirements). Bank j's expenses, in turn, are unaffected. The ECB benefits from the underestimation of liquidity needs, since its revenue rises by $(m - r)|v|$.

These results are reverted when liquidity needs are overestimated ($v > 0$). Then equilibrium bids satisfy $b_i = \frac{\bar{l}_i}{\bar{l}_j + v} b_j$, bank i is allocated with its full needs, and bank j receives v in excess, which it will deposit with the ECB. Expenses of bank i are left unchanged as compared with the case with $v = 0$, but bank j's expenses rise. ECB's revenues are again larger than without estimation errors. The ECB always benefits from $v \neq 0$, because the estimation error will have to be corrected through the "standing facilities."

LEMMA 4. *When the ECB underestimates (overestimates) the system's liquidity needs, net demanders (suppliers) in the interbank market bear the burden of this estimation error.*

4. Variable-Rate Tenders

In variable-rate tenders, banks have two decision variables: the amount they propose to buy (as in a fixed-rate auction) and the interest rate they propose to pay.

The bank offering the highest interest rate wins the auction and receives a liquidity amount equal to its bid. The loser is left with the remaining liquidity (or, more precisely, with the minimum between the remaining liquidity and available collateral). I assume that each bank pays its own interest rate bid—multiple-rate auction, the procedure adopted by the Eurosystem.

Consider that the ECB wishes to allocate $\bar{l}_i + \bar{l}_j + v$, as before, with $v < 0$ corresponding to an underestimation of the system's needs. Denote by r_i (r_j) the interest rate bid of bank i (j). Then, liquidity allocated to each counterpart is the following:[12]

[12]When $r_i = r_j$ (both banks bid the same price), each one receives a proportion of the liquidity injection corresponding to the quantity bid made, as in a fixed-rate tender without full allotment.

$$l_i = \begin{cases} b_i \Leftarrow r_i > r_j \\ \bar{l}_i + \bar{l}_j + \upsilon - b_j \Leftarrow r_i < r_j \end{cases}$$

$$l_j = \begin{cases} \bar{l}_i + \bar{l}_j + \upsilon - b_i \Leftarrow r_i > r_j \\ b_j \Leftarrow r_i < r_j. \end{cases}$$

Liquidity in short for bank i is

$$\bar{l}_i - l_i = \begin{cases} \bar{l}_i - b_i \Leftarrow r_i > r_j \\ b_j - \bar{l}_j - \upsilon \Leftarrow r_i < r_j \end{cases}$$

and in excess for bank j is

$$l_j - \bar{l}_j = \begin{cases} \bar{l}_i + \upsilon - b_i \Leftarrow r_i > r_j \\ b_j - \bar{l}_j \Leftarrow r_i < r_j. \end{cases}$$

When bank i wins the auction $(r_i > r_j)$, demand and supply in the interbank market (with no collateral restrictions for i) are

$$Q^D = \begin{cases} \bar{l}_i - b_i \Leftarrow o \leq m \\ 0 \Leftarrow o > m \end{cases}$$

$$Q^S = \begin{cases} 0 \Leftarrow o < d \\ \gamma(\bar{l}_i - b_i + \upsilon)\frac{o-d}{m-d} \Leftarrow o \geq d, \end{cases}$$

so the equilibrium overnight rate is

$$o^* = \begin{cases} d + \frac{m-d}{\gamma}\frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \Leftarrow \gamma > \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \\ m \Leftarrow \gamma \leq \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \end{cases}$$

and quantity traded is

$$Q^* = \begin{cases} \bar{l}_i - b_i \Leftarrow \gamma > \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \\ \gamma(\bar{l}_i - b_i + \upsilon) \Leftarrow \gamma \leq \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon}. \end{cases}$$

When the winner is bank j $(r_i < r_j)$, the following changes occur:

$$Q^D = \begin{cases} b_j - \bar{l}_j - \upsilon \Leftarrow o \leq m \\ 0 \Leftarrow o > m \end{cases}$$

$$Q^S = \begin{cases} 0 \Leftarrow o < d \\ \gamma(b_j - \bar{l}_j)\frac{o-d}{m-d} \Leftarrow o \geq d, \end{cases}$$

so the equilibrium overnight rate is

$$o^* = \begin{cases} d + \frac{m-d}{\gamma}\frac{b_j-\bar{l}_j-\upsilon}{b_j-\bar{l}_j} \Leftarrow \gamma > \frac{b_j-\bar{l}_j-\upsilon}{b_j-\bar{l}_j} \\ m \Leftarrow \gamma \leq \frac{b_j-\bar{l}_j-\upsilon}{b_j-\bar{l}_j} \end{cases}$$

and quantity traded is

$$Q^* = \begin{cases} b_j - \bar{l}_j - \upsilon \Leftarrow \gamma > \frac{b_j-\bar{l}_j-\upsilon}{b_j-\bar{l}_j} \\ \gamma(b_j - \bar{l}_j) \Leftarrow \gamma \leq \frac{b_j-\bar{l}_j-\upsilon}{b_j-\bar{l}_j}. \end{cases}$$

Therefore bank i has to resort to the lending facility for

$$\bar{l}_i - b_i - Q^* = \begin{cases} 0 \Leftarrow \gamma > \frac{\bar{l}_i-b_i}{\bar{l}_i-b_i+\upsilon} \\ (1-\gamma)(\bar{l}_i - b_i) - \gamma\upsilon \Leftarrow \gamma \leq \frac{\bar{l}_i-b_i}{\bar{l}_i-b_i+\upsilon} \end{cases}$$

when $r_i > r_j$, and for

$$\bar{l}_i - b_i - Q^* = \begin{cases} 0 \Leftarrow \gamma > \frac{b_j-\bar{l}_j-\upsilon}{b_j-\bar{l}_j} \\ (1-\gamma)(b_j - \bar{l}_j) - \upsilon \Leftarrow \gamma \leq \frac{b_j-\bar{l}_j-\upsilon}{b_j-\bar{l}_j} \end{cases}$$

when $r_i < r_j$. Bank j, in turn, deposits

$$l_j - \bar{l}_j - Q^* = \begin{cases} \upsilon \Leftarrow \gamma > \frac{\bar{l}_i-b_i}{\bar{l}_i-b_i+\upsilon} \\ (1-\gamma)(\bar{l}_i - b_i + \upsilon) \Leftarrow \gamma \leq \frac{\bar{l}_i-b_i}{\bar{l}_i-b_i+\upsilon} \end{cases}$$

when $r_i > r_j$, and

$$l_j - \bar{l}_j - Q^* = \begin{cases} \upsilon \Leftarrow \gamma > \frac{b_j - \bar{l}_j - \upsilon}{b_j - \bar{l}_j} \\ (1-\gamma)(b_j - \bar{l}_j) \Leftarrow \gamma \leq \frac{b_j - \bar{l}_j - \upsilon}{b_j - \bar{l}_j} \end{cases}$$

when $r_i < r_j$.

Consider the case of a pure variable-rate tender, in which bids are distributed in the interval $[d, m]$.[13] In a subsection below, these results will be particularized to variable-rate tenders with a minimum bid higher than d, the scenario adopted by the ECB. As we will see, qualitative findings do not change.

If one assumes a uniform distribution for interest rate bids, the probability for bank i of winning the auction is given by $\Pr(r_j < r_i) = \frac{r_i - d}{m - d}$. For bank j, in turn, the probability of winning is $\frac{r_j - d}{m - d}$. Each bank minimizes in its own price and quantity bids a weighted average of the expenses in the winning and in the losing cases. The weights are the probabilities above.

The solution of each bank's optimization problem gives rise to the optimal b_i, b_j, r_i, and r_j, which, once replaced in the equilibrium expressions above for the interbank market and for the standing facilities, allow us to obtain the complete solution of the reserve-keeping problem. As in the analysis of fixed-rate tenders, I begin with the case of $\upsilon = 0$.

4.1 Absence of Estimation Errors

When the ECB has a precise estimation of the system's liquidity needs ($\upsilon = 0$), the conditions imposed on b_i and b_j such that in equilibrium bank i is not a supplier in the interbank market and bank j is not a demander imply that $b_i \in (0, \bar{l}_i]$ and $b_j \in [\bar{l}_j, \bar{l}_i + \bar{l}_j]$. Note that, contrary to what happens in a fixed-rate auction, b_i and b_j have finite superior limits. This prevents the allotment ratio from falling to very low levels.

Interior solutions for r_i and r_j require that aggregate bidding behavior exceeds aggregate liquidity needs ($b_i + b_j > \bar{l}_i + \bar{l}_j$), which

[13] It does not pay to bid an interest rate higher than m, because the bank can obtain liquidity at m through the lending facility. Also, the Central Bank is not willing to accept bids below d, since banks can deposit liquidity at d.

corresponds to rationing at the marginal rate, a situation that occurred in almost all variable-rate tenders conducted.

When $\upsilon = 0$ the cut-off level for γ so that the equilibrium overnight rate does not rise above m is simply equal to 1. The optimization problems of banks i and j for $\gamma > 1$ ($o^* < m$) resume to

$$
\begin{aligned}
&min_{b_i,r_i}\left(\frac{r_i-d}{m-d}\right)\left(r_ib_i+\left(d+\frac{m-d}{\gamma}\right)(\bar{l}_i-b_i)\right)\\
&\quad+\left(\frac{m-r_i}{m-d}\right)\left(r_i(\bar{l}_i+\bar{l}_j-b_j)+\left(d+\frac{m-d}{\gamma}\right)(b_j-\bar{l}_j)\right)\\
&min_{b_j,r_j}\left(\frac{m-r_j}{m-d}\right)\left(r_j(\bar{l}_i+\bar{l}_j-b_i)-\left(d+\frac{m-d}{\gamma}\right)(\bar{l}_i-b_i)\right)\\
&\quad+\left(\frac{r_j-d}{m-d}\right)\left(r_jb_j-\left(d+\frac{m-d}{\gamma}\right)(b_j-\bar{l}_j)\right).
\end{aligned}
$$

For $\gamma \leq 1$ these optimization problems are

$$
\begin{aligned}
&min_{b_i,r_i}\left(\frac{r_i-d}{m-d}\right)(r_ib_i+m(\bar{l}_i-b_i))\\
&\quad+\left(\frac{m-r_i}{m-d}\right)(r_i(\bar{l}_i+\bar{l}_j-b_j)+m(b_j-\bar{l}_j))\\
&min_{bj,r_j}\left(\frac{m-r_j}{m-d}\right)(r_j(\bar{l}_i+\bar{l}_j-b_i)-(m\gamma+d(1-\gamma))(\bar{l}_i-b_i))\\
&\quad+\left(\frac{r_j-d}{m-d}\right)(r_jb_j-(m\gamma+d(1-\gamma))(b_j-\bar{l}_j)).
\end{aligned}
$$

There are four equilibria for the variable-rate tender when $\gamma > 1$. The only equilibrium with both types of banks participating in the MRO and interior solutions for the price bids is[14]

[14]There are three other equilibria, which do not comply with the condition $b_i + b_j > \bar{l}_i + \bar{l}_j$, so do not have rationing at the marginal rate.

$$b_i^* = \bar{l}_i,\ r_i^* = d + \frac{m-d}{2\gamma}$$
$$b_j^* = \bar{l}_i + \bar{l}_j,\ r_j^* = \begin{cases} d + \frac{m-d}{2\gamma} - \frac{(m-d)\bar{l}_j}{2\bar{l}_i} \Leftarrow \bar{l}_j \leq \frac{\bar{l}_i}{\gamma} \\ d \Leftarrow \bar{l}_j > \frac{\bar{l}_i}{\gamma}. \end{cases}$$

In this equilibrium bank i wins the auction ($r_i > r_j\ \forall\ \bar{l}_i, \bar{l}_j$)[15] and is allocated with all the liquidity it needs. Bank j also receives all the liquidity required (it receives $\bar{l}_i + \bar{l}_j - \bar{l}_i = \bar{l}_j$, even though it bids higher) but pays a lower interest rate, declining in its own liquidity needs and rising in the rival's, such that when $\bar{l}_i$ is sufficiently small relative to $\bar{l}_j$, bank j is not willing to make a price offer above the minimum (d). The bank that expects to have a demand position in the interbank market is the one that makes the lowest quantity bid; however, it is the one that wins, so bidding an amount equal to its needs is the optimal behavior.

For $\gamma \leq 1$, the relevant case under a financial markets crisis scenario, the only equilibrium with interior solutions for the interest rate bids is[16]

$$b_i^* = \bar{l}_i,\ r_i^* = \frac{m+d}{2}$$
$$b_j^* = \bar{l}_i + \bar{l}_j,\ r_j^* = \begin{cases} d + \frac{(m-d)\gamma}{2} - \frac{(m-d)\bar{l}_j}{2\bar{l}_i} \Leftarrow \bar{l}_j \leq \gamma\bar{l}_i \\ d \Leftarrow \bar{l}_j > \gamma\bar{l}_i. \end{cases}$$

Bank i is the winner and pays an interest rate independent of γ, because unsatisfied liquidity will have to be paid at m. The price offered by j declines as γ increases, for the more inelastic is supply, the less bank j will be able to sell in the money market (at a rate equal to m), and hence the more it will have to deposit at d. In the limit, as γ tends to zero (interbank market vanishing), r_j^* will tend to d, the minimum bid allowed in this scenario (see subsection 4.4 for a higher minimum bid).

[15] The bank with the worst expectations about its position in the secondary market is the one that bids the highest price.

[16] There is another equilibrium, which does not comply with the condition $b_i + b_j > \bar{l}_i + \bar{l}_j$.

These results allow us to state the following lemma, which contrasts with lemma 1.

LEMMA 5. *The equilibrium quantity bid in a variable-rate tender is finite. The equilibrium allotment ratio is equal to* $\frac{\bar{l}_i+\bar{l}_j}{2\bar{l}_i+\bar{l}_j}$, *which is growing in the liquidity needs of banks that expect not to be demanders in the secondary market, and decreasing in the liquidity needs of banks that expect not to be suppliers. In the context of this model, the allotment ratio is higher than* $\frac{1}{2}$.

Furthermore, the finite number of equilibrium quantity bids in a variable-rate tender is a clear difference as compared with the outcome of fixed-rate tenders (without full allotment). The informational role of quantity bids (in conjunction with price bids) regarding liquidity needs is preserved.

4.2 Collateral Shortage

Assume now that i is limited by the collateral amount c_i, such that $c_i < \bar{l}_i$. Under these circumstances bank i will bid in such a way that available collateral is fully used in the primary market, as in the fixed-rate tender. The overnight rate may rise above m. I consider that collateral is restrictive both when i wins and when it loses the auction.[17] For all γ, demand in the interbank market is

$$Q^D = \bar{l}_i - c_i,$$

while supply is given by

$$Q^S = \begin{cases} 0 \Leftarrow o < d \\ \gamma(\bar{l}_i - c_i)\frac{o-d}{m-d} \Leftarrow o \geq d \end{cases}$$

if i wins the auction, and by

$$Q^S = \begin{cases} 0 \Leftarrow o < d \\ \gamma(b_j - \bar{l}_j)\frac{o-d}{m-d} \Leftarrow o \geq d \end{cases}$$

if the winner is j.

[17] So, when i loses, it does not buy $\bar{l}_i + \bar{l}_j - b_j$, but just c_i, exactly the same amount bought when it wins.

In the equilibrium of this game, the interest rate paid by j is higher the more i is restricted by collateral. When i wins the auction, its shortage of funds is solved in the secondary market at an interest rate equal to $d+\frac{m-d}{\gamma}$;[18] when it loses, the interbank interest rate is higher.

LEMMA 6. *In the equilibrium of a variable-rate tender with collateral restrictions for bank i, interest rate bids of j in the primary market are rising in i's shortage of collateral.*

There are multiple equilibria for the quantity bidding behavior of i. The focal equilibrium is $b_i^* = c_i$, under which the allotment ratio is higher than with no collateral restrictions, because now i bids below its needs. The choice of $b_i^* = c_i$ is also related to the fact that, in the practice of the Eurosystem, bids which show up to be impossible to cover with collateral are highly penalized.

4.3 Credit Rationing in the Interbank Market

Consider now that, due to information problems, the liquidity supplier in the secondary market, j, decides to ration credit at the interest rate t, with $d < t < m$. Rationing may also occur because of uncertainty related to future own liquidity needs. Then the equilibrium of the game with both banks participating in the auction and interior solutions for the interest rate bids, $v = 0$, and no collateral restrictions becomes

$$b_i^* = \bar{l}_i,\ r_i^* = \frac{m+d}{2} - \gamma\frac{(t-d)(m-t)}{2(m-d)}$$

$$b_j^* = \bar{l}_i + \bar{l}_j,\ r_j^* = d + \gamma\frac{(t-d)^2}{2(m-d)} - \frac{(m-d)\bar{l}_j}{2\bar{l}_i}.$$

Note that $\frac{\partial r_i^*}{\partial t} < 0$ is true only for $t < \frac{m+d}{2}$. This inequality is valid whenever $\gamma > 2$, so for high-enough elasticity of supply, the loss in terms of quantity acquired in the interbank market (and that

[18]The value of c_i has no influence on o^* because there is a shift in supply associated with the shift in demand due to collateral shortage, so that o^* remains unchanged.

has to be transferred to the lending facility at a penalty rate) is important and induces a more aggressive price-bidding behavior in the MRO. If supply is not so elastic, then the reduction in quantity traded is more modest and the rise in the price bid of i occurs only if rationing is sufficiently relevant. Comparing with the corresponding equilibrium with no rationing, we state the following:

LEMMA 7. *Credit rationing in the secondary market gives rise to a decline in the interest rate bid of the bank that is a supplier in that market. The price bid made by the demander may rise or fall, depending on the elasticity of supply being high or low, respectively.*

Of course, under extreme credit rationing such as in the recent crisis, t is lower than $\frac{m+d}{2}$, so the interest rate bid of i-banks will tend to be very high. Observation of marginal rates in MROs actually shows a gap relative to the minimum bid that is larger during the turbulence than before it.

4.4 Minimum Bid for the Interest Rate

In the analysis conducted so far, the lower bound for the price offers has been d. When the change to variable-rate tenders occurred in June 2000, the minimum bid was set at the fixed rate in use in the MROs of the Eurosystem at the time of the change—that is, r in our notation. In these circumstances r_i and r_j are now free to fluctuate in the interval $[r, m]$ instead of $[d, m]$. The equilibrium for $\gamma > 1$ with both types of banks participating in the MRO and with interior solutions for the price bids becomes

$$b_i^* = \bar{l}_i,\ r_i^* = \begin{cases} \frac{d+r}{2} + \frac{m-d}{2\gamma} \Leftarrow \gamma > \frac{m-d}{r-d} \\ r \Leftarrow \gamma \geq \frac{m-d}{r-d} \end{cases}$$

$$b_j^* = \bar{l}_i + \bar{l}_j,\ r_j^* = \begin{cases} \frac{d+r}{2} + \frac{m-d}{2\gamma} - \frac{(m-r)\bar{l}_j}{2\bar{l}_i} \Leftarrow \bar{l}_j < \frac{(m-d-\gamma(r-d))\bar{l}_i}{\gamma(m-r)} \\ r \Leftarrow \bar{l}_j \geq \frac{(m-d-\gamma(r-d))\bar{l}_i}{\gamma(m-r)}. \end{cases}$$

The interest rates paid by both banks rise as compared with the no-minimum-bid situation. Hence their expenses rise as well, as do the ECB's revenues. The winning price bid is mostly influenced by the minimum level r, the interest rate on the deposit facility being the one with the lowest impact on r_i^* if the elasticity of supply is not too high ($\gamma < 2$); otherwise, the lending facility rate has the lowest impact.

When $\gamma \leq 1$, this equilibrium is

$$b_i^* = \bar{l}_i,\ r_i^* = \frac{m+r}{2}$$

$$b_j^* = \bar{l}_i + \bar{l}_j,\ r_j^* = \begin{cases} \frac{d+r}{2} + \frac{(m-d)\gamma}{2} - \frac{(m-r)\bar{l}_j}{2\bar{l}_i} \Leftarrow \bar{l}_j \leq \frac{((m-d)\gamma-(r-d))\bar{l}_i}{m-r} \\ r \Leftarrow \bar{l}_j > \frac{((m-d)\gamma-(r-d))\bar{l}_i}{m-r}. \end{cases}$$

Interest bids are again higher than in the absence of a minimum bid. The winning bank offers a price equal to the midpoint of the allowed interval, just as in the no-minimum-bid case. The value of γ does not influence this bid, because the interbank rate is predetermined at m, contrary to what happens when $\gamma > 1$.

A variable-rate tender with a minimum price bid has no qualitative difference from a pure variable-rate tender. When the signaling effect of the minimum offer as to the monetary policy stance can be achieved through some other vehicle (for instance, by announcing an operational target), the choice of the instrument is irrelevant.

4.5 Estimation Errors

When the ECB fails to correctly estimate the system's liquidity needs,[19] it can be seen that bank i will find it more expensive to comply with reserve requirements if $\upsilon < 0$ (underestimation), and cheaper if $\upsilon > 0$ (overestimation). The interest rate paid rises in the former situation as compared with the correct estimation case, and declines in the latter. There is an additional reason for bank i's expenses to grow in the underestimation case—the fact that it will have to resort to the lending facility for the amount $|\upsilon|$.

As to bank j, a negative estimation error has no effect on the price paid, but $\upsilon > 0$ implies a lower price offer. This bank makes use of the deposit facility (for an amount equal to the estimation error) when $\upsilon > 0$.

[19] As in fixed-rate tenders, this wrong estimation is related to the absorption or injection of liquidity by "autonomous factors."

The equilibria characterized by interior solutions for the overnight rate and by both banks participating in the MRO and making interior price bids (with no minimum rate) are as follows. For $\upsilon < 0$

$$b_i^* = \bar{l}_i + \upsilon,\ r_i^* = d + \frac{m-d}{2\gamma} - \frac{\upsilon(m-d)}{2\bar{l}_i}$$

$$b_j^* = \bar{l}_i + \bar{l}_j,\ r_j^* = \begin{cases} d + \frac{m-d}{2\gamma} - \frac{(m-d)\bar{l}_j}{2\bar{l}_i} \Leftarrow \bar{l}_j \leq \frac{\bar{l}_i}{\gamma} \\ d \Leftarrow \bar{l}_j > \frac{\bar{l}_i}{\gamma} \end{cases}$$

and for $\upsilon > 0$

$$b_i^* = \bar{l}_i,\ r_i^* = d + \frac{(m-d)\bar{l}_i}{2\gamma(\bar{l}_i + \upsilon)}$$

$$b_j^* = \bar{l}_i + \bar{l}_j + \upsilon,\ r_j^* = \begin{cases} d + \frac{(m-d)\bar{l}_i}{2\gamma(\bar{l}_i+\upsilon)} - \frac{(m-d)(\bar{l}_j+\upsilon)}{2\bar{l}_i} \Leftarrow \bar{l}_j \leq \frac{\bar{l}_i^2}{\gamma(\bar{l}_i+\upsilon)} - \upsilon \\ d \Leftarrow \bar{l}_j > \frac{\bar{l}_i^2}{\gamma(\bar{l}_i+\upsilon)} - \upsilon. \end{cases}$$

The bank which expects not to be a liquidity supplier in the secondary market reduces its quantity bid as compared with the no-estimation-error case when $\upsilon < 0$, and the bank which expects not to be a liquidity demander raises its bid when $\upsilon > 0$. In these circumstances expectations as to net positions in the interbank market are fulfilled. Deviations as to liquidity needs are fully corrected through the ECB's standing facilities. As stated above, bank i is harmed, in terms of expenses, by underestimation, and benefits from overestimation. Bank j is unaffected by underestimation; when there is overestimation, results are ambiguous and depend, among other factors, on the magnitude of υ (on the one hand, the price paid in the MRO is reduced as compared with the $\upsilon = 0$ case, but liquidity acquired is higher, and there is a surplus to be deposited at d).

In these equilibria the allotment ratio is $\frac{\bar{l}_i+\bar{l}_j+\upsilon}{2\bar{l}_i+\bar{l}_j+\upsilon}$. So, underestimation of the system's liquidity needs decreases this ratio, whereas overestimation raises it.

LEMMA 8. *An underestimation (overestimation) by the ECB of the system's liquidity needs raises (decreases) the interest rate bid of* *banks which expect to be liquidity demanders in the secondary market and their expenses, and decreases (raises) the allotment ratio.*

5. Fixed- versus Variable-Rate Tenders

The model presented is a simple three-stage game in which banks try to obtain the liquidity needed by making use of open-market operations of the ECB, the interbank money market, and "standing facilities." Although simple, the model replicates empirical facts observed since the beginning of Stage Three of the EMU, confirms some results in the literature, and allows us to address the possible implications of some facts deriving from the recent credit crisis, such as collateral shortage, credit rationing in the interbank market, and wrong estimation by the ECB of the system's true liquidity needs. This section summarizes some of the results obtained.

Confirming observation and previous literature results, low allotment ratios were shown to be inherent to fixed-rate tenders (without full allotment), since bids may reach very high levels. On the contrary, variable-rate tenders allow for reasonable levels of this ratio. Overbidding may be present in both types of auction; however, it may be a much more serious problem in the former than in the latter (the change from fixed- to variable-rate procedures in June 2000 has indeed led to clearly higher allotment ratios).

The informational content of quantity bids as regards true liquidity needs is lost in fixed-rate tenders (without full allotment), due to multiplicity of equilibria. However, it is preserved in variable-rate tenders.

In a variable-rate tender banks have a higher probability of being allocated with all the liquidity needed in the MRO.[20] This is especially important for those institutions that expect to be in a demand position in the interbank market and even more when this market is highly restricted or does not even work. Actually, in a fixed-rate tender liquidity allocated to each counterpart is a function of its own quantity bid and also of the rival's, so that a perception error about the latter may leave the bank in a weak position thereafter; on the

[20] Again, this is as compared with a fixed-rate tender without full allotment. If fixed-rate tenders have full allotment, the probability of being allocated with all the liquidity needed is one.

contrary, in a variable-rate tender the bank may guarantee itself the desired liquidity as long as it makes a sufficiently high price bid.

Suppose that i believes that j will bid b_{j1} and therefore, in a fixed-rate tender (with no distortions—binding collateral and other), chooses $b_{i1} = \frac{\bar{l}_i}{\bar{l}_j} b_{j1}$, but j actually bids $b_{j2} > b_{j1}$. Then bank i will receive $\frac{\bar{l}_i+\bar{l}_j}{b_{i1}+b_{j2}} b_{i1} < \frac{\bar{l}_i+\bar{l}_j}{b_{i1}+b_{j1}} b_{i1}$ and j will obtain $\frac{\bar{l}_i+\bar{l}_j}{b_{i1}+b_{j2}} b_{j2} > \frac{\bar{l}_i+\bar{l}_j}{b_{i1}+b_{j1}} b_{j1}$. What j receives in excess is exactly what i is short of; this misallocation is corrected in the secondary market at an (interior) interbank rate equal to $d + \frac{m-d}{\gamma}$. For sufficiently low γ ($\gamma < \frac{m-d}{r-d}$), j benefits in terms of expenses and i is harmed, the reverse being true for high γ. In any case, the ECB's revenues are unaffected by this perception error of bank i.

In a variable-rate tender (again with no distortions) the winning bank is not harmed by a wrong perception of the rival's bid. The loser, however, may suffer from an incorrect perception of the other's liquidity needs (and hence quantity bid), both in what concerns allotted amount and price paid.

For banks restricted by collateral, a relevant scenario under the recent financial turmoil, bidding mistakes (above the equilibrium) are more harmful under fixed-rate procedures than under variable-rate procedures. As we have seen, the equilibrium quantity bid for bank i is $\frac{c_i}{\bar{l}_i+\bar{l}_j-c_i} b_j$ in a fixed-rate tender and c_i in a variable-rate one. Bidding above the equilibrium implies less liquidity for bank j in the fixed-rate auction, and hence a rise in the interbank market rate; in a variable-rate auction, on the contrary, liquidity received by j is unaffected by b_i, so in this sense bank i's situation is not worsened.

Shortage of collateral to cover liquidity needs may benefit banks on the supply side of the secondary market. The revenues of the ECB are unaffected by collateral shortage in a fixed-rate tender but may rise in variable-rate tenders, because price bids of unrestricted banks are higher.

The existence of credit rationing in the interbank market, another relevant scenario under intensified market tensions, accompanied by shortage of collateral of the bank which will be in a demand position in that market, leaves this institution with unsatisfied liquidity needs in both procedures. In turn, there is excess liquidity to be absorbed by the ECB, as evidence on the high frequency of fine-tuning operations during the credit crisis has confirmed. Under a variable-rate procedure, and with interbank credit rationing, banks will tend to bid high interest rates in the primary market, therefore leading to an enlarged differencial between the marginal rate and the minimum bid, as also observed.

Estimation errors by the ECB have clearer effects on banks' expenses under a fixed-rate procedure than under a variable-rate one. In the former, banks with a demand position in the secondary market are hurt by underestimation of liquidity needs and are unaffected by overestimation; the reverse happens for banks with a supply position. In a variable-rate auction, underestimation has the same implications as in the fixed-rate auction; however, banks with a demand position may benefit from an ECB's overestimation of the system's needs, while the impact of this mistake on the expenses of the banks with a supply position is ambiguous.

Taking into account the variety of procedures that have been used to stabilize the markets during the recent turmoil, the debate on the virtues and drawbacks of fixed- and variable-rate tenders gained new relevance. However, as we have seen, the results in this paper still favor the choice of variable-rate procedures as opposed to no-full-allotment fixed-rate tenders.

6. Conclusion

Open-market operations are an important instrument of the ECB's monetary policy. Most of them have been historically conducted through variable-rate tenders, although fixed-rate operations have already been in practice and have never been definitely excluded. Actually, the Central Bank returned to the fixed-rate format, but with full allotment, to conduct its MROs from October 9, 2008 on. During the financial market turmoil that emerged in the summer of 2007 and intensified later on, the ECB made use of several liquidity management instruments, and a comparative analysis of both types of tenders is pertinent, under relevant scenarios such as the possibility that collateral is binding, rationing by suppliers in the interbank market (due to uncertainty as to other institutions' solvency and own need of funds), and estimation errors of the system's true needs by the monetary authority. Recovering some previous results in the literature, this paper has compared fixed- and variable-rate tenders in what concerns bidding behavior of the counterparts and induced allotment ratios in the primary market, functioning of the interbank market, and resorting to "standing facilities." Overbidding is again found to be inherent to fixed-rate tenders (without full allotment), but can be very mitigated under a variable-rate procedure. Furthermore, variable-rate tenders allow keeping some informational content of quantity bids, as opposed to fixed-rate tenders (again without full allotment).

The paper has also shown that (i) bidding mistakes are more harmful for banks restricted by collateral when the liquidity auction takes place at a fixed interest rate than when this rate is variable, (ii) credit rationing in the interbank market may lead to the placement of high amounts of liquidity with the ECB, reinforces the opportunity for fine-tuning operations, and enlarges the gap between the marginal rate and the minimum bid in a variable-rate auction, and (iii) a non-acute estimation of the system's liquidity needs has stronger implications for banks' expenses (to comply with reserve requirements) under primary market fixed-rate tenders than under variable-rate tenders. The paper also provides some rationale for the enlargement in the set of assets eligible for collateral purposes. Altogether, our results reinforce the use of variable-rate tenders as a means to provide liquidity to credit institutions, as opposed to fixed-rate tenders without full allotment, especially in times of pronounced market turbulence.

References

Ayuso, J., and R. Repullo. 2001. "Why Did the Banks Overbid? An Empirical Model of the Fixed Rate Tenders of the European Central Bank." *Journal of International Money and Finance* 20 (6): 857–70.

———. 2003. "A Model of the Open Market Operations of the European Central Bank." *Economic Journal* 113 (490): 883–902.

Bindseil, U. 2002. "Equilibrium Bidding in the Eurosystem's Open Market Operations." ECB Working Paper No. 137.

Catalão-Lopes, M. 2001. "Financing in the Eurosystem: Fixed versus Variable Rate Tenders." Working Paper No. 5-01, Banco de Portugal.

European Central Bank. 2000. "The Switch to Variable Rate Tenders in the Main Refinancing Operations." *Monthly Bulletin* (July).

———. 2003. *Annual Report 2002*.

———. 2008a. "The Implementation of Monetary Policy in the Euro Area: General Documentation on Eurosystem Monetary Policy Instruments and Procedures." (November).

———. 2008b. "EU Banking Structures." Report. (October).

Nautz, D., and J. Oechssler. 2003. "The Repo Auctions of the European Central Bank and the Vanishing Quota Puzzle." *Scandinavian Journal of Economics* 105 (2): 207–20.

———. 2006. "Overbidding in Fixed Rate Tenders—An Empirical Assessment of Alternative Explanations." *European Economic Review* 50 (3): 631–46.

Scalia, A., M. Ordine, and G. Bruno. 2005. "Banks' Participation in the Eurosystem Auctions and Money Market Integration." Bank of Italy Discussion Paper No. 562.