Discretion Rather Than Rules? When Is Discretionary Policymaking Better Than the Timeless Perspective?*

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Discretionary monetary policy produces a dynamic loss in the New Keynesian model in the presence of cost-push shocks. The possibility to commit to a specific policy rule can increase welfare. A number of authors since Woodford (1999) have argued in favor of a timeless-perspective rule as an optimal policy. The short-run costs associated with the timeless perspective are neglected in general, however. Rigid prices, relatively impatient households, a high preference of policymakers for output stabilization, and a deviation from the steady state all worsen the performance of the timeless-perspective rule and can make it inferior to discretion.

JEL Code: E5.

1. Introduction

Kydland and Prescott (1977) showed that rule-based policymaking can increase welfare. The timeless perspective proposed by Woodford (1999) represents a prominent modern form of such a rule in

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monetary policy analysis. It helps to overcome not only the traditional inflation bias in the sense of Barro and Gordon (1983a, 1983b) but also the stabilization bias, a dynamic loss stemming from cost-push shocks in the New Keynesian model as described in Clarida, Galí, and Gertler (1999). It is, however, associated with short-run costs that may be larger than the long-run gains from commitment.

After deriving a formal condition for the superiority of discretion over the timeless-perspective rule, this paper investigates the influence of structural and preference parameters on the performance of monetary policy both under discretion and the timeless perspective in the sense of Woodford (1999). Discretion gains relative to the timeless-perspective rule—i.e., the short-run losses become relatively more important—if the private sector behaves less forward looking or if the monetary authority puts a greater weight on output-gap stabilization. For empirically reasonable values of price stickiness, the relative gain from discretion rises with stickier prices. A fourth parameter which influences the relative gains is the persistence of shocks: Introducing serial correlation into the model only strengthens the respective relative performance of policies in the situation without serial correlation in shocks. In particular, we show conditions for each parameter under which discretion performs strictly better than the timeless-perspective rule.

Furthermore, the framework of short-run losses and long-run gains also allows explaining why an economy that is sufficiently far away from its steady state suffers rather than gains from implementing the timeless-perspective rule. In general, this paper uses unconditional expectations of the loss function as welfare criterion, in line with most of the literature. The analysis of initial conditions, however, requires reverting to expected losses conditional on the initial state of the economy because unconditional expectations of the loss function implicitly treat the economy’s initial conditions as stochastic. Altogether, in the normal New Keynesian model, all conditions for the superiority of discretion need not be as adverse as one might suspect—in particular, if the initial output-gap situation is taken into account.

The following section 2 presents the canonical New Keynesian model. Section 3.1 explains the relevant welfare criteria. The analytical solution in section 3.2 is followed by simulation results and a thorough economic interpretation of the performance of policies
under discretion and the timeless perspective. Section 3.4 completes the discussion of Woodford’s timeless perspective by looking at the effects of initial conditions before section 4 concludes.

2. New Keynesian Model

The New Keynesian or New Neoclassical Synthesis model has become the standard toolbox for modern macroeconomics. While there is some debate about the exact functional forms, the standard setup consists of a forward-looking Phillips curve, an intertemporal IS curve, and a welfare function. Following, e.g., Walsh (2003), the New Keynesian Phillips curve based on Calvo (1983) pricing is given by

\[ \pi_t = \beta E_t \pi_{t+1} + \alpha y_t + u_t \]  

with

\[ \alpha \equiv \frac{(1 - \zeta)(1 - \beta \zeta)}{\zeta}. \]  

\( \pi_t \) denotes inflation, \( E_t \) the expectations operator conditional on information in period \( t \), \( y_t \) the output gap, and \( u_t \) a stochastic shock term that is assumed to follow a stationary AR(1) process with AR parameter \( \rho \) and innovation variance \( \sigma^2 \). While the output gap refers to the deviation of actual output from natural or flexible-price output, \( u_t \) is often interpreted as a cost-push shock term that captures time-varying distortions from consumption or wage taxation or markups in firms’ prices or wages. It is the source of the stabilization bias. \( 0 < \beta < 1 \) denotes the (private sector’s) discount factor and \( 0 \leq \zeta < 1 \) is the constant probability that a firm is not able to reset its price in period \( t \). A firm’s optimal price depends on current and (for \( \zeta > 0 \)) future real marginal costs, which are assumed to be proportional to the respective output gap.\(^2\) Hence, \( \zeta \) and \( \alpha \) reflect the degree of price rigidity in this model, which is increasing in \( \zeta \) and decreasing in \( \alpha \).

\(^1\)Depending on the purpose of their paper, some authors directly use an instrument rule or a targeting rule without explicitly maximizing some welfare function.

\(^2\)In (1), the proportionality factor is set equal to 1.
The policymaker’s objective at an arbitrary time $t = 0$ is to minimize

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t L_t \quad \text{with} \quad L_t = \pi_t^2 + \omega y_t^2,$$

(3)

where $\omega \geq 0$ reflects the relative importance of output-gap variability in policymaker preferences. We assume zero to be the target value of both inflation and the output gap. While the former assumption is included only for notational simplicity and without loss of generality, the latter is crucial for the absence of a traditional inflation bias in the sense of Barro and Gordon (1983a, 1983b).

The New Keynesian model also includes an aggregate demand relationship based on consumers’ intertemporal optimization in the form of

$$y_t = E_t y_{t+1} - b(R_t - E_t \pi_{t+1}) + v_t,$$

(4)

where $R_t$ is the central bank’s interest rate instrument and $v_t$ is a shock to preferences, government spending, or the exogenous natural-rate value of output, for example.\(^3\) The parameter $b > 0$ captures the output-gap elasticity with respect to the real interest rate. Yet, for distinguishing between the timeless-perspective and discretionary solutions, it is sufficient to assume that the central bank can directly control $\pi_t$ as an instrument. Hence, the aggregate demand relationship can be neglected below.\(^4\)

2.1 Model Solutions

If the monetary authority neglects the impact of its policies on inflation expectations and reoptimizes in each period, it conducts monetary policy under discretion. This creates both the Barro and Gordon (1983a, 1983b) inflation bias for positive output-gap targets and the Clarida, Galí, and Gertler (1999) stabilization bias caused by cost-push shocks. To concentrate on the second source of dynamic losses

\(^3\) $v_t$ is generally referred to as a demand shock. But in this model, $y_t$ reflects the output gap and not output alone. Hence, shocks to the flexible-price level of output are also included in $v_t$. See, e.g., Woodford (2003, p. 246).

\(^4\) Formally, adding (4) as a constraint to the optimization problems below gives a value of zero to the respective Lagrangian multiplier.
in this model, a positive inflation bias is ruled out by assuming an output-gap target of zero in the loss function (3). Minimizing (3) subject to (1) and to given inflation expectations $E_t \pi_{t+1}$ results in the Lagrangian

$$\Lambda_t = \pi_t^2 + \omega y_t^2 - \lambda_t (\pi_t - \beta E_t \pi_{t+1} - \alpha y_t - u_t) \quad \forall \ t = 0, 1, 2, \ldots .$$

The first-order conditions

$$\frac{\partial \Lambda_t}{\partial y_t} = 2 \omega y_t + \alpha \lambda_t = 0$$

$$\frac{\partial \Lambda_t}{\partial \pi_t} = 2 \pi_t - \lambda_t = 0$$

imply

$$\pi_t = \frac{-\omega}{\alpha} y_t.$$  

(6)

If instead the monetary authority takes the impact of its actions on expectations into account and possesses an exogenous possibility to credibly commit itself to some future policy, it can minimize the loss function (3) over an enhanced opportunity set. Hence, the resulting commitment solution must be at least as good as the one under discretion. The single-period Lagrangian (5) changes to

$$\Lambda = E_0 \sum_{t=0}^{\infty} \beta^t \left[ (\pi_t^2 + \omega y_t^2) - \lambda_t (\pi_t - \beta \pi_{t+1} - \alpha y_t - u_t) \right].$$

(7)

This yields as first-order conditions

$$\frac{\partial \Lambda}{\partial y_t} = 2 \omega y_t + \alpha \lambda_t = 0, \quad t = 0, 1, 2, \ldots ,$$

$$\frac{\partial \Lambda}{\partial \pi_t} = 2 \pi_t - \lambda_t = 0, \quad t = 0,$$

$$\frac{\partial \Lambda}{\partial \pi_{t+1}} = 2 \pi_t - \lambda_t + \lambda_{t-1} = 0, \quad t = 1, 2, \ldots ,$$
implying

\[
\pi_t = \frac{\omega}{\alpha} y_t, \quad t = 0 \quad \text{and} \quad (8)
\]

\[
\pi_t = -\frac{\omega}{\alpha} y_t + \frac{\omega}{\alpha} y_{t-1}, \quad t = 1, 2, \ldots \quad (9)
\]

The commitment solution improves the short-run output/inflation trade-off faced by the monetary authority because short-run price dynamics depend on expectations about the future. Since the authority commits to a history-dependent policy in the future, it is able to optimally spread the effects of shocks over several periods. The commitment solution also enables the policymaker to reap the benefits of discretionary policy in the initial period without paying the price in terms of higher inflation expectations, since these are assumed to depend on the future commitment to (9). Indeed, optimal policy is identical under commitment and discretion in the initial period. Nevertheless, the equilibrium outcomes for inflation and the output gap under the two policy regimes differ, as they also depend on inflation expectations and thus the prevailing policy regime. In a recent paper, Dennis and Söderström (2006) compare the welfare gains from commitment over discretion under different scenarios.

However, the commitment solution suffers from time inconsistency in two ways: First, by switching from (9) to (6) in any future period, the monetary authority can exploit given inflation expectations and gain in the respective period. Second, the monetary authority knows at \( t = 0 \) that applying the same optimization procedure (7) in the future implies a departure from today’s optimal plan, a feature McCallum (2003, p. 4) calls “strategic incoherence.”

To overcome the second form of time inconsistency and thus gain true credibility, many authors since Woodford (1999) have proposed the concept of policymaking under the timeless perspective: The optimal policy in the initial period should be chosen such that it would have been optimal to commit to this policy at a date far in the past, not exploiting given inflation expectations in the initial period.\(^5\) This implies neglecting (8) and applying (9) in all periods, not just in \( t = 1, 2, \ldots \):

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\(^5\)Woodford (1999) compares this “commitment” to the “contract” under John Rawls’s veil of uncertainty. In a recent contribution, Loisel (2008) endogenizes
\[ \pi_t = \frac{-\omega}{\alpha} y_t + \frac{\omega}{\alpha} y_{t-1}, \quad t = 0, 1, \ldots \] (10)

Hence, the only difference to the commitment solution lies in the different policy in the initial period, unless the economy starts from its steady state with \( y_{-1} = 0 \). But since the commitment solution is by definition optimal for (7), this difference causes a loss of the timeless-perspective policy compared with the commitment solution. If this loss is greater than the gain from the commitment solution (COM) over discretion, rule-based policymaking under the timeless perspective (TP) causes larger losses than policy under discretion (DIS):

\[ L_{TP} - L_{COM} > L_{DIS} - L_{COM} \iff L_{TP} > L_{DIS}. \] (11)

The central aim of the rest of this paper is to compare the losses from TP and DIS.

2.2 Minimal State Variable (MSV) Solutions

Before we are able to calculate the losses under the different policy rules, we need to determine the particular equilibrium behavior of the economy, which is given by the New Keynesian Phillips curve (1) and the respective policy rule, i.e., DIS (6) or TP (10). Following McCallum (1999), the minimal state variable (MSV) solution to each model represents the rational-expectations solution that excludes bubbles and sunspots.

Under discretion, \( u_t \) is the only relevant state variable in (1) and (6):

\[ \pi_t = \beta E_t \pi_{t+1} + \alpha y_t + u_t \]
\[ \pi_t = -\frac{\omega}{\alpha} y_t, \]

the timeless perspective for certain calibrated parameter values following the reputation mechanism in Barro and Gordon (1983b).

\(^6\)Due to the history dependence of (10), the different initial policy has some influence on the losses in subsequent periods, too.

\(^7\)Without loss of generality but to simplify the notation, the MSV solutions are derived based on (1) without reference to (2). The definition of \( \alpha \) in (2) is substituted into the MSV solutions for the simulation results in section 3.3.
so the conjectured solution is of the form

\[ \pi_t,_{DIS} = \phi_1 u_t \]
\[ y_t,_{DIS} = \phi_2 u_t. \]

Since \( E_t \pi_{t+1} = \phi_1 \rho u_t \) in this case, the MSV solution is given by

\[ \pi_t,_{DIS} = \frac{\omega}{\omega(1 - \beta \rho) + \alpha^2} u_t \]
\[ y_t,_{DIS} = \frac{-\alpha}{\omega(1 - \beta \rho) + \alpha^2} u_t. \] (12) (13)

Under the timeless perspective, \( y_{t-1} \) and \( u_t \) are the relevant state variables from (1) and (10):

\[ \pi_t = \beta E_t \pi_{t+1} + \alpha y_t + u_t \]
\[ \pi_t = -\frac{\omega}{\alpha} y_t + \frac{\omega}{\alpha} y_{t-1}. \]

Hence, the conjectured solution becomes

\[ \pi_t,_{TP} = \phi_{11} y_{t-1} + \phi_{12} u_t \]
\[ y_t,_{TP} = \phi_{21} y_{t-1} + \phi_{22} u_t. \] (14) (15)

After some calculations, the resulting MSV solution is described by

\[ \pi_t,_{TP} = \frac{\omega(1 - \delta)}{\alpha} y_{t-1} + \frac{1}{\gamma - \beta(\rho + \delta)} u_t \]
\[ y_t,_{TP} = \delta y_{t-1} - \frac{\alpha}{\omega(\gamma - \beta(\rho + \delta))} u_t, \] (16) (17)

with \( \gamma \equiv 1 + \beta + \frac{\alpha^2}{\omega} \) and \( \delta \equiv \frac{\gamma - \sqrt{\gamma^2 - 4\beta}}{2\beta} \). Given these MSV solutions, we are now able to evaluate the relative performance of monetary policy under discretion and the timeless-perspective rule.

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8These calculations include a quadratic equation in \( \phi_{21} \), of which only one root, \( 0 < \delta < 1 \), is relevant according to both the stability and MSV criteria.
3. Policy Evaluation

3.1 Welfare Criteria

3.1.1 Unconditional Expectations

The standard approach to evaluate monetary policy performance is to compare average values for the period loss function—i.e., values of the unconditional expectations of the period loss function in (3), denoted as $E[L_t]$. We follow this approach for the analysis of the influence of preference and structural parameters mainly because it is very common in the literature\(^9\) and allows an analytical solution. However, it includes several implicit assumptions.

First, $\pi_t$ and $y_t$ need to be covariance stationary. This is not a problem in our setup since $u_t$ is stationary by assumption and $0 < \delta < 1$ is chosen according to the stability criterion; see footnote 8. Second, using unconditional expectations of $L_t$ implies treating the initial conditions as stochastic (see, e.g., King and Wolman 1999, p. 377) and thus averages over all possible initial conditions. Third, the standard approach treats all periods in the same way as $E[L_t] = E[L_{t+j}]$ for all $j \geq 0$. This may influence the precise parameter values for which DIS performs better than TP in section 3.3, but it only strengthens the general argument with respect to the influence of $\beta$, as will be shown below.

3.1.2 Conditional Expectations

At the same time, using unconditional expectations impedes an investigation of the effects of specific initial conditions and transitional dynamics to the steady state on the relative performance of policy rules. For this reason and to be consistent with the microfoundations of the New Keynesian model, Kim and Levin (2005), Kim et al. (2008), and Schmitt-Grohé and Uribe (2004) argue in favor of conditional expectations as the relevant welfare criterion. If future outcomes are discounted—i.e., $\beta < 1$—the use of conditional expectations...
expectations—i.e., $L$ in (3)—as welfare criterion implies that short-run losses from TP become relatively more important to the long-run gains compared with the evaluation with unconditional expectations.

Both concepts can be used to evaluate the performance of monetary policy under varying parameter values, and the results are qualitatively equivalent. Besides its popularity and analytical tractability, the choice of unconditional expectations as the general welfare measure has a third advantage: by implicitly averaging over all possible initial conditions and treating all periods the same, we can evaluate policies for all current and future periods and thus consider the policy problem from a “truly timeless” perspective in the sense of Jensen (2003), which does not bias our results in favor of discretionary policymaking. Only the analysis of the effects of different initial conditions requires reverting to conditional expectations.

3.2 Analytical Solution

In principle, the relative performance of DIS and TP can be solved analytically if closed-form solutions for the unconditional expectations of the period loss function are available. This is possible, since

$$L_i = E[L_{t,i}] = E[\pi_{t,i}^2] + \omega E[y_{t,i}^2], \quad i \in \{DIS, TP\} \quad (18)$$

from (3) and the MSV solutions in section 2.2 determine the unconditional variances $E[\pi_{t,i}^2]$ and $E[y_{t,i}^2]$. The MSV solution under discretion, (12) and (13) with $u_t$ as the only state variable and $E[u_t^2] = \frac{1}{1-\rho^2} \sigma^2$, gives the relevant welfare criterion

$$L_{DIS} = \left[ \frac{\omega}{\omega(1-\beta \rho) + \alpha^2} \right]^2 \frac{1}{1-\rho^2} \sigma^2 + \omega \left[ \frac{-\alpha}{\omega(1-\beta \rho) + \alpha^2} \right]^2 \frac{1}{1-\rho^2} \sigma^2$$

$$= \frac{\omega(\omega + \alpha^2)}{[\omega(1-\beta \rho) + \alpha^2]^2} \cdot \frac{1}{1-\rho^2} \sigma^2. \quad (19)$$

For the timeless perspective, the MSV solution (16) and (17) depends on two state variables, $y_{t-1}$ and $u_t$. From the conjectured solution in (14) and (15), we have
\[ E[\pi_{t, TP}^2] = \phi_{11}^2 E[y_{t-1}^2] + \phi_{12}^2 E[u_t^2] + 2\phi_{11}\phi_{12} E[y_{t-1}u_t] \]
\[ E[y_{t, TP}^2] = \phi_{21}^2 E[y_{t-1}^2] + \phi_{22}^2 E[u_t^2] + 2\phi_{21}\phi_{22} E[y_{t-1}u_t]. \] (20)

These two equations are solved and plugged into (18) in the appendix. The result is

\[ L_{TP} = \frac{2\omega(1-\delta)(1-\rho) + \alpha^2(1+\delta\rho)}{\omega(1-\delta^2)(1-\delta\rho)[\gamma - \beta(\delta + \rho)]^2} \cdot \frac{1}{1-\rho^2} \sigma^2. \] (21)

Hence, discretion is superior to the timeless-perspective rule if

\[ L_{DIS} < L_{TP} \iff \frac{\omega(\omega + \alpha^2)}{[\omega(1-\beta\rho) + \alpha^2]^2} < \frac{2\omega(1-\delta)(1-\rho) + \alpha^2(1+\delta\rho)}{\omega(1-\delta^2)(1-\delta\rho)[\gamma - \beta(\delta + \rho)]^2} \iff RL \equiv L_{TP}/L_{DIS} - 1 > 0. \] (22)

Equation (22) allows analytical proofs of several intuitive arguments: First, the variance of cost-push shocks \( \frac{1-\rho^2}{1-\rho^2}\sigma^2 \) affects the magnitude of absolute losses in (19) and (21) but has no effect on the relative loss \( RL \) because it cancels out in (22). Second, economic theory states that with perfectly flexible prices, i.e., \( \zeta = 0 \) and \( \alpha \to \infty \), the short-run Phillips curve is vertical at \( y_t = 0 \). In this case, the short-run output/inflation trade-off and hence the source of the stabilization bias disappears completely and no difference between DIS, COM, and TP can exist.

Third, if the society behaves as an “inflation nutter” (King 1997) and only cares about inflation stabilization—i.e., \( \omega = 0 \)—inflation deviates from the target value neither under discretion nor under rule-based policymaking. This behavior eliminates the stabilization bias because the effect of shocks cannot be spread over several periods. Shocks always enter the contemporaneous output gap completely, but they do not cause welfare losses for an inflation nutter. Furthermore, the initial conditions do not matter, since \( y_{t-1} \) receives a weight of 0 in (10) and no short-run loss arises. The last two statements are summarized in the following proposition.

**Proposition 1.** Discretion and Woodford’s timeless perspective are equivalent for
(i) perfectly flexible prices or
(ii) inflation-nutter preferences.

Proof.

(i) \( \lim_{\alpha \to \infty} RL = 0 \).
(ii) \( \lim_{\omega \to 0} RL = 0 \).

Finally, proposition 2 states that discretion is not always inferior to Woodford’s timeless perspective. If the private sector discounts future developments at a larger rate—i.e., \( \beta \) decreases—firms care less about optimal prices in the future when they set their optimal price today. Hence, the potential to use future policies to spread the effects of a current shock via the expectations channel decreases. Therefore, the loss from the stabilization bias under DIS, where this potential is not exploited (i.e., the long-run gains \( L_{DIS} - L_{COM} \)), also decreases with smaller \( \beta \), while the short-run costs from TP, \( L_{TP} - L_{COM} \), remain unaffected under rule (10). In the extreme case of \( \beta = 0 \), expectations are irrelevant in the Phillips curve (1) and the source of the stabilization bias disappears. If the reduction in the long-run gain is sufficiently large, conditions (11) and (22) are fulfilled.

Proposition 2. There exists a discount factor \( \beta \) small enough such that discretion is superior to Woodford’s timeless perspective as long as some weight is given to output stabilization and prices are not perfectly flexible.

Proof. \( RL \) is continuous in \( \beta \) because stability requires \( 0 \leq \delta, \rho < 1 \). Furthermore, \( \lim_{\beta \to 0} RL = \frac{[\alpha^2+2(1-\rho)\omega+(1+\rho)\omega](\alpha^2+\omega)}{(\alpha^2+2\omega)[\alpha^2+(1-\rho)\omega]} - 1 > 0 \) for \( \omega > 0 \land \alpha < \infty \).

In principle, (22) could be used to look at the influence of structural \( (\zeta, \rho) \) and preference \( (\beta, \omega) \) parameters on the relative performance of monetary policy under discretion and the timeless-perspective rule more generally.\(^{10}\) Unfortunately, (22) is too complex

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\(^{10}\)Please note that, conceptually, it would be nonsense to compare one policy over several values of a preference parameter. Here, however, we always compare two policies (DIS and TP), holding all preference and structural parameters constant.
Table 1. Parameter Values Used in the Benchmark Model and Common in the Literature

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\beta$</th>
<th>$\omega$</th>
<th>$\zeta$</th>
<th>$\alpha$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark Model</td>
<td>0.99</td>
<td>0.0625</td>
<td>0.8722</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>Range in the Literature</td>
<td>0.97–1</td>
<td>0.01–0.25</td>
<td>0.73–0.91</td>
<td>0.01–0.1</td>
<td>0–0.95</td>
</tr>
</tbody>
</table>

to be analytically tractable. Hence, we have to turn to results from simulations.

### 3.3 Simulation Results

Preference ($\beta, \omega$) and structural ($\zeta, \rho$) parameters influence the relative performance of monetary policy under discretion and the timeless-perspective rule. To evaluate each effect separately, we start from a benchmark model with parameter values presented in table 1 and then vary each parameter successively.

If one period in the model reflects one quarter, the discount factor of $\beta = 0.99$ corresponds to an annual real interest rate of 4 percent. Setting $\omega = 1/16$ implies an equal weight on the quarterly variances of annualized inflation and the output gap. For $\beta = 0.99$, $\zeta = 0.8722$ corresponds to $\alpha = 0.02$, the value used in Jensen and McCallum (2002) based on empirical estimates in Galí and Gertler (1999).\(^{11}\)

To put the benchmark model into perspective, table 1 also reports the range of parameter values commonly used or estimated in the literature. For example, Ljungqvist and Sargent (2004) calibrate their model to $\beta$ between 0.97 and 0.99.\(^{12}\) Furthermore, Rudebusch and Svensson (1999) justify the use of unconditional expectations as their welfare criterion with the notion that conditional expectations of the total loss function (3), scaled by $(1 - \beta)$, and unconditional expectations of the period loss function, $E[L_t]$, converge for $\beta \to 1$.

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\(^{11}\) $\zeta$ and $\alpha$ are linked through the definition of $\alpha$ in (2).

\(^{12}\) Note that Galí, Gertler, and López-Salido (2001) estimate discount factors in the Phillips curve as low as 0.91 for the euro area and 0.92 for the United States. The implied steady-state real interest rate of approximately 12 percent per annum for $\beta = 0.97$ is, however, substantially above the empirically observed rate of about 2 percent per annum in developed economies.
Figure 1. Variation of Discount Factor $\beta$, TP vs. DIS

(see also Dennis 2004). Hence, values between 0.97 and 1 are used for $\beta$ in the literature, while $\beta = 0.99$ represents the most common figure. The ranges for $\omega$ (0.01 to 0.25), $\alpha$ (0.01 to 0.1), and thus via (2) also $\zeta$ (0.73 to 0.91) are taken from the discussion of the literature in Walsh (2003, p. 527). Regarding the serial correlation of the cost-push shock, $\rho$, the literature covers a very broad range between 0 and 0.95 (see, e.g., Kuester, Müller, and Stötting 2007). Figures 1–5 in the remaining part of this section illustrate the parameter of the benchmark model as a dashed, vertical line and the range of parameters in the literature as a gray-shaded area.

3.3.1 Discount Factor $\beta$

Figure 1 presents the results for the variation of the discount factor $\beta$ as the loss from the timeless perspective relative to discretionary policy, $RL$. A positive (negative) value of $RL$ means that the loss from the timeless-perspective rule is greater (smaller) than the loss under discretion, while an increase (decrease) in $RL$ implies a relative gain (loss) from discretion.

The simulation shows that $RL$ increases with decreasing $\beta$; i.e., DIS gains relative to TP if the private sector puts less weight on the future. This pattern reflects proposition 2 in the previous section.
Figure 2. Variation of Discount Factor $\beta$ Using Conditional Expectations of Loss Function, TP vs. DIS

Since the expectations channel becomes less relevant with smaller $\beta$, the stabilization bias and thus the long-run gains from commitment also decrease in $\beta$, whereas short-run losses remain unaffected. In particular, DIS becomes superior to TP in the benchmark model for $\beta < 0.839$, but, e.g., with $\omega = 1$ already for $\beta < 0.975$.$^{13}$ Differentiating between the central bank’s and the private sector’s discount factor $\beta$ (see McCallum 2005; Sauer 2007) shows that the latter drives $RL$ because it enters the Phillips curve, while the former is irrelevant due to the use of unconditional expectations as the welfare criterion, as discussed in section 3.1. But since using the unconditional expectations of the loss function treats all periods the same and hence gives greater weight to future periods than actually valid for $\beta < 1$, this effect only strengthens the general argument.

This can be shown with the value of the loss function (3), $\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t L_t$, conditional on expectations at $t = 0$ instead of the unconditional expectations $E[L]$. As figure 2 demonstrates,

$^{13}$The threshold of $\beta = 0.975$ for $\omega = 1$ is still a rather small value, as it implies a steady-state real interest rate of approximately 10 percent per annum.
the general impact of $\beta$ on $RL$ is similar to figure 1.\textsuperscript{14} The notable difference is the absolute superiority of DIS over TP in our benchmark model, independently of $\beta$. In order to get a critical value of $\beta$ for which DIS and TP produce equal losses, other parameters of the benchmark model have to be adjusted such that they favor TP, e.g., by reducing $\omega$ as explained below. Hence, figure 2 provides evidence that the use of unconditional expectations does not bias the results toward lower losses for discretionary policy. For reasons presented in section 3.1, we focus on unconditional expectations in this section.

3.3.2 Output-Gap Weight $\omega$

In Barro and Gordon (1983b), the traditional inflation bias increases in the weight on the output gap, while the optimal stabilization policies are identical both under discretion and under commitment. In our intertemporal model without structural inefficiencies, however, the optimal stabilization policies are different under DIS and COM/TP. The history dependence of TP in (10) improves the monetary authority’s short-run output/inflation trade-off in each period because it makes today’s output gap enter tomorrow’s optimal policy with the opposite sign, but with the same weight $\omega/\alpha$ in both periods. Hence, optimal current inflation depends on the change in the output gap under TP, but only on the contemporaneous output gap under DIS. This way, rule-based policymaking eliminates the stabilization bias and reduces the relative variance of inflation and output gap, which is a prominent result in the literature.\textsuperscript{15}

The short-run costs from TP arise because the monetary authority must be tough on inflation in the initial period. These short-run costs increase with the weight on the output gap $\omega$.\textsuperscript{16} The long-run gains from TP are caused by the size of the stabilization bias and the importance of its elimination given by the preferences in the loss function. Equation (10) shows that increasing $\omega$ implies a softer policy on inflation today but is followed by a tougher policy tomorrow. Although the effect of tomorrow’s policy is discounted by the private sector with $\beta$, the size of the stabilization bias—i.e., the neglection of

\textsuperscript{14}The use of conditional expectations requires setting the initial conditions—i.e., $y_{-1}$ and $u_0$—to specific values. In figure 2, $y_{-1} = -0.02$ and $u_0 = 0$.

\textsuperscript{15}See, e.g., Dennis and Söderström (2006) and Woodford (1999).

\textsuperscript{16}The optimal output gap $y_t$ under DIS is decreasing in $\omega$; see equation (6).
the possibility to spread shocks over several periods—appears to be largely independent from $\omega$. However, the reduction in the relative variance of inflation due to TP becomes less important the larger the weight on the variance of the output gap in the loss function—i.e., the long-run gains from TP decrease in $\omega$.

In the benchmark model of figure 3, $RL$ initially decreases from 0 for $\omega = 0$ with increasing $\omega$.\(^{17}\) But for reasonable values of $\omega$—i.e., $\omega > 0.0009$ in the benchmark model—$RL$ increases in the preference for output stabilization since short-run costs increase and long-run gains decrease in the weight on the output gap (increasing $\omega$). DIS even outperforms TP for $\omega > 5.28$, which is an extraordinarily large value empirically, however.\(^ {18}\)

### 3.3.3 Price Rigidity $\zeta$

Proposition 1 states that DIS and TP are equivalent for perfectly flexible prices; i.e., $\zeta = 0$ or $\alpha \to \infty$, respectively. Increasing price  

\(^{17}\)Note the magnifying glass in figure 3.  

\(^{18}\) $RL$ may approach 0 again for $\omega \to \infty$, the (unreasonable) case of an “employment nutter.”
rigidity—i.e., increasing $\zeta$—has two effects: First, firms’ price setting becomes more forward looking because they have fewer opportunities to adjust their prices. This effect favors TP over DIS for increasing $\zeta$ because TP optimally incorporates forward-looking expectations. Second, more rigid prices imply a flatter Phillips curve, and thus the requirement of TP to be tough on inflation already in the initial period becomes more costly. Hence, the left-hand side of (11), the short-run losses from TP over DIS, increases. Figure 4 demonstrates that for $\zeta > 0.660$, the second effect becomes more important and for $\zeta > 0.959$, the second effect even dominates the first one.\(^\text{19}\)

Based on the discussion of the literature in Walsh (2003), common estimates for price rigidity lie within $\alpha \in [0.01; 0.10]$; i.e., $\zeta \in [0.909; 0.733]$. In this range, figure 4 shows that $RL$ increases with the firms’ probability of not being able to reset their price, $\zeta$, and exceeds 0 for $\zeta > 0.959$ or $\alpha < 0.002$.

\(^{19}\)Since the relationship between $\zeta$ and $\alpha$ given by equation (2) also depends on $\beta$, there is a qualitatively irrelevant and quantitatively negligible difference between varying the probability of no change in a firm’s price, $\zeta$, and directly varying the output-gap coefficient in the Phillips curve, $\alpha$.  

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**Figure 4. Variation of Degree of Price Rigidity $\zeta$, TP vs. DIS**

![Image of Figure 4](image-url)
3.3.4 Correlation of Shocks \( \rho \)

The analysis of the influence of serial correlation in cost-push shocks, \( \rho \), is more complex. \( L_{DIS} \) exceeds \( L_{TP} \) in the benchmark model with \( \rho = 0 \), and raising \( \rho \) ceteris paribus strengthens the advantage of TP as demonstrated in the solid line in figure 5. If shocks become more persistent, their impact on future outcomes increases, and thus TP gains relative to DIS because it accounts for these effects in a superior way. The long-run gains from TP dominate its short-run losses, and \( RL \) decreases with \( \rho \).

However, the relationship between \( \rho \) and \( RL \) is not independent of the other parameters in the model, while the relationships between \( RL \) and \( \beta \), \( \zeta \), and \( \omega \), respectively, appear to be robust to alternative specifications of other parameters. Broadly speaking, as long as \( L_{DIS} > L_{TP} \) for \( \rho = 0 \), varying \( \rho \) results in a diagram similar to the solid line in figure 5; i.e., \( L_{DIS} > L_{TP} \) for all \( \rho \in [0; 1) \), and \( RL \) decreases in \( \rho \).

On the contrary, an appropriate combination of \( \beta \), \( \zeta \), and \( \omega \) can lead to \( L_{DIS} \leq L_{TP} \) for \( \rho = 0 \). For example, the combination of a low discount factor with rigid prices and a high preference for output-gap stabilization—such as the values \( \beta = 0.97 \), \( \zeta = 0.91 \), and \( \omega = 0.25 \) reported from the literature in table 1—results in \( RL > 0 \).
In this case, a picture symmetric to the horizontal axis emerges, as shown by the dashed line in figure 5. That means that a higher degree of serial correlation only strengthens the dominance of either TP or DIS already present without serial correlation. Hence, serial correlation on its own does not seem to be able to overcome the result of the trade-off between short-run losses and long-run gains from TP implied by the other parameter values.

3.4 Effects of Initial Conditions

As argued in section 3.1, we have to use conditional expectations of $\mathcal{L}$ in (3) in order to investigate the effects of the initial conditions—i.e., the previous output gap $y_{-1}$ and the current cost-push shock $u_0$—on the relative performance of policy rules. Figure 6 presents the relative loss $\hat{RL} = \mathcal{L}_{TP}/\mathcal{L}_{DIS} - 1$ conditional on $y_{-1}$ and $u_0$ in the range between $-0.05$ and $0.05$ from different viewpoints.

Starting from the steady state with $y_{-1} = u_0 = 0$ where $\hat{RL} = -0.0666$ in the benchmark model, increasing the absolute value of the initial lagged output gap $|y_{-1}|$ increases the short-run cost from following TP instead of DIS and leaves long-run gains unaffected: While $\pi_{0,DIS} = y_{0,DIS} = 0$ from (12) and (13), $\pi_{0,TP}$ and $y_{0,TP}$ deviate from their target values as can be seen from the history dependence of (10) or the MSV solution (16) and (17). Hence, TP becomes suboptimal under conditional expectations for sufficiently large $|y_{-1}|$. Note also that this short-run cost is symmetric to the steady-state value $y_{-1} = 0$ (see figures 6A and 6B).

If in addition to $|y_{-1}| > 0$ a cost-push shock $|u_0| > 0$ hits the economy, the absolute losses under both DIS and TP increase. Since TP allows an optimal combination of the short-run cost from TP, the inclusion of $|y_{-1}| > 0$ in (10), with the possibility to spread the

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20 For parameter combinations that result in $L_{DIS}$ in the neighborhood of $L_{TP}$ for $\rho = 0$, increasing $\rho$ has hardly any influence on $\hat{RL}$, but for high degrees of serial correlation from about $\rho > 0.8$, $\hat{RL}$ increases rapidly.

21 This shows that the results in McCallum and Nelson (2004, p. 48), who only report the relationship visible from the solid line in figure 5, do not hold in general.

22 Figure 6A offers the complete three-dimensional perspective of $\hat{RL}$ with an additional plane marking $\hat{RL} = 0$; figure 6B turns to the $y_{-1} - \hat{RL}$ perspective and figure 6C to the $u_0 - \hat{RL}$ perspective; figure 6D provides the birds-eye view, with the shaded area signaling $\hat{RL} < 0$. 

impact of the initial shock $|u_0| > 0$ over several periods, a larger shock $u_0$ alleviates the short-run cost from TP. Hence, the relative loss $\hat{RL}$ from TP decreases in $|u_0|$ for any given $|y_{-1}| > 0$ (see figures 6A and 6C).

However, this effect is weaker the closer $|y_{-1}|$ is to 0, as can be seen from the lines in figure 6C: the smaller $\hat{RL}$ is at $u_0 = 0$ because of a smaller given $|y_{-1}|$, the less bent is the respective line depicting $\hat{RL}$. If $y_{-1} = 0$, the size of $|u_0|$ has no influence on $\hat{RL}$ anymore since DIS and TP do not differ in $t = 0$.\(^{23}\) In this case, $\hat{RL}$ is parallel

\(^{23}\)To be precise, the policy “rules” (6) and (10) do not differ in $t = 0$, but the losses differ because of the more favorable output/inflation trade-off through the impact of TP on $E_0\pi_1$ in (1). This benefit of TP is part of the long-run gains, however, because it is also present under COM.
to the $u_0$-axis. While $u_0$ still influences the absolute loss values $L$ under both policies and how these losses are spread over time under TP, it has no influence on the relative gain from TP as measured by $RL$, which is solely determined by the long-run gains from TP for $y_{-1} = 0$.

The shaded area in figure 6D summarizes the previous information, as it illustrates all combinations of $u_0$ and $y_{-1}$ for which $RL < 0$. DIS is superior to TP for all other combinations of $u_0$ and $y_{-1}$.

Note that $RL$ is symmetric both to $y_{-1} = 0$ for any given $u_0$ and to $u_0 = 0$ for any given $y_{-1}$. Under DIS, $y_{-1}$ has no impact because (6) is not history dependent and $u_0$ only influences the respective period loss $L_0$, which is the weighted sum of the variances $\pi_0^2$ and $y_0^2$. Hence, $L_{DIS}$ is independent of $y_{-1}$ and symmetric to $u_0 = 0$.

Under TP, however, the history dependence of (9) makes $y_{-1}$ and $u_0$ influence current and future losses. While the transitional dynamics differ with the relative sign of $u_0$ and $y_{-1}$, the total absolute loss $L_{TP}$ does not for any given combination of $|y_{-1}|$ and $|u_0|$. If the economy was in a recession ($y_{-1} < 0$) in the previous period, for example, the price to pay under TP is to decrease $\pi_0$ through dampening $y_0$ with a ceteris paribus restrictive monetary policy that lowers aggregate demand.

**Scenario 1.** If additionally a negative cost-push shock $u_0 < 0$ hits the economy—i.e., with the same sign as $y_{-1} < 0$—this shock lowers $\pi_0$ further as the Phillips curve (1) is shifted downward from its steady-state locus. At the same time, $u_0 < 0$ increases $y_0$ ceteris paribus,\(^25\) brings $y_0$ closer to the target of 0, and thus reduces the price to pay under TP in the next periods $t = 1, \ldots$. The anticipation of this policy in turn lowers inflation expectations $E_0\pi_1$ compared with the steady state. Therefore, the Phillips curve shifts even further down and the output gap $y_0$ closes even more in the resulting equilibrium.

\(^{24}\)The following arguments run in a completely analogous manner for $y_{-1} > 0$ (see Sauer 2010).

\(^{25}\)Formally, partial derivatives of (16) and (17) with respect to both state variables $(y_{t-1}, u_t)$ show that both have the same qualitative effect on $\pi_t$ and an opposing effect on $y_t$: $\partial\pi_t/\partial y_{t-1} = \frac{\omega(1-\delta)}{\alpha} > 0$ and $\partial\pi_t/\partial u_t = \frac{1}{\gamma-\beta(\rho+\delta)} > 0$ while $\partial y_t/\partial y_{t-1} = \delta > 0$ and $\partial y_t/\partial u_t = \frac{-\alpha}{\omega(\gamma-\beta(\rho+\delta))} < 0$. 

Scenario 2. If, however, the initial cost-push shock $u_0$ is positive—i.e., of opposite sign to $y_{-1} < 0$—the transitional dynamics are reversed. The Phillips curve (1) is shifted upward. In contrast to scenario 1 with $u_0 < 0$, this reduces the negative impact of $y_{-1}$ on $\pi_0$ but increases $y_0$. Hence, the price to pay under TP in $t = 1$ is larger than in scenario 1, which in turn also lowers inflation expectations $E_0 \pi_1$ by more. The additional shift of the Phillips curve downward is thus larger than for $u_0 < 0$.

Figure 7 presents the discounted period losses under TP for both cases in the benchmark model. The behavior of the economy as described above causes a larger loss in the initial period for the first scenario with sign$(y_{-1}) = \text{sign}(u_0)$ compared with the case with sign$(y_{-1}) = -\text{sign}(u_0)$ because the expectations channel has a smaller impact; but it causes a reversal of the magnitude of losses for $t \geq 1$ because the price to pay for TP then is larger until the period loss converges to its unconditional value. Since the sum of the discounted losses, however, is equal in both scenarios, $L_{TP}$ is symmetric to $u_0 = 0$ given $y_{-1}$ and to $y_{-1} = 0$ given $u_0$.

To summarize, figure 6 presents the influences of the initial conditions on the relative performance of TP and DIS and the rest.
of this section provides intuitive explanations of the effects present in the model. \( \hat{RL} \) becomes positive—i.e., DIS performs better than TP—in the benchmark model for quite realistic values of the initial conditions, e.g., \( \hat{RL} > 0 \) for \( |y_{-1}| = 0.015 \) and \( |u_0| = 0.01 \). Hence, it may not be welfare increasing for an economy to switch from DIS to TP if it is not close to its steady state.

The previous sections 3.2 and 3.3 demonstrated, based on unconditional expectations as welfare criterion, the possibility that DIS outperforms TP for some rather extreme combinations of parameters. This section has highlighted three points: First, the use of conditional expectations as a welfare criterion may have important consequences for the evaluation of different policies (see also Kim et al. 2008; Schmitt-Grohé and Uribe 2004, for example). Second, the relevance of the results presented in this paper may be non-negligible in practice given that already small deviations from the steady state suffice to make discretion superior to the timeless-perspective rule. Third, other papers have shown that a timeless-perspective rule exists that is optimal for all combinations of parameters under unconditional expectations as welfare criterion (Blake 2001; Damjanovic, Damjanovic, and Nolan 2008; Jensen and McCallum 2002; Sauer 2007). However, this rule results in a diagram that is similar to figure 6 when its performance is evaluated for different initial conditions under conditional expectations (see Sauer 2007). For any timeless rule, initial conditions and hence the short-run costs can be sufficiently adverse to make the rule inferior to discretion.

4. Conclusion

This paper explores the theoretical implications of the timeless-perspective policy rule and discretionary policy under varying parameters in the New Keynesian model. With the comparison of short-run gains from discretion over rule-based policy and long-run losses from discretion, we have provided a framework in which to think about the impact of different parameters on monetary policy rules versus discretion. This framework allows intuitive economic explanations of the effects at work.

Already Blake (2001), Jensen (2003), and Jensen and McCallum (2002) provide evidence that a policy rule following the timeless
perspective can cause larger losses than purely discretionary modes of monetary policymaking in special circumstances. But none of these contributions considers an economic explanation for this rather unfamiliar result, let alone analyzes the relevant parameters as rigorously as this paper.

What recommendations for economic policymaking can be derived? Most importantly, the timeless perspective in its standard formulation is not optimal for all economies at all times. Considering each parameter separately, the critical values obtained in this paper require a lower discount factor, a greater degree of price rigidity, or a higher preference for output stabilization than calibrated or estimated in most of the literature. But if an economy features a combination of these characteristics and in particular a sufficiently large deviation from its steady state, the long-run losses from discretion may be less relevant than previously thought. In this case, discretionary monetary policy appears preferable over the timeless perspective.

In an overall laudatory review of Woodford (2003), Walsh (2005) argues that Woodford’s book “will be widely recognized as the definitive treatise on the new Keynesian approach to monetary policy.” He criticizes the book, however, for its lack of an analysis of the potential short-run costs of adopting the timeless-perspective rule. Walsh (2005) sees these short-run costs arising from incomplete credibility of the central bank. Our analysis has completely abstracted from such credibility effects and still found potentially significant short-run costs from the timeless perspective. Obviously, if the private sector does not fully believe in the monetary authority’s commitment, the losses from sticking to a rule relative to discretionary policy are even greater than in the model used in this paper. One way to incorporate such issues is to assume that the private sector has to learn the monetary policy rule. Evans and Honkapohja (2001) provide a convenient framework to analyze this question in more detail.

Appendix. Derivation of $L_{TP}$

The unconditional loss for the timeless perspective, equation (21), can be derived in several steps. The MSV solution (16) and (17)
depends on two state variables, \( y_{t-1} \) and \( u_t \). From the conjectured solution in (15), we have

\[
E[y_t^2] = \phi_{21}^2 E[y_{t-1}^2] + \phi_{22}^2 E[u_t^2] + 2\phi_{21}\phi_{22} E[y_{t-1}u_t].
\] (23)

\( E[y_{t-1}u_t] \) can be calculated from (15) with \( u_t = \rho u_{t-1} + \epsilon \) as

\[
E[y_{t-1}u_t] = E[(\phi_{21}y_{t-2} + \phi_{22}(\rho u_{t-2} + \epsilon_{t-1})) (\rho u_{t-1} + \epsilon_t)]
\]

\[
= E \left[ \phi_{21} \rho \underbrace{y_{t-2}u_{t-1}}_{=E[y_{t-1}u_t]} + \phi_{22} \left( \rho^2 \underbrace{u_{t-1}u_{t-2}}_{=\rho \sigma_u^2} + \rho \underbrace{u_{t-1} \epsilon_{t-1}}_{=\sigma^2} \right) \right] + 3 \cdot 0,
\] (24)

since the white-noise shock \( \epsilon_t \) is uncorrelated with anything from the past. Solving for \( E[y_{t-1}u_t] \) with \( \sigma_u^2 = \frac{1}{1-\rho^2} \sigma^2 \) gives

\[
E[y_{t-1}u_t] = \frac{\phi_{22}\rho}{1-\phi_{21}\rho} \cdot \frac{1}{1-\rho^2} \sigma^2.
\] (25)

Plugging this into (23), using \( E[y_t^2] = E[y_{t-1}^2] = E[y^2] \) and \( \phi_{21}, \phi_{22} \) from the MSV solution (17) leaves

\[
E[y^2] = \frac{1}{1-\phi_{21}^2} \left( \phi_{22}^2 + \frac{2\phi_{21}\phi_{22}^2}{1-\phi_{21}\rho} \right) \frac{1}{1-\rho^2} \sigma^2
\]

\[
= \frac{\alpha^2(1+\delta\rho)}{\omega^2(1-\delta^2)(1-\delta\rho)[\gamma-\beta(\delta+\rho)]^2} \cdot \frac{1}{1-\rho^2} \sigma^2.
\] (26)

From the conjectured solution in (14), we have

\[
E[\pi_t^2] = \phi_{11}^2 E[y_{t-1}^2] + \phi_{12}^2 E[u_t^2] + 2\phi_{11}\phi_{12} E[y_{t-1}u_t].
\] (27)

Combining this with the previous results and the MSV solution (16) results in

\[
E[\pi^2] = \frac{2(1-\rho)}{(1+\delta)(1-\delta\rho)[\gamma-\beta(\delta+\rho)]^2} \cdot \frac{1}{1-\rho^2} \sigma^2.
\] (28)

Hence, \( L_{TP} \) as the weighted sum of \( E[\pi^2] \) and \( E[y^2] \) is given by

\[
L_{TP} = \frac{2\omega(1-\delta)(1-\rho) + \alpha^2(1+\delta\rho)}{\omega(1-\delta^2)(1-\delta\rho)[\gamma-\beta(\delta+\rho)]^2} \cdot \frac{1}{1-\rho^2} \sigma^2.
\] (29)
References


