The Role of Asset Prices in Best-Practice Monetary Policy*

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I study the role of asset prices in the conduct of monetary policy under the commitment equilibrium. The findings lend support to the lean-against-the-wind strategy in that it is optimal for the central bank to set interest rates to respond to asset-price movements. The gain from responding to asset prices comes from the fact that asset-price movements can provide a signal about the development in the state of the economy. The paper also suggests that prior to and during the subprime mortgage crisis of 2007, it would have been optimal for the Federal Reserve to increase the weight of asset prices in its rate-setting decision.

JEL Codes: E44, E52.

1. Introduction

The issue of asset prices in the conduct of monetary policy has come back right to the fore of the debate following the subprime mortgage crisis of 2007. After the sustained increase in house prices between 2001 and 2005, the booming housing market halted abruptly in the late summer of 2005, which caused house prices to be flat for the rest of 2005 and throughout 2006.[1] At the beginning of 2007, house prices

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[1]During 2001–05, the average year-on-year growth rate of the S&P/Case-Shiller Home Price Index was 12.4 percent, with the number of existing homes sold growing at an annual rate of 6.2 percent. The annualized number of existing homes sold peaked at 6.3 million in September 2005. Subsequently, the number of existing homes sold declined at an annual rate of 8.7 percent.

started to decline, and the downward movement in house prices continued during 2007–08.[2] This led to a sharp rise in the default rate, especially on subprime mortgage loans made to high-risk borrowers. Mortgage lenders who underwrote such loans and financial institutions who bought collateralized debt obligations (CDOs) backed by mortgage payments in turn suffered huge losses. In response to the crisis, the Federal Reserve lowered the target for the federal funds rate 50 basis points to 4.75 percent in September 2007, after maintaining the target rate at 5.25 percent for fifteen months. Given that the subprime mortgage crisis might have repercussions for other sectors and might cause the economy to slip into a recession, the Federal Reserve continued to lower the target federal funds rate in the two remaining Federal Open Market Committee (FOMC) meetings in 2007.

This raises a question about how the central bank should conduct monetary policy amid asset-price booms and busts. One prominent approach suggested by several authors is for the central bank to respond to asset-price movements only insofar as the latter affect the forecast of inflation.[3] Under this notion, an appropriate monetary policy strategy is to set short-term interest rates to respond strongly to inflation, and there is little gain from responding directly to asset prices.[4] This also implies that the central bank should not attempt to strike down an asset-price boom. Instead, the central bank should follow the "mop-up-after" strategy by being alert and easing the monetary policy stance to alleviate the adverse effect from a sharp fall in asset prices.

An alternative approach, proposed by the so-called lean-against-the-wind camp, is that the central bank can improve macroeconomic

[2]From the second half of 2006, house prices, as measured by the S&P/Case-Shiller Index, were flat, growing at 4.5 percent a year. However, since January 2007, the S&P/Case-Shiller Index has declined in every month. The median home sale price collected by the National Association of Realtors, another measure of house prices, peaked at $230,900 in June 2006 and has declined in every month since at an annual rate of 2.5 percent.

[3]See Bernanke and Gertler (1999, 2001), Gilchrist and Leahy (2002), Tetlow (2005), Gilchrist and Saito (2006), and Faia and Monacelli (2007). This idea is also consistent with what was proposed by Alan Greenspan. See Greenspan (2002, 2005).

[4]To respond *strongly* to inflation means that the extent to which the central bank raises the nominal interest rate is much larger than the increase in inflation.

performance by setting short-term interest rates to respond to asset-price movements. This implies that the central bank should raise short-term interest rates as asset prices go up, even though other target variables, such as inflation forecasts and employment, are roughly on target.[5]

This paper examines how the central bank should take into account asset-price movements in its interest-rate-setting process. What distinguishes this paper from previous studies is that this paper examines the role of asset prices in the conduct of monetary policy in the commitment equilibrium.[6] Under the goal of maintaining price stability and full employment, optimal policy under commitment yields the best possible outcome that the central bank is capable of implementing. The result in this paper thus provides an implication on which strategy, mopping up after or leaning against the wind, is optimal policy.

The present paper finds that the leaning-against-the-wind strategy is optimal. In particular, from the standpoint of a central bank whose objective is to maintain price stability and full employment, it is optimal to set interest rates to respond to asset-price movements. The reason, according to the analysis in this paper, is that asset-price movements can provide signals about the development in the state of the economy that the central bank cannot perfectly observe. The results in this paper also suggest that before and during the subprime mortgage crisis of 2007, in which it may have been difficult for the Federal Reserve to perfectly verify creditworthiness and risk premiums of U.S. households, the Federal Reserve should have paid even more attention to asset-price movements. The usefulness of asset prices in the rate-setting process nonetheless depends on the degree to which the central bank can verify the nature of asset-price misalignments.

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[5]See, for instance, Cecchetti et al. (2000) and Bordo and Jeanne (2002a, 2002b).

[6]The earlier literature conducts the analysis by relying on some preselected sets of Taylor-type interest rate rules. Thus, the earlier literature is silent on what is the optimal strategy amid asset-price booms and busts. Bordo and Jeanne (2002a, 2002b) do not conduct their analysis on standard New Keynesian-type DSGE models. Rather, they conduct their analysis in a stylized model with a finite number of periods and thus do not derive the rational-expectations equilibrium.

The remainder of the paper proceeds in the following manner. Section 2 sets up the model used for the analysis. Section 3 presents the policy problem of a central bank who can commit, and provides an algorithm for computing optimal policy under commitment. Section 4 considers whether it is optimal for the central bank to respond to asset prices in a hypothetical world in which the central bank can perfectly observe the state of the economy. Section 5 considers more realistic scenarios in which the assumption of full information is relaxed. Under the partial-information case, the central bank solves an optimal filtering problem, using asset prices as an indicator. Section 6 examines whether the main findings in this paper depend on some important assumptions. Section 7 concludes.

2. The Model Economy

The model used in the analysis is the one presented in Gilchrist and Saito (2006) (henceforth, GS). The GS model is essentially a standard New Keynesian model augmented to include credit-market frictions through the financial accelerator mechanism described in Bernanke, Gertler, and Gilchrist (1999) (henceforth, BGG). The model consists of six sectors: households, entrepreneurs, retailers, capital producers, the government, and the central bank. Households consume, hold money, save in one-period riskless bonds, and supply labor to entrepreneurs. Entrepreneurs manage the production of wholesale goods, which requires capital constructed by capital producers and labor supplied by both households and entrepreneurs. Entrepreneurs purchase capital and finance the expenditures of capital with their net worth and debt. Entrepreneurs sell wholesale goods to monopolistically competitive retailers who differentiate the product slightly at zero resource cost. Each retailer then sets its price and sells its differentiated product to households, capital producers, entrepreneurs, and the government.

Rather than work through the details of the derivation, which are readily available in GS, I instead directly introduce the log-linearized version of the aggregate relationships of the model.

Table 1 provides a summary of the variables in the model. Throughout, steady-state levels of the variables are in lower case without time subscripts while log-deviations from the steady-state

Table 1. Summary of the Model Variables

Variable	Explanation
c_t	Consumption
z_t	Productivity Growth
i_t	Nominal Interest Rate
π_t	Inflation
r_t^k	Real Rate of Return on Capital
y_t	Output
k_{t+1}	Capital at the End of Period t
mc_t	Real Marginal Cost
q_t	Price of Capital
s_t	External Finance Premium
n_{t+1}	Net Worth at the End of Period t
inv_t	Investment
h_t	Labor Supply
ε_t	Transitory Shock to Productivity
d_t	Persistent Component of Productivity
υ_t	Persistent Shock to Productivity
$\widehat{y}_t$	Output Gap
r_t^*	Natural Interest Rate

are in lower case with time subscripts. The corresponding hypothetical levels of the variables in the frictionless economy are denoted by an asterisk. Greek letters and lowercase Roman letters without subscripts denote fixed parameters. Table 2 provides a summary of the parameters as well as their baseline calibration.

The first equation is the log-linearized version of the national income identity:

$$y_t = \frac{c}{y} c_t + \frac{\mathrm{inv}}{y} \mathrm{inv}_t. \tag{1}$$

Note that in the baseline calibration of the GS model, entrepreneurs' consumption and government spending are normalized to zero. Model simulations conducted under the original BGG framework imply that these simplifications are reasonable.

Households' consumption is determined by a standard Euler equation summarizing households' optimal consumption-savings allocation:

Table 2. Baseline Calibration of the Model Parameters and the Steady-State Level of Some Key Variables

Parameter	Explanation	Baseline Calibration
β	Discount Factor	0.984
α	Labor Share	2/3
γ	Inverse of Labor Supply Elasticity	0.8
δ	Depreciation Rate	0.025
η_k	Elasticity of Asset Prices	0.25
$\varepsilon/(\varepsilon-1)$	Steady-State Markup	1.1
υ	Calvo Parameter	0.75
$k/n-1$	Steady-State Leverage Ratio	0.8
χ	Elasticity of the Finance Premium	0.05
μ	Mean Technology Growth Rate	0.00427
σ_ε	Standard Deviation of the Transitory Shock	0.01×100
σ_υ	Standard Deviation of the Persistent Shock	0.001×100
ρ_d	AR(1) Coefficient of the Persistent Shock	0.95

$$-c_t = -\mathrm{E}_t c_{t+1} - \mathrm{E}_t z_{t+1} + i_t - \mathrm{E}_t \pi_{t+1}. \tag{2}$$

z_t, the growth of productivity, enters the Euler equation, as well as several other equations in the model, because the levels of consumption, investment, output, capital stock, and net worth are normalized by the level of technology, in order to make these real quantities stationary.

Households also make a decision on labor supply. Labor demand, on the other hand, is derived from entrepreneurs' profit maximization problem. In an equilibrium, labor supply equals labor demand. Using the labor-demand condition to eliminate wages from the labor-supply equation yields the following labor-market equilibrium condition:

$$y_t + \mathrm{mc}_t - c_t = (1+\gamma)h_t. \tag{3}$$

mc_t enters (3) because we use the definition of mc_t, $\mathrm{mc}_t = p_{w,t} - p_t$, to eliminate $p_{w,t} - p_t$, where $p_{w,t}$ is the wholesale price and p_t is the price level of the economy. Equation (3) thus is the equation that defines mc_t in the system.

On the production side, entrepreneurs have access to a Cobb-Douglas technology:

$$y_t = \alpha h_t + (1-\alpha)k_t - (1-\alpha)z_t. \tag{4}$$

Capital k_t is purchased by the entrepreneurs at the end of period $t-1$. The expected real rate of return on capital, $\mathrm{E}_t r^k_{t+1}$, is given by

$$\begin{aligned}\mathrm{E}_t r^k_{t+1} &= \frac{\mathrm{mc}(1-\alpha)\frac{y}{k}Z}{\mathrm{mc}(1-\alpha)\frac{y}{k}Z + (1-\delta)}(\mathrm{E}_t y_{t+1} - k_{t+1} + \mathrm{E}_t z_{t+1} + \mathrm{E}_t \mathrm{mc}_{t+1}) \\ &\quad + \frac{1-\delta}{\mathrm{mc}(1-\alpha)\frac{y}{k}Z + (1-\delta)}\mathrm{E}_t q_{t+1} - q_t. \end{aligned} \tag{5}$$

Intuitively, the expected real rate of return on capital depends on the marginal profit from the production of wholesale goods, which (log-linearized) is given by

$$\frac{\mathrm{mc}(1-\alpha)\frac{y}{k}Z}{\mathrm{mc}(1-\alpha)\frac{y}{k}Z + (1-\delta)}(\mathrm{E}_t p_{w,t+1} - \mathrm{E}_t p_{t+1} + \mathrm{E}_t y_{t+1} - k_{t+1}).$$

$\mathrm{E}_t y_{t+1} - k_{t+1}$ is derived from log-linearizing the marginal product of capital. Substituting the real marginal cost (for the retailers), $\mathrm{mc}_t = p_{w,t} - p_t$, we derive the first part of the right-hand side of (5). The second part, $\frac{1-\delta}{\mathrm{mc}(1-\alpha)\frac{y}{k}Z+(1-\delta)}\mathrm{E}_t q_{t+1} - q_t$, is the capital gain. Summing the marginal profit and the capital gain, we derive the real rate of return on capital.

To finance their capital expenditures, the entrepreneurs employ internal funds, net worth, but also need to acquire loans from financial intermediaries. In the presence of credit-market frictions, the financial intermediaries can verify the return on the entrepreneurial investment only through the payment of a monitoring cost. The financial intermediaries and the entrepreneurs design loan contracts to minimize the expected agency cost. The nature of the contracts is that the entrepreneurs need to pay a premium above the riskless rate, which in this model is the opportunity cost for the financial intermediaries. The external finance premium in turn depends on the financial position of the entrepreneurs. In particular, the external finance premium increases when a smaller fraction of the capital expenditures are financed by the entrepreneurs' net worth:

$$s_t = \chi(q_t + k_{t+1} - n_{t+1}). \tag{6}$$

In a competitive financial market, the expected cost of borrowing is equated to the expected return on capital:

$$\mathrm{E}_t r_{t+1}^k = i_t - \mathrm{E}_t \pi_{t+1} + s_t, \tag{7}$$

where $i_t - \mathrm{E}_t \pi_{t+1}$ is the (real) riskless rate.

The rest of the capital expenditures are financed by entrepreneurial net worth, which is determined by

$$n_{t+1} = \frac{k}{n} r_t^k - \left(\frac{k}{n} - 1\right) \mathrm{E}_{t-1} r_t^k + n_t - z_t.$$

That is, the aggregate net worth of the entrepreneurs at the end of period t is the sum of the net worth from the previous period, n_t, and $\frac{k}{n} r_t^k - (\frac{k}{n} - 1)\mathrm{E}_{t-1} r_t^k$, the operating profit of the entrepreneurs earned during period t. $\frac{k}{n} r_t^k$ is the (log-linearized) realized return on investment. $(\frac{k}{n} - 1)\mathrm{E}_{t-1} r_t^k$ is the (log-linearized) entrepreneurs' marginal cost of external funds that is predetermined in period t by the financial intermediaries. Using the definition of the external finance premium, $\mathrm{E}_{t-1} r_t^k = s_{t-1} + i_t - \mathrm{E}_{t-1}\pi_t$, we have

$$n_{t+1} = \frac{k}{n} r_t^k - \left(\frac{k}{n} - 1\right)(s_{t-1} + i_t - \mathrm{E}_{t-1}\pi_t) + n_t - z_t. \tag{8}$$

The entrepreneurs purchase capital from capital producers who combine investment and depreciated capital stock. This activity entails physical adjustment costs, with the corresponding CRS production. The aggregate capital accumulation equation is thus given by

$$k_{t+1} = \frac{(1-\delta)}{Z}(k_t - z_t) + \left(1 - \frac{1-\delta}{Z}\right)\mathrm{inv}_t. \tag{9}$$

Capital producers maximize profit subject to the adjustment cost, yielding the following first-order condition:

$$q_t = \eta_k(\mathrm{inv}_t - k_t + z_t). \tag{10}$$

Equation (10) can be interpreted as an equilibrium condition for the investment-good market. That is, the demand for investment from entrepreneurs equals the investment goods supplied by capital producers. This determines the price of capital, which in this model is interpretable as asset prices. Equation (10) implies that investment increases as asset prices rise.

The retailers set prices in a staggered fashion, as in Calvo (1983). This gives rise to a standard Phillips curve:

$$\pi_t = \kappa \mathrm{mc}_t + \beta \mathrm{E}_t \pi_{t+1}. \tag{11}$$

It is practical to use (11) to write the dynamics of net worth as

$$n_{t+1} = \frac{k}{n} r_t^k - \left(\frac{k}{n} - 1\right)\left(s_{t-1} + i_t - \frac{\pi_{t-1}}{\beta} + \frac{\kappa}{\beta}\mathrm{mc}_{t-1}\right) + n_t - z_t. \tag{12}$$

The growth of productivity has both transitory and persistent components:

$$z_t = d_t + \varepsilon_t. \tag{13}$$

The persistent component follows an AR(1) process:

$$d_t = p_d d_{t-1} + \upsilon_t, \tag{14}$$

where shocks to the transitory and persistent components are

$$\varepsilon_t \sim i.i.d.N(0, \sigma_\varepsilon^2)$$

and

$$\upsilon_t \sim i.i.d.N(0, \sigma_\upsilon^2).$$

Finally, since one of the central bank's target variables is the output gap, which is the deviation of output from its hypothetical level in the frictionless economy, we also have the following set of equations defining the frictionless economy:

$$y_t^* - c_t^* = (1 + \gamma) h_t^* \tag{15}$$

$$y_t^* = \alpha h_t^* + (1 - \alpha) k_t^* - (1 - \alpha) z_t \tag{16}$$

$$y_t^* = \frac{c}{y} c_t^* + \frac{\mathrm{inv}}{y} \mathrm{inv}_t^* \tag{17}$$

$$k_{t+1}^* = \frac{1-\delta}{Z}(k_t^* - z_t) + (1 - \frac{1-\delta}{Z})\text{inv}_t^* \quad (18)$$

$$q_t^* = \eta_k(\text{inv}_t^* - k_t^* + z_t) \quad (19)$$

$$r_t^* = \frac{\text{mc}(1-\alpha)\frac{y}{k}Z}{\text{mc}(1-\alpha)\frac{y}{k}Z + (1-\delta)}(\text{E}_t y_{t+1}^* - k_{t+1}^* + \text{E}_t z_{t+1}) + \frac{1-\delta}{\text{mc}(1-\alpha)\frac{y}{k}Z + (1-\delta)}\text{E}_t q_{t+1}^* - q_t^* \quad (20)$$

$$-c_t^* = -\text{E}_t c_{t+1}^* - \text{E}_t z_{t+1} + r_t^* \quad (21)$$

$$\widehat{y}_t = y_t - y_t^*. \quad (22)$$

Thus, I define the frictionless variables conditional on the hypothetical level of capital stock that exists when the economy has been under flexible prices and without credit-market frictions, as in Neiss and Nelson (2003). I also conducted the analysis in this paper by defining the frictionless economy conditional on the actual level of capital stock, as in Woodford (2003). All of the conclusions in this paper remain valid under the Woodford approach. I follow Neiss and Nelson because this approach allows me to illustrate my results in a particularly sharp way.[7]

For ease of presentation, write the model economy in the state-space format, as in Svensson (2006):

$$\begin{bmatrix} X_{t+1} \\ Hx_{t+1|t} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bi_t + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}, \quad (23)$$

[7]Like Gilchrist and Saito (2006), in the frictionless economy, there are no nominal rigidities and credit-market frictions. The reason that I define the frictionless economy as the flex-price economy in the absence of credit-market frictions as opposed to in the presence of credit-market frictions is because it can be argued that a goal of the central bank is to lead the economy as close as possible to the "distortion-free" state. In this model economy, credit-market frictions are distortions in the form of asymmetric information in financial markets that in turn gives rise to fluctuations in the external finance premium. As will be shown later, the central bank can in fact stabilize the external finance premium. At the micro level, unlike the distortions arising from monopolistic competition that are beyond central banks' authority to deal with, most central banks, including the Federal Reserve, are capable of dealing with the distortions in financial markets. As pointed out by Bernanke (2002), "The Fed has been entrusted with the responsibility of helping to ensure the stability of the financial system . . . by supporting such objectives as more transparent accounting and disclosure practice and working to improve the financial literacy and competence of investors."

where X_t is an n_X-vector of predetermined variables, x_t is an n_x-vector of non-predetermined variables, i_t is an n_i-vector of instruments, and ε_t is an n_ε-vector of exogenous zero-mean i.i.d. shocks. The matrices A, B, C, and H are of dimension $(n_X+n_x)\times(n_X+n_x)$, $(n_X+n_x)\times n_i$, $n_X\times n_\varepsilon$, and $n_x\times n_x$, respectively. For any vector z_t, $z_{t+1|t}$ denotes the rational expectations $\mathrm{E}_t z_{t+1}$.

It is practical to partition A and B conformably with X_t and x_t,

$$A\equiv\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix},\qquad B\equiv\begin{bmatrix}B_1\\ B_2\end{bmatrix}.$$

Under this format, the system includes nine predetermined variables, twenty non-predetermined variables, two shocks, and one instrument. Appendix 1 includes the detail on how to present the GS model into the canonical format (23).

3. Optimal Policy under Commitment

Let the intertemporal loss function in period 0 be

$$\mathrm{E}_0\sum_{t=0}^{\infty}(1-\delta)\delta^t L_t, \tag{24}$$

where

$$L_t=\frac{1}{2}[\pi_t^2+\lambda\widehat{y}_t^2+\nu(i_t-i_{t-1})^2]. \tag{25}$$

This is a flexible inflation-targeting loss function where $\delta(0<\delta<1)$ denotes the constant discount factor, $\lambda>0$ a relative weight on output-gap variability, and $\nu>0$ a relative weight on interest rate smoothing.

Consider minimizing (24) under commitment once and for all in period $t=0$, subject to (23) for $t\geq 0$ and $X_0=\overline{X}_0$ where $\overline{X}_0$ is given. The Lagrangian of the dynamic optimization can be written as

$$\mathcal{L}_0=\mathrm{E}_0\sum_{t=0}^{\infty}(1-\delta)\delta^t\begin{bmatrix}L_t+\Xi_t'(Hx_{t+1}-A_{21}X_t-A_{22}x_t-B_2i_t)\\ +\xi_{t+1}'(X_{t+1}-A_{11}X_t-A_{12}x_t-B_1i_t-C\varepsilon_{t+1})\end{bmatrix}$$
$$+\frac{1-\delta}{\delta}\xi_0'(X_0-\overline{X}_0), \tag{26}$$

where ξ_{t+1} and Ξ_t are vectors of n_X and n_x Lagrange multipliers of the upper and lower blocks, respectively, of the canonical system (23).

Following Svensson (2006), I solve this problem by the recursive saddlepoint method of Marcet and Marimon (1998). Appendix 2 provides an algorithm on how to solve this problem using the recursive saddlepoint method. It can be shown that the solution, or the commitment equilibrium, takes the form of the following policy function and law of motion of the state of the economy:

$$\begin{bmatrix} x_t \\ i_t \\ \gamma_t \end{bmatrix} = F \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix}$$

$$\begin{bmatrix} X_{t+1} \\ \Xi_t \end{bmatrix} = M \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix} + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}. \tag{27}$$

Thus, it follows that the instruments and the non-predetermined variables depend on the state of the economy according to

$$i_t = F_i \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix} \tag{28}$$

$$x_t = F_x \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix}, \tag{29}$$

where F_i and F_x are appropriate rows in F.

4. Should the Central Bank Respond to Asset Prices? The Full-Information Case

As shown in (28), the reaction function of the central bank can be described as

$$i_t = F_i \widetilde{X}_t. \tag{30}$$

Since q_t, the asset price, is non-predetermined, under the commitment equilibrium, q_t satisfies

$$q_t = F_q \widetilde{X}_t,$$

where F_q is an appropriate row vector in F_x.

It follows that $q_t - F_q\widetilde{X}_t = 0$ and thus

$$i_t = F_i\widetilde{X}_t + \theta(q_t - F_q\widetilde{X}_t), \quad \text{for any } \theta.$$

Or rearranging further,

$$i_t = (F_i - \theta F_q)\widetilde{X}_t + \theta q_t. \tag{31}$$

Thus, one can choose any feedback coefficient on q_t and (31) is still satisfied. In other words, in the commitment equilibrium, it is irrelevant to discuss whether the interest rate instrument should be set to respond directly to asset prices. The reason is that asset prices are fully determined by the state of the economy.[8] Hence, if the central bank can perfectly observe the state of the economy and has already set the interest rate instrument to respond to the state of the economy as in (30), there will be no additional gain from responding to asset prices directly. Specifically, in an environment with full information in which the central bank can perfectly observe the state of the economy, all of the information necessary for the monetary policymaking process has already been contained in the state of the economy. Movements in asset prices do not reveal any extra information.

Table 3 reports the coefficients of (30). These are the feedback coefficients on the state of the economy, as represented by the predetermined variables X_t as well as the Lagrange multipliers on the non-predetermined block of (23), Ξ_{t-1}.

The interpretation of the predetermined variables X_t is straightforward. The Lagrange multipliers can be interpreted as "promises" of the central bank to deliver the level of non-predetermined variables that it has committed from the previous period. To derive the promise for a specific variable, the promise terms in the dual-period loss function (38) need to be rearranged so that all terms with the variable in question are grouped together.[9] For instance, the promise on the level of inflation will be $-\frac{\Xi_{1t-1}}{\beta} + \frac{\Xi_{9t-1}}{\beta} + \Xi_{3t-1}$. $-\frac{\Xi_{1t-1}}{\beta} + \frac{\Xi_{9t-1}}{\beta} + \Xi_{3t-1} > 0$, for instance, implies that the central

[8] In this framework, the state of the economy consists of the predetermined variables and the Lagrange multipliers of the non-predetermined equations.

[9] This is because most non-predetermined equations have expectations terms on more than one variable.

Table 3. The Feedback Coefficients of the Reaction Function

Variables	Optimal Feedback Coefficients
k_t	−0.0997
s_{t-1}	0
n_t	0
π_{t-1}	0
ε_t	0.0099
d_t	0.4196
k_t^*	0.0898
i_{t-1}	0.1165
mc_{t-1}	0
Ξ_{1t-1}	0.3640
Ξ_{2t-1}	0
Ξ_{3t-1}	−0.1125
Ξ_{4t-1}	−0.0893
Ξ_{5t-1}	0
Ξ_{6t-1}	0
Ξ_{7t-1}	0
Ξ_{8t-1}	0
Ξ_{9t-1}	0.3155
Ξ_{10t-1}	0
Ξ_{11t-1}	0
Ξ_{12t-1}	0
Ξ_{13t-1}	0
Ξ_{14t-1}	0
Ξ_{15t-1}	0
Ξ_{16t-1}	0
Ξ_{17t-1}	0
Ξ_{18t-1}	0
Ξ_{19t-1}	0
Ξ_{20t-1}	0

bank made a promise in the previous period that today it will deliver an inflation rate lower than if it did not make the promise, no matter how the state of the economy is realized today. This is a hallmark of optimal policy under commitment in the sense that promises made in the past do constrain current policy in an optimal way.

Table 3 provides another interesting implication. As noted, in the full-information scenario in which the central bank can perfectly observe the state of the economy, there is no additional gain from responding to today's asset prices. However, it is possible to be optimal for the central bank to respond to *last period's* asset prices. The reason is that s_{t-1}, the external finance premium in the last period, is a predetermined variable. Last period's external finance premium is predetermined because it is a factor that determines *today*'s borrowing cost. Under the Townsend (1979)-style optimal loan contract, the borrowing cost in period t is determined in period $t-1$, whereby the lenders formed expectations on the state of the economy and, in particular, the possibility of defaults in period t. The lenders then marked up the borrowing cost over their opportunity cost to compensate for the default risk. Thus, the borrowing cost in the current period is in fact a predetermined variable that characterizes the state of the economy. s_{t-1} then enters X_t because it is a factor that determines the borrowing cost, $\mathrm{E}_{t-1} r_t^k$, in period t. That is why mc_{t-1} and π_{t-1}, the other factors that determine $\mathrm{E}_{t-1} r_t^k$, also enter X_t.

In this way, since movements in asset prices can affect the value of capital owned by entrepreneurs and thereby their net worth, which is the reason why the external finance premium can be affected by asset prices, it may be optimal for the central bank to respond to the lagged asset prices if there is a rationale for the bank to respond to last period's external finance premium.

Table 3, however, suggests that it may not be optimal for the central bank to respond to last period's external finance premium and thereby to the lagged asset prices. More precisely, the table implies that it is optimal for the central bank *not* to respond to the cost of borrowing, as evident from the fact that the feedback coefficients on all other factors that determine the borrowing cost, namely mc_{t-1} and π_{t-1}, are also zero.

Intuitively, the cost of borrowing for period t is determined solely by the expectations, which were formed in period $t-1$, on the state of the economy in period t. However, when the central bank is setting the interest rate instrument in period t, it has already had a chance to observe the realized state of the economy, including especially the productivity shock, in that period. Thus, the central bank can simply utilize the information that has already been observed in the period. The expectations on the state of the economy, which has already been realized and is fully observable, provide no additional useful information. In other words, it is inefficient for the central bank to respond to a (past) prediction of the outcome of an event when the outcome has already been known to the central bank.

5. Should the Central Bank Respond to Asset Prices? The Partial-Information Case

The analysis in the previous section provides a benchmark scenario in which policy should be conducted when the state of the economy is perfectly observable. This is, however, a hypothetical setup that is unlikely to be true in practice. In the real world, central banks will not be able to observe most economic variables, or aggregate economic activities, directly. In the policymaking process, central banks must rely on economic indicators that are constructed to reflect aggregate economic activities. Economic indicators, however, are not perfect measures of aggregate economic activities, since each economic indicator arguably is subject to some forms of errors.[10] This implies that real-world central banks are required to make interest rate decisions while facing imperfect knowledge about the state of the economy.

Thus, consider a scenario in which the state of the economy is imperfectly observable, as analyzed in Svensson and Woodford (2003). That is, some or all of the elements in X_t, x_t, and Ξ_{t-1} are assumed to be unobservable or partially observable. Furthermore, information is assumed to be symmetric in the sense that the private agents and the central bank have the same information set.

Let the vector of n_z observable variables, or indicators, Z_t, be given by

[10]Evidence has found that most economic indicators are subject to imperfection in the form of measurement errors and data revisions. Most notably among others, Orphanides (2001, 2003) has pointed out that in reality central banks need to make a decision based on error-prone data that usually are substantially revised even a year after their releases. For instance, the 1996 Boskin Commission report estimated that the U.S. CPI was upward biased by 1.1 percent per year. Most recently, following the release of the U.S. second-quarter GDP in 2006 were downward revisions to real growth in each of the past three years. Goldman Sachs calculated that the average adjustment to U.S. quarterly GDP growth between 1999 and 2004 was –0.4 percentage point.

$$Z_t = \widetilde{D} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + v_t, \tag{32}$$

where v_t, the vector of noise, is i.i.d. with mean zero and covariance matrix Σ_{vv}. Other setups remain the same as those in section 3.

Under this setting, Svensson and Woodford (2003) show that in the commitment equilibrium, certainty equivalence holds. That is, the optimal policy is the same as if the state of the economy were fully observable except that one responds to an efficient estimate of the state vector rather than to its actual value. Furthermore, a separation principle applies, according to which the selection of the optimal policy and the estimation of the current state of the economy can be treated as separate problems.

It can be shown that in the partial-information economy the central bank can set the interest rate instrument to respond to an infinite sum of current and lagged indicators, according to the following reaction function:

$$i_t = \widetilde{F}\left(\sum_{\tau=0}^{\infty} Q_\tau K Z_{t-\tau}\right) + \Phi\left(\sum_{\tau=0}^{\infty} \Sigma^\tau S \sum_{s=0}^{\infty} Q_s K Z_{t-1-\tau-s}\right). \tag{33}$$

Appendix 3 provides an algorithm for computing Q, K, S, and Σ.

5.1 Optimal Weights on Indicators under Partial Information

The effect of incomplete information on the role of asset prices can be best shown by a quantitative exercise. In particular, I will derive in this section the optimal weights, or the optimal feedback coefficients, on current and lagged indicators as in (33) under the base-case calibration of the GS model, for a given set of indicators and a calibrated covariance Σ_{vv}.

Consider a scenario in which the central bank employs the following set of indicators:[11]

[11] My conclusions, however, do not depend on what variables are included in the indicator set. I examine the indicator set (34) as my baseline analysis because this setup is fairly general but at the same time allows me to get my message across in a sharp way. The results on other sets of indicators are available upon request.

$$Z_t = \left\{ \begin{matrix} k_t, n_t, \varepsilon_t, d_t, k_t^*, z_t, \pi_t, r_t^k, y_t, \text{mc}_t, q_t, \text{inv}_t, \\ h_t, k_{t+1}, n_{t+1}, y_t^*, h_t^*, c_t^*, k_{t+1}^*, \text{inv}_t^*, q_t^*, r_t^* \end{matrix} \right\}. \quad (34)$$

Thus, the central bank employs all variables that are necessary to compute every key aspect of the economy. Given that c_t can be inferred from y_t and inv_t, including c_t is redundant and thus it is excluded from the set. The same reason goes for s_t and $\widehat{y}_t$ since s_t can be inferred from r_t^k and r_t, and $\widehat{y}_t$ can be inferred from y_t and y_t^*. From a preliminary analysis, c_t, s_t, and $\widehat{y}_t$ receive zero weights when they are included in the indicator set.

It should be noted that the central bank cannot perfectly observe the fundamental values of assets. What the central bank can observe are the market values of assets, which are readily available from the financial markets. Thus, q_t in the indicator set denotes market prices of assets. The movements in q_t capture the movements in the fundamental values of assets as well as in the asset-price misalignments or bubbles.

In the base-case analysis, the variance of productivity growth and all of the hypothetical frictionless variables, or *noisy indicators*, is calibrated to 0.1.[12] This is consistent with the fact that, in general, real-world central banks cannot perfectly verify exogenous disturbances and hypothetical frictionless variables, such as potential output. Nonetheless, central banks often include their estimates of these variables in their monetary policymaking process.[13]

The variance of all other indicators, or *reliable indicators*, is taken to be 0.0001, or $\frac{1}{1000}$ of that of the noisy indicators. That is, these indicators are quite accurately measured but still contain some unavoidable noise. This setup is intended to simulate the real-world

[12] The main findings here do not depend on the calibration of the variance of noise to indicators. Section 6 provides a robustness test on whether the results in this paper remain valid for different assumptions on the variance.

[13] Orphanides and van Norden (2002), for example, point out that although it is necessary to estimate potential output and thus the output gap for the policymaking process, the reliability of such estimates in real time tends to be quite low. The August 18, 2006 issue of *Financial Times* also reports that following a series of data revisions, economists may have overestimated the growth rate of potential output by as much as 60 basis points.

situation in which most economic data, including those used as the reliable indicators in (34), are subject to errors.[14]

5.2 Leaning against the Wind by Employing Asset Prices as an Indicator

Table 4 reports the optimal feedback coefficients on the current values of the indicators in (34) and those on their lags up to two periods, under the base-case calibration in which the variance of the noise to the noisy indicators is 0.1 and that to the reliable indicators is 0.0001.[15]

As evident from the table, when the central bank cannot observe the state of the economy perfectly, the feedback coefficients on asset prices become strictly different from zero. In this sense, table 4 implies that when the central bank cannot perfectly verify the state of the economy, the central bank should take asset prices into account in determining the path of the interest rate instrument. Row 11 of the table also implies that asset prices in the current period are relatively more informative than lagged asset prices.

As a basis for comparison, table 5 provides the optimal feedback coefficients under a hypothetical scenario in which the variance of the noise to the noisy indicators, including shocks and frictionless variables, is assumed to be equal to that of the reliable indicators, or equal to 0.0001.

Tables 4 and 5 show that when it becomes more uncertain to verify the state of the economy, particularly productivity growth and the frictionless variables, it is optimal for the central bank to utilize more information revealed by the more reliable indicators, including asset prices. In particular, the weights given to asset prices rise further, compared with the scenario when all indicators are relatively accurate.

[14]Economic indicators, by default, should be subject to at least some small amounts of errors. The reason is that most economic indicators are constructed from survey data which, by construction, are subject to sampling errors. Those indicators taken from the national income account or the balance of payments are also error prone, in the form of statistical discrepancies due to data collection, data entry, and unreported activities. Finally, even asset prices observed in the markets—such as equity prices, house prices, and foreign exchange rates—are considered noisy if what central banks really need to measure are the fundamental values of these assets.

[15]Note that in the table $y\mathrm{E} - 0Z$ denotes $y \cdot 10^{-Z}$.

Table 4. Optimal Feedback Coefficients on a Distributed Lag of the Indicators when the Variance of the Noisy Indicator is 0.1

Indicator	Current Period	One-Period Lag	Two-Period Lag
k_t	4.806E-03	2.485E-03	1.316E-03
n_t	4.458E-03	2.305E-03	1.221E-03
ε_t	−6.122E-02	−3.449E-02	−1.967E-02
d_t	6.126E-02	3.450E-02	1.967E-02
k_t^*	5.801E-04	1.196E-03	1.075E-03
z_t	3.040E-05	7.497E-06	4.625E-07
π_t	1.696E-03	3.320E-04	−5.654E-05
r_t^k	1.332E-03	3.230E-04	1.491E-05
y_t	−2.129E-03	−4.169E-04	7.100E-05
mc_t	1.942E-02	3.802E-03	−6.474E-04
q_t	1.837E-03	3.596E-04	−6.124E-05
inv_t	−1.825E-02	−3.574E-03	6.085E-04
h_t	9.604E-03	1.881E-03	−3.203E-04
k_{t+1}	−2.538E-02	−4.970E-03	8.463E-04
n_{t+1}	−2.355E-02	−4.610E-03	7.851E-04
y_t^*	2.164E-04	4.680E-04	4.229E-04
h_t^*	4.979E-05	1.077E-04	9.729E-05
c_t^*	1.268E-04	2.742E-04	2.478E-04
k_{t+1}^*	5.598E-04	1.211E-03	1.094E-03
inv_t^*	8.950E-04	1.935E-03	1.749E-03
q_t^*	8.631E-05	1.867E-04	1.687E-04
r_t^*	−1.268E-04	−2.742E-04	−2.478E-04

These results suggest that asset prices can provide information regarding the developments in the state of the economy. The more uncertainty surrounding the true state of the economy, the greater the role of asset prices should be in the rate-setting procedure. This is a reason why it is desirable for the central bank to respond to asset prices.

The present paper thus lends support to the lean-against-the-wind camp that the central bank should respond to movements in the market values of assets. According to the analysis in this paper, this is an optimal strategy that will prevent asset prices from persistently deviating from the trend and thus eliminating asset-price booms and busts.

Table 5. Optimal Feedback Coefficients on a Distributed Lag of the Indicators when All Variances Are Set to 0.0001

Indicator	Current Period	One-Period Lag	Two-Period Lag
k_t	1.197E-02	3.038E-03	2.226E-04
n_t	1.110E-02	2.818E-03	2.065E-04
ε_t	−1.708E-01	−4.248E-02	−2.744E-03
d_t	2.041E-01	4.884E-02	1.722E-03
k_t^*	1.305E-02	4.067E-03	9.743E-04
z_t	3.336E-02	6.360E-03	−1.022E-03
π_t	1.417E-03	2.201E-04	−8.246E-05
r_t^k	1.433E-03	2.700E-04	−4.645E-05
y_t	−1.779E-03	−2.763E-04	1.035E-04
mc_t	1.622E-02	2.520E-03	−9.442E-04
q_t	1.534E-03	2.383E-04	−8.930E-05
inv_t	−1.525E-02	−2.369E-03	8.874E-04
h_t	8.025E-03	1.246E-03	−4.670E-04
k_{t+1}	−2.121E-02	−3.294E-03	1.234E-03
n_{t+1}	−1.967E-02	−3.056E-03	1.145E-03
y_t^*	−7.993E-03	−9.027E-04	7.860E-04
h_t^*	−1.839E-03	−2.077E-04	1.808E-04
c_t^*	−4.683E-03	−5.289E-04	4.605E-04
k_{t+1}^*	−2.067E-02	−2.335E-03	2.033E-03
inv_t^*	−3.305E-02	−3.733E-03	3.250E-03
q_t^*	−3.187E-03	−3.600E-04	3.134E-04
r_t^*	4.683E-03	5.289E-04	−4.605E-04

5.3 Partial Information in the Credit Market

Among the reliable indicators, one may argue that some indicators may contain more noise than others. For instance, it may be more difficult for the central bank to observe the true values of the variables that reflect credit-market frictions, including net worth and the real return on capital. Thus, consider increasing the variance of these friction-related variables to 0.001. The variance of other reliable indicators, including capital stock, inflation, consumption, output, asset prices, investment, and labor supply, remains at 0.0001. The variance of productivity growth and the frictionless variables

Table 6. Optimal Feedback Coefficients on a Distributed Lag of the Indicators when the Variance of the Credit-Market-Friction-Related Indicators Is 0.001

Indicator	Current Period	One-Period Lag	Two-Period Lag
k_t	6.978E-03	3.641E-03	1.899E-03
n_t	6.473E-04	3.378E-04	1.761E-04
ε_t	−6.122E-02	−3.402E-02	−1.878E-02
d_t	6.126E-02	3.403E-02	1.878E-02
k_t^*	5.825E-04	9.600E-04	8.283E-04
z_t	4.102E-05	1.024E-05	4.740E-07
π_t	2.255E-03	4.373E-04	−9.439E-05
r_t^k	1.796E-04	4.402E-05	1.291E-06
y_t	−2.831E-03	−5.491E-04	1.185E-04
mc_t	2.582E-02	5.007E-03	−1.081E-03
q_t	2.442E-03	4.736E-04	−1.022E-04
inv_t	−2.427E-02	−4.706E-03	1.016E-03
h_t	1.277E-02	2.477E-03	−5.346E-04
k_{t+1}	−3.375E-02	−6.545E-03	1.413E-03
n_{t+1}	−3.131E-03	−6.072E-04	1.311E-04
y_t^*	2.132E-04	3.739E-04	3.259E-04
h_t^*	4.904E-05	8.602E-05	7.497E-05
c_t^*	1.249E-04	2.191E-04	1.910E-04
k_{t+1}^*	5.514E-04	9.671E-04	8.430E-04
inv_t^*	8.816E-04	1.546E-03	1.348E-03
q_t^*	8.502E-05	1.491E-04	1.300E-04
r_t^*	−1.249E-04	−2.191E-04	−1.910E-04

is still fixed at 0.1. This calibration is meant to capture a scenario in which the friction-related indicators are less accurately measured than other reliable indicators, but more accurately measured than the noisy indicators.

Table 6 provides the optimal feedback coefficients under this scenario. It is evident that the optimal feedback coefficient on asset prices becomes larger than those under the scenario in which credit-market frictions are perfectly observable. This implies that the role of asset prices, in the rate-setting process, should become even greater when there is uncertainty regarding the true state of the credit market.

This result also provides an implication on optimal policy responses prior to and during the subprime mortgage crisis in the United States, which began with housing bubbles during 2001–05, followed by the collapse of house prices and a sharp rise in home foreclosure since the summer of 2005. In this case, net worth, n_t in the model, can be interpreted as creditworthiness of U.S. households. Note also that s_t, the external finance premium, can be inferred from r_t^k, the real rate of return on capital; therefore, r_t^k can be interpreted as the U.S. households' risk premium. The result here thus suggests that if the central bank cannot verify the true state of creditworthiness and risk premiums of the U.S. households, movements in house prices will become an important indicator in predicting where the economy is heading. Under this condition, it would be optimal for the central bank to respond to movements in house prices particularly when the house prices *started* to rise in 2001 or *started* to fall in 2005.

5.4 Fundamental Values and Misalignments

Finally, notice that in every scenario under which optimal feedback coefficients are computed, asset prices are given larger weights than other price variables, such as inflation, but lower weights than most real-sector indicators, including labor supply. Even the one-period lag of labor supply is given a roughly equal weight to the asset prices in the current period. This result may provide a rationale for the belief held by financial markets that the FOMC and the Federal Reserve staff may pay some serious attention to labor-market indicators, such as non-farm payroll, and for the fact that the markets too are highly sensitive to any surprises in the monthly release of non-farm payroll figures.[16]

[16] A recent example is following the release of non-farm payroll on November 3, 2006 in which the actual release fell short of the market expectation for 13,000 jobs, the S&P 500 index jumped immediately by more than five points. See also Ramchander, Simpson, and Chaudhry (2005). Non-farm payroll is considered to be the most important economic indicator in financial markets because its release is very timely; it is released seven days following the end of the month being reviewed. Furthermore, non-farm payroll reflects household income and thus consumers' spending power, which accounts for two-thirds of the economy's total output. This is why financial markets consider non-farm payroll to be most informative in helping forecast future economic activity.

The question is, are there any scenarios in which it may be rational for the central bank to attach more weights to asset prices in the rate-setting process? One possibility is if asset prices are in fact more accurately measured. To examine this scenario, I run the optimal filtering problem by lowering the variance of asset prices to 0.000001, or $\frac{1}{100}$ of that of labor supply. The variances of all other reliable indicators and the noisy indicators remain at 0.0001 and 0.1, respectively.

Thus, this scenario corresponds to a situation when asset prices contain less noise than labor-market indicators. Arguably, it will be sensible to motivate this scenario if certain conditions are met. After all, it is extremely difficult, if not impossible, for the central bank to accurately observe the aggregate labor-market activities. As such, the central bank needs to rely on a number of labor-market indicators, all of which are derived from surveys. Thus, by default, labor-market indicators always contain at least some measurement errors, which in this case are captured by the variance of 0.0001 attached to them. Asset prices, however, can be readily observed from the markets. If what the central bank really needs to observe is the fundamental values, asset-price data retrieved from the markets can be fairly accurate, especially in the times when there is no bubble, or when market prices truly reflect the fundamentals.

As is evident from table 7, when this condition is met and therefore asset prices contain smaller noise than other indicators, the weights on asset prices in the reaction function rise significantly. In particular, the optimal feedback coefficient on asset prices in the current period is 0.20, the largest among all indicators in any horizons.

This result suggests that the usefulness of asset prices in the rate-setting process depends on the degree to which the central bank understands the nature of asset-price misalignments, or bubbles. Movements in the market values of assets should be given a prominent role in the rate-setting process when the central bank can distinguish the fundamental values from the misalignments. Under the scenario that there is uncertainty regarding whether movements in the market values of assets are driven by misalignments, it would be optimal for the central bank to also rely on other economic indicators.

Table 7. Optimal Feedback Coefficients on a Distributed Lag of the Indicators when Asset Prices Are More Accurately Measured Than Other Reliable Indicators

Indicator	Current Period	One-Period Lag	Two-Period Lag
k_t	6.068E-03	7.075E-03	4.317E-03
n_t	5.629E-04	6.563E-04	4.005E-04
ε_t	−6.122E-02	−6.936E-02	−4.209E-02
d_t	6.126E-02	6.936E-02	4.209E-02
k_t^*	5.814E-04	1.023E-04	4.628E-07
z_t	3.397E-05	6.526E-06	−2.321E-06
π_t	1.848E-03	−3.636E-05	−4.398E-04
r_t^k	1.486E-04	2.600E-05	−1.219E-05
y_t	−2.321E-03	4.565E-05	5.522E-04
mc_t	2.116E-02	−4.163E-04	−5.036E-03
q_t	2.002E-01	−3.937E-03	−4.763E-02
inv_t	−1.989E-02	3.913E-04	4.733E-03
h_t	1.047E-02	−2.059E-04	−2.491E-03
k_{t+1}	−2.766E-02	5.442E-04	6.582E-03
n_{t+1}	−2.566E-03	5.048E-05	6.106E-04
y_t^*	2.156E-04	3.769E-05	1.096E-06
h_t^*	4.959E-05	8.671E-06	2.521E-07
c_t^*	1.263E-04	2.209E-05	6.421E-07
k_{t+1}^*	5.575E-04	9.749E-05	2.834E-06
inv_t^*	8.913E-04	1.559E-04	4.532E-06
q_t^*	8.596E-05	1.503E-05	4.370E-07
r_t^*	−1.263E-04	−2.209E-05	−6.421E-07

6. Robustness

6.1 Variance of Noise to Indicators

In the analysis in the previous section, I set the variance of noise to the reliable indicators to 0.0001. The main results here, however, do not depend on the assumptions on the variance of noise. For example, consider varying the variance of noise to the reliable indicators to 0.001 and 0.00001. Table 8 reports the optimal feedback coefficients on the current-period asset prices and labor supply in these two cases. It is evident that with different variance of noise,

Table 8. Robustness Test on the Variance of Noise to Reliable Indicators

Scenario	Noisy Productivity Growth	Noisy Credit Friction	Asset Prices More Accurate
Var = 0.001			
h_t	5.218E-03	6.938E-03	5.686E-03
q_t	9.977E-04	1.327E-03	1.087E-01
Var = 0.00001			
h_t	1.818E-02	2.418E-02	1.981E-02
q_t	3.476E-03	4.623E-03	3.789E-01

the main results in this paper remain valid. That is, when it is more difficult for the central bank to verify the true state of the economy, the optimal feedback coefficients on asset prices become larger. Furthermore, across different scenarios in which labor supply and asset prices are equally accurate, it is optimal for the central bank to pay more attention to labor supply. Asset prices become more important only when labor supply is more noisy.

The same is true for the relative accuracy between the reliable and the noisy indicators. In the base-case analysis, the variance of noise to the reliable indicators is set to $\frac{1}{1000}$ of that to the noisy indicators. Table 9 reports the results from the robustness test in which the ratio is varied to $\frac{1}{100}$ and $\frac{1}{10000}$. With different degrees of

Table 9. Robustness Test on the Relative Accuracy between Reliable and Noisy Indicators

Scenario	Noisy Productivity Growth	Noisy Credit Friction	Asset Prices More Accurate
Relative Accuracy $\frac{1}{100}$			
h_t	5.218E-03	6.956E-03	5.699E-03
q_t	9.977E-04	1.330E-03	1.090E-01
Relative Accuracy $\frac{1}{10000}$			
h_t	1.785E-02	2.360E-02	1.942E-02
q_t	3.414E-03	4.512E-03	3.714E-01

Table 10. Robustness Test on Stabilization Weights

Scenario	Noisy Productivity Growth	Noisy Credit Friction	Asset Prices More Accurate
Varying λ to 0			
h_t	2.363E-04	3.143E-04	2.576E-04
q_t	4.519E-05	6.009E-05	4.925E-03
Varying λ to 2			
h_t	9.628E-03	1.280E-02	1.049E-02
q_t	1.841E-03	2.448E-03	2.007E-01
Varying ν to 0			
h_t	1.185E-02	1.576E-02	1.291E-02
q_t	2.266E-03	3.013E-03	2.470E-01
Varying ν to 0.5			
h_t	8.160E-03	1.085E-02	8.893E-03
q_t	1.560E-03	2.075E-03	1.701E-01

the relative accuracy between the reliable and the noisy indicators, we still derive the same conclusions as in section 5.

6.2 Stabilization Weights

In the base-case analysis, I follow Rudebusch and Svensson (1999) by setting the stabilization weights on the output gaps and interest rate smoothing to 1 and 0.2. Nevertheless, this assumption does not affect the main findings in this paper. Table 10 reports the results from the robustness test on the stabilization weights. On rows 1 through 6, λ, the weight on the output gap, is varied to 0 and 2 while ν is fixed at 0.2. On rows 7 through 12, ν is varied to 0 and 0.5 while λ is kept constant at unity. It is evident from the table that the main findings in this paper remain valid that the optimal feedback coefficient on asset prices becomes strictly different from zero when there is uncertainty regarding the state of the economy. The more difficult it is for the central bank to observe the state of the economy, the larger the optimal feedback coefficient on asset prices is. Finally, labor supply receives larger weights than asset prices when both are equally accurate; the optimal feedback coefficient on asset prices becomes larger only when labor supply contains more noise.

7. Conclusions

This paper examines the role of asset prices in the commitment equilibrium. It can be shown that asset prices are fully determined by the state of the economy. Hence, in the full-information case whereby the central bank can observe and respond to the state of the economy directly, there will be no further gain from setting the interest rate instrument to respond to asset prices.

In reality, however, it is unlikely that the central bank can perfectly observe the state of the economy. The paper finds that under the partial-information case, asset prices should enter the bank's rate-setting process with a weight strictly different from zero. There is a gain from responding to asset-price movements, because asset prices can provide signals on the developments in the state of the economy. The gain from responding to asset-price movements nonetheless depends on the degree to which the central bank understands the nature of asset-price misalignments. When there is uncertainty regarding whether movements in the market values of assets are driven by misalignments, it is advisable for the central bank to also rely on other economic indicators, including labor-market indicators.

In my future research, I plan to extend the analysis in this paper in several directions. First, it would be interesting to compare the implications regarding the role of asset prices under the commitment equilibrium with those when the central bank does not have access to a commitment device and thus follows discretionary policy.

Second, the present paper assumes that information is symmetric in the sense that private agents and the central bank have the same information set. In practice, this may not be true, and some may argue that private agents are more informed. I plan to examine whether the main findings in this paper are robust to such informational structure.

Finally, financial markets and financial instruments can be modeled explicitly and incorporated into the framework. This will allow us to examine a wider range of questions, including, for instance, the implications of value-at-risk and low-probability, extreme events on the role of asset prices.

Appendix 1. Presenting the GS Model into the State-Space Format

The GS model (1), (2), (3), (4), (5), (6), (7), (9), (10), (11), (12), (13), (14), and (15)–(22) can be presented into the canonical system (23), by defining the following sets of predetermined and non-predetermined variables:

$$X_t = \{k_t, s_{t-1}, n_t, \pi_{t-1}, \varepsilon_t, d_t, k_t^*, i_{t-1}, \mathrm{mc}_{t-1}\} \tag{35}$$

$$x_t = \left\{ \begin{matrix} c_t, z_t, \pi_t, r_t^k, y_t, \mathrm{mc}_t, q_t, \mathrm{inv}_t, s_t, h_t, k_{t+1}, n_{t+1}, \widehat{y}_t, y_t^*, \\ h_t^*, c_t^*, k_{t+1}^*, \mathrm{inv}_t^*, q_t^*, r_t^* \end{matrix} \right\}. \tag{36}$$

The elements of the corresponding matrices A, B, and C are available upon request.

Note that a key to make the analysis of optimal policy in this paper work is to define k_{t+1} and n_{t+1} as non-predetermined variables. This classification, however, is not out of the ordinary. Remember that under the GS model, k_{t+1} and n_{t+1} are in fact determined in period t. When one solves a rational-expectations model on dynare or gensys, a convention in these programs is that variables dated t are always known at t.[17] Thus, to assemble the model into these programs, one needs to write k_{t+1} and n_{t+1} as k_t and n_t, or treat k_{t+1} and n_{t+1} in the same way as all other variables dated t.

Appendix 2. Solving Optimal Policy under Commitment

Notice that problem (26) is not recursive, because non-predetermined variables, x_t, depend on expected future non-predetermined variables Hx_{t+1}. Thus, the practical dynamic-programming method cannot be used directly.

Nonetheless, as pointed out in Svensson (2006), this problem can be solved using the recursive saddlepoint method of Marcet

[17] See Collard and Juillard (2003) for an introduction to dynare and Sims (2002) for the algorithm used in gensys.

and Marimon (1998) by introducing a fictitious vector of Lagrange multipliers, Ξ_{-1}, equal to zero,

$$\Xi_{-1} = 0. \tag{37}$$

Then, the discounted sum of the upper term in the Lagrangian can be written

$$\mathrm{E}_0 \sum_{t=0}^{\infty} (1-\delta)\delta^t [L_t + \Xi_t'(Hx_{t+1} - A_{21}X_t - A_{22}x_t - B_2 i_t)]$$
$$= \sum_{t=0}^{\infty} (1-\delta)\delta^t [L_t + \Xi_t'(-A_{21}X_t - A_{22}x_t - B_2 i_t) + \frac{1}{\delta}\Xi_{t-1}' Hx_t].$$

It follows that the loss function (25) can be rewritten in terms of the dual-period loss:

$$\widetilde{L}_t \equiv L_t + \gamma_t'(-A_{21}X_t - A_{22}x_t - B_2 x_t) + \frac{1}{\delta}\Xi_{t-1}' Hx_t, \tag{38}$$

where Ξ_{t-1} is a new predetermined variable in period t and γ_t is introduced as a new control. Ξ_{t-1} and γ_t are related by the dynamic equation,

$$\Xi_t = \gamma_t. \tag{39}$$

The optimal policy under commitment problem can then be reformulated as the recursive dual saddlepoint problem:

$$\max_{\{\gamma_t\}_{t\geq 0}} \min_{\{x_t, i_t\}_{t\geq 0}} \mathrm{E}_0 \sum_{t=0}^{\infty} (1-\delta)\delta^t \widetilde{L}_t$$

subject to (39) and

$$X_{t+1} = A_{11}X_t + A_{12}x_t + B_1 i_t + C\varepsilon_{t+1}. \tag{40}$$

Notice that the recursive dual saddlepoint problem is recursive where $\{x_t, i_t, \gamma_t\}$ are controls and $\{X_t, \Xi_{t-1}\}$ are predetermined. Here, we can use the standard solution for the linear quadratic regulator (LQR) problem.

Appendix 3. An Algorithm for Solving the Optimal Filtering Problem

Given that certainty equivalence holds and a separation principle applies, the optimal policy under commitment amid partial information satisfies

$$i_t = \widetilde{F} X_{t|t} + \Phi \Xi_{t-1} \tag{41}$$

$$x_{t|t} = G X_{t|t} + \Gamma \Xi_{t-1} \tag{42}$$

$$\Xi_t = S X_{t|t} + \Sigma \Xi_{t-1}, \tag{43}$$

where $\widetilde{F}$ and Φ, G and Γ, and S and Σ are appropriate matrices in F_i, F_x, and M, respectively, which are derived in section 3. $X_{t|t}$ denotes the central bank's estimate of the predetermined variables, given the information available in period t.

It should be noted that in this case, Ξ_{t-1} is the vector of the central bank's estimate of its promises from the past. Ξ_{t-1} is an estimate because it depends on the estimates of past predetermined variables,

$$\Xi_{t-1} = \sum_{\tau=0}^{\infty} \Sigma^{\tau} S X_{t-1-\tau|t-1-\tau}, \tag{44}$$

where (44) is derived from solving (43) backward.

Given that, the lower block of (23) implies

$$A_{21}(X_t - X_{t|t}) + A_{22}(x_t - x_{t|t}) = 0.$$

Substituting in (42) for $x_{t|t}$ leads to

$$x_t = G^1 X_t + G^2 X_{t|t} + \Gamma \Xi_{t-1}, \tag{45}$$

where

$$G^1 = -(A_{22})^{-1} A_{21}$$

$$G^2 = G - G^1.$$

Substituting (41) and (45) into the first row of (23) yields

$$X_{t+1} = T X_t + J X_{t|t} + \Psi \Xi_{t-1} + C \varepsilon_{t+1}, \tag{46}$$

where

$$T = A_{11} + A_{12}G^1$$
$$J = B_1\widetilde{F} + A_{12}G^2$$
$$\Psi = A_{12}\Gamma + B_1\Phi.$$

Equations (43) and (45)–(46) then describe the evolution of the predetermined and non-predetermined variables, X_t and x_t, once we determine the evolution of the estimates $X_{t|t}$ of the predetermined variables.

Note that (46) also implies

$$X_{t+1|t} = (T + J)X_{t|t} + \Psi\Xi_{t-1}. \tag{47}$$

Substituting (45) into (32), we obtain a measurement equation for an optimal filtering problem,

$$Z_t = LX_t + MX_{t|t} + \Lambda\Xi_{t-1} + v_t,$$

where

$$L = \widetilde{D}_1 + \widetilde{D}_2G^1$$
$$M = \widetilde{D}_2G^2$$
$$\Lambda = \widetilde{D}_2\Gamma$$

and $\widetilde{D}_1$ and $\widetilde{D}_2$ are appropriate matrices in $\widetilde{D}$.

The optimal linear prediction of X_t is then given by a Kalman filter,

$$X_{t|t} = X_{t|t-1} + K(Z_t - LX_{t|t-1} - MX_{t|t} - \Lambda\Xi_{t-1}). \tag{48}$$

Svensson and Woodford (2003) show that the Kalman gain matrix K is given by

$$K = PL^{'}(LPL^{'} + \Sigma_{vv})^{-1}, \tag{49}$$

where the matrix $P \equiv \text{Cov}\ [X_t - X_{t|t-1}]$ is the covariance matrix for the prediction errors $X_t - X_{t|t-1}$ and fulfills

$$P = T[P - PL^{'}(LPL^{'} + \Sigma_{vv})^{-1}LP]T^{'} + CC^{'}. \tag{50}$$

To find optimal weights on the indicators Z_t in the estimation of the state of the economy, use the prediction equation (48) to solve for $X_{t|t}$ to get

$$X_{t|t} = (I + KM)^{-1}[(I - KL)X_{t|t-1} - K\Lambda\Xi_{t-1} + KZ_t].$$

We can then use (43) and (47) to express the dynamic equation for $X_{t|t}$ in terms of $X_{t-1|t-1}$ and Ξ_{t-2},

$$X_{t|t} = (I + KM)^{-1}\{[(I - KL)(T + J) - K\Lambda S]X_{t-1|t-1} + [(I - KL)\Psi - K\Lambda\Sigma]\Xi_{t-2} + KZ_t\}.$$

Solving the system consisting of this equation and (43) backwards, $X_{t|t}$ can be expressed as the weighted sum of current and past indicators,

$$X_{t|t} = \sum_{\tau=0}^{\infty} Q_\tau KZ_{t-\tau},$$

where Q_τ is the upper-left submatrix of the matrix

$$\begin{bmatrix} (I + KM)^{-1}[(I - KL)(T + J) - K\Lambda S] & (I + KM)^{-1}[(I - KL)\Psi - K\Lambda\Sigma] \\ S & \Sigma \end{bmatrix}^{\tau}.$$

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