Discussion of "Limitations on the Effectiveness of Forward Guidance at the Zero Lower Bound"

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1. Introduction

What should the monetary authority do when prices are sticky? One answer to this question is provided by the standard New Keynesian model of sticky prices. It says that optimal monetary policy should fully stabilize the long-term price level in many environments. For example, if all fluctuations are in the "IS curve," then the optimal monetary policy under commitment is simply for the short-term interest rate to mirror the "natural rate of interest" and maintain a strict policy of price-level constancy. Output is then maintained close to the "natural rate."

However, when the monetary authority faces a zero lower bound on the nominal interest rate, then the committed monetary authority faces a different set of trade-offs: as stressed by Eggertsson and Woodford (2003), it is typically optimal for the monetary authority to plan a temporary period of inflation.

In this paper, Levin et al. (2009) (hereafter, LLNY) provide—among many other results—an important reminder that the optimal policy for the price level incorporates "base drift" whenever the zero lower bound is binding, with the long-run price level rising as a result of the inflation that is necessary as part of a desirable stabilization program.

In my discussion, I lay out the basic mechanics of this result and explain why it is somewhat surprising vis-à-vis the "case for price stability" that is present in this class of models.

I also relate the difference between these two policies to alternative monetary policy statements that might be issued by a monetary authority during a period of low interest rates so as to produce outcomes that are close to the optimal monetary policy under commitment.

I also identify two research questions that seem important to address based on the literature exemplified by LLNY and the statements of Federal Reserve policymakers. First, what is the nature of optimal policy if there is an exogenous constraint that the price level must be expected to return to an invariant long-run level within a specified number of years—five, for example—as part of a stabilization policy package? Bernanke (2003) has advocated an approach of "constrained discretion" with respect to the conduct of monetary policy, and this price-level constraint is a readily understandable one from the standpoint of the public. As a related matter, it is of interest to understand the cost of this constraint during a zero-bound episode to an optimizing policymaker under commitment. Second, Kohn (2009) suggests that the policies suggested by the "theoretical proscription" of the zero-bound literature—that a temporary period of high inflation is part of overcoming the floor on the nominal interest rate—have not been followed due to a concern about destabilizing long-term inflation expectations. I call for the development of models of the zero bound with imperfect credibility which can address this concern.

2. The Optimality of Price Stabilization

The analysis of LLNY starts with a linear-quadratic macroeconomic model of the variety described by Clarida, Galí, and Gertler (1999) and extensively developed by Woodford (2003). There are two key equations governing the relationship between inflation (π_t) and the output gap (x_t). First, the monetary authority seeks to maximize the objective,

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2+\lambda(x_t-\xi)^2\right],$$

where β is a discount factor, λ is a trade-off parameter, and ξ is a level of the output gap that a fully unconstrained monetary authority would select. The monetary authority is constrained by a forward-looking Phillips curve of the form

$$\pi_t=\beta E_t\pi_{t+1}+\kappa x_t,$$

where κ is a positive parameter.

In this setting, the optimal monetary policy can be determined as follows. One forms the Lagrangian,

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2+\lambda(x_t-\xi)^2\right]+\sum_{j=0}^{\infty}\beta^t\phi_t[\pi_t-\beta\pi_{t+1}-\kappa x_t],$$

where $\phi_t \geq 0$ is sometimes called the commitment multiplier. The first-order conditions imply

$$\pi_t = \phi_t - \phi_{t-1}$$
$$x_t = \xi - \frac{\kappa}{\lambda}\phi_t.$$

The first of these conditions has been augmented with a lagged multiplier at date 0 so that there is a time-invariant difference equation system.[1]

If monetary policy has been optimal for a long time or if the timeless perspective of Woodford (2003) is employed, then the constant policy consistent with these conditions and the Phillips curve is

$$\pi = 0$$
$$x = 0$$
$$\phi = \frac{\lambda}{\kappa}\xi.$$

That is, in the steady state, optimal policy is zero inflation in the absence of shocks. Further, since $\pi_t = P_t - P_{t-1}$ by definition, with P being the log price level, and since the inflation rate is related to the commitment multiplier via $\pi_t = \phi_t - \phi_{t-1}$, it follows that the long-run price level is $P = \phi$ (or differs by an inessential constant). That is, the optimal policy is to commit to a particular long-run price level.

[1]Generally, an optimal policy involves $\phi_{-1} = 0$ and an initial period of high inflation, which is sometimes called the "startup problem." However, my discussion is abstracting from that aspect of monetary policy until the final sections.

3. Shocks That Affect the Price Level

It is possible to add other shocks to the model without affecting the implication that there is an invariant long-run price level. For example, Clarida, Galí, and Gertler (1999) add a "cost-push shock" to the Phillips curve. If such a shock is added as an autoregressive process ($u_t = \rho u_{t-1} + e_t$), then the equilibrium solution for the multiplier is modified to

$$\phi_t - \phi = \mu(\phi_{t-1} - \phi) + \gamma u_t,$$

where μ is the stable eigenvalue of the dynamic system and γ is a separate parameter which is tied down by the analysis.

However, the efficiency condition for the monetary authority is not modified: it remains $\pi_t = \phi_t - \phi_{t-1}$. Accordingly, the price level in this model evolves as a stationary stochastic process:

$$P_t - P = \mu(P_{t-1} - P) + \gamma u_t.$$

That is, under optimal policy, shocks to the Phillips curve call for temporary movements in the price level, but do not themselves lead to permanent variations in the price level.

4. A "Fluctuating" Natural Rate of Interest

Notice that the above results are all derived without specification of interest rates, real or nominal, or any specification of monetary policy. Suppose next that the model is augmented with a "New Keynesian IS curve" that incorporates the "Fisher equation," so that

$$E_t x_{t+1} - x_t = \sigma\left[i_t - E_t\pi_{t+1} - r_t^n\right],$$

where i is the nominal interest rate, r^n is the natural rate of interest, and σ is a positive parameter. Let's abstract from all other sources of shocks, but assume—as in LLNY—that the natural real interest rate is given by the simple process

$$r_{t+1}^n = r^n + \rho\left(r_t^n - r^n\right).$$

This is an autoregressive process, but the procedure in LLNY is to look at a one-time shock at period 0 so that I do not add an error term. If the initial condition on the real interest rate, r_0, is positive, then the optimal inflation rate $\pi = 0$ is compatible with a nominal interest rate

$$i_t = r_t^n = r^n + \rho^t\left(r_0^n - r^n\right).$$

This simple finding is a consequence of the more general point that there is an interest rate path which can support a zero optimal inflation rate and output-gap series in the New Keynesian model. For small aggregate demand shocks, the optimal monetary policy is simply to mimic the movements in the natural rate of interest in general. In the current context of a one-time shock, the policy is simply to have a period of low nominal interest rate, mirroring the decline in the real interest rate.

5. The Zero Lower Bound

Now, suppose that the initial natural interest rate is quite negative. Then, the zero lower bound on the nominal interest rate is binding,

$$i_t \geq 0.$$

Accordingly, it is no longer possible for the central bank to obtain the zero-inflation solution. Instead, it faces the constraint,

$$i_t = \pi_{t+1} + r_t^n + \frac{1}{\sigma}[x_{t+1} - x_t] \geq 0,$$

so that its future inflation as well as current and future output gaps are tied together for some time. As with other studies in the literature, LLNY augment the Lagrangian above with an additional term, which we can write as

$$\sum_{j=0}^{\infty} \beta^t \theta_t [\pi_{t+1} + r_t^n + \frac{1}{\sigma}[x_{t+1} - x_t],$$

where the Lagrange multiplier θ_t will be positive during periods where the constraint is binding. Let's label this additional multiplier as the zero-lower-bound multiplier.

During the first period, the monetary authority chooses

$$\pi_t = \phi_t - \phi_{t-1}.$$

But during any later period when the zero lower bound is binding, then it will have also been in the prior period, so that the efficiency condition for inflation is modified to

$$\pi_t = \phi_t - \phi_{t-1} + \frac{1}{\beta}\theta_{t-1}$$

during such periods. Further, the behavior of efficient output is given by

$$\lambda(x_t - \xi) = -\alpha\phi_t - \frac{1}{\sigma}\theta_t + \frac{1}{\beta\sigma}\theta_{t-1}.$$

During those periods without a binding zero-lower-bound constraint, the zero-lower-bound multiplier is equal to zero so that inflation and output are governed by the same conditions as in the previous section. However, notice that both inflation and output should be higher—relative to their relationship to the change in the commitment multiplier ϕ—as the economy exits the zero lower bound. That is, in the formulae above, when $\theta_t = 0$ there is still a term involving θ_{t-1} that makes for expansion of output and inflation.

6. Inflation and Output during the Zero-Lower-Bound Period

When the zero lower bound is binding, then current output is given by

$$x_t = \sigma\left(\pi_{t+1} + r_t^n\right) + x_{t+1}$$

and inflation is given by

$$\pi_t = \kappa\left[\sigma\left(\pi_{t+1} + r_t^n\right) + x_{t+1}\right] + \beta\pi_{t+1}$$

in general. Thus, the demand shock $r_t^n < 0$ will reduce output and inflation, holding expectations fixed.

However, as Eggertsson and Woodford (2003) stress, a smaller output loss can be brought about by a policy that leads to higher real output in the future and higher inflation in the future. So, optimal commitment policy stabilizes current activity by manipulating expectations about future inflation and real activity. A policy of stimulating both real activity and inflation after the main effects of the natural rate shock wear away has the effect of stimulating current activity, because both the IS curve and the Phillips curve are forward looking.

7. Interest Rate Dynamics

The optimal policy developed in Eggertsson and Woodford (2003) can be illustrated in the LLNY setting. In particular, suppose that the difference equation above is consistent with it taking τ periods for the real natural rate of interest to become exactly zero:

$$r_\tau^n = \mu + \rho^\tau \left(r_0^n - \mu\right).$$

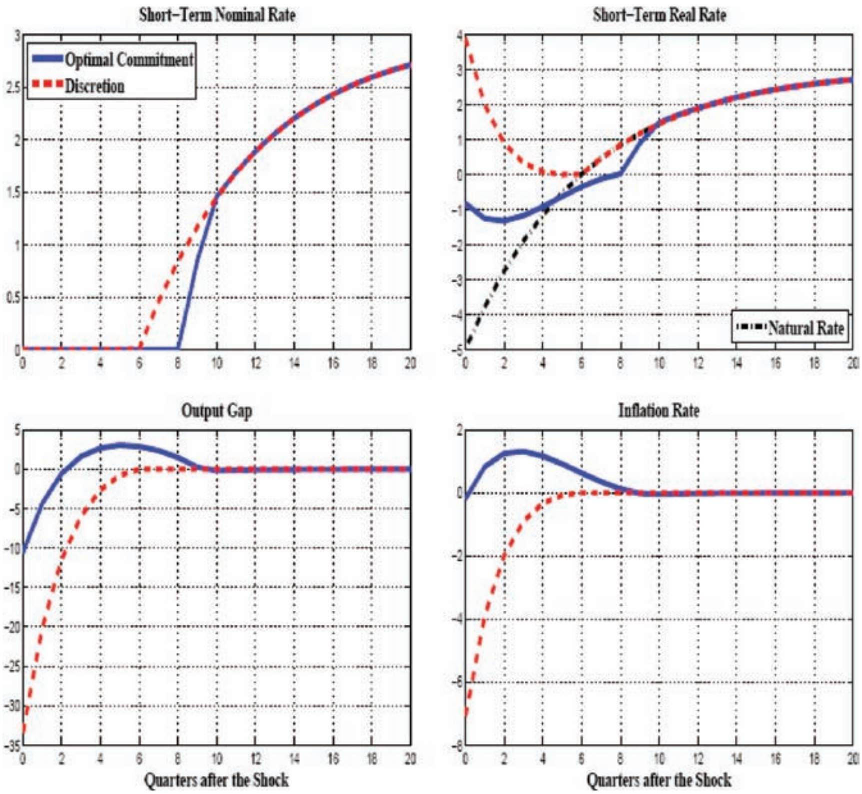
Then, the optimal policy is for the nominal rate to be zero for a larger number of periods, say m, because such a policy has the effect of being consistent with an optimal profile of inflation and real activity.

Intuitively, a policy of having positive inflation during the zero-bound interval allows the real interest rate to drop toward its natural level, thus cushioning the economy against the demand shock. However, the surprising result developed in Eggertsson and Woodford (2003) and illustrated in LLNY (figure 5 in their paper, reproduced below) is that the optimal policy involves staying at the zero lower bound after it is no longer necessarily a constraint (that is, at date τ when r_t^n is no longer negative: this looks to be about six quarters in the "Great Recession" case of LLNY). The zero lower bound is binding for periods τ through m because this is consistent with stimulating both inflation and output during that time, which also has the effect of raising real activity during earlier periods (0 to $\tau - 1$) during which the zero-lower-bound period was necessarily binding. In the LLNY experiment reproduced below, this interval of a zero interest rate looks to be about eight quarters.

8. Price-Level Dynamics

But is the temporary stimulation of the economy via higher inflation and output gaps consistent with a return to the long-run price level

Figure 5. "Great Recession"-Style Shock



Note: This simulation was performed using the baseline parameterization, $\beta = 0.9925$, $\kappa = 0.0024$, and $\sigma = 6$. The natural rate shock follows an AR(1) process with first-order autocorrelation $\rho = 0.85$. The short-term nominal interest rate, the short-term real interest rate, and the inflation rate are each expressed at annual rates in percent; the output gap is expressed in percentage points.
Source: Levin et al. (2010).

which would have obtained if no natural rate of interest shock had taken place?

To address this question, I suppose that the initial conditions on the multipliers are $\phi_{-1} = \phi$ and $\theta_{-1} = 0$. That is, before the shock, it is assumed that the economy has settled down to an initial zero rate of inflation, with the Phillips curve a binding constraint on its actions, and there was no zero bound in the prior period (-1).

As above, the inflation rate is related to the price level (in logarithms) via $\pi_t = P_t - P_{t-1}$. Combining this expression with the optimal policy formula ($\pi_t = \phi_t - \phi_{t-1} + \frac{1}{\beta}\theta_{t-1}$), we can determine that

$$P_t = \phi_t + \frac{1}{\beta}\sum_{j=0}^{t-1}\theta_j.$$

That is, in the zero-lower-bound case, the price-level path now depends on two multipliers, but in somewhat different ways. It depends on the Phillips-curve commitment multiplier, just as before, but it depends on the *sum* of the zero-bound multipliers from the onset of the plan until the current point in time.

The multiplier must be non-negative and it must be positive when the zero-lower-bound constraint is binding. Suppose that it is so for only m periods as in the discussion above ($m = 8$ in the LLNY "Great Recession" experiment; see their figure 6). Looking at the limit of this expression, as t goes to infinity, we have that

$$\lim_{t\to\infty} P_t = \lim_{t\to\infty}\phi_t + \sum_{j=0}^{n}\theta_j.$$

Accordingly, so long as the long-run value of the multiplier is ϕ, then the price level is permanently and positively affected by the optimal policy response to a zero lower bound.

The behavior of the price level just discussed is consistent with various figures in LLNY (as illustrated in figure 5). It does rely on the idea that the multiplier ϕ is invariant in the long run, but that is a clear implication of the stationarity requirements for the commitment optimum problem. In stochastic versions of this model, such as those studied in detail by Adam and Billi (2006, 2007) and Nakov (2008), simulations of the inflation rate also seem to have a positive average, so that the sort of "price-level base drift" found in the LLNY analysis appears to be a general property of this class of models.[2]

[2]Goodfriend (1987) discusses how interest rate smoothing can lead to price-level base drift, but that drift is of a symmetric form in contrast to the result with a zero lower bound on the nominal interest rate.

9. Policy Statements

To bring the matter into sharp relief, it is useful to imagine the types of policy announcements that a central bank might make to guide private agents about the process of inflation. If there were a policy of price-level targeting without base drift, then a statement might read as follows:

> In the face of credit market developments, a zero level of the interest rate will be maintained for some time, well beyond the period of current financial market turmoil. *Some higher inflation will arise after the initial deflationary stimulus, but the price level will return to the previous target path within five years' time.* Short-term interest rate settings will be compatible with this objective and with moderating declines in real economic activity. Some declines in real economic activity, however, will likely be necessary as consumption, investment, and employment are altered in response to changes in financial market conditions.

By contrast, the statement for the economy as analyzed by LLNY might read as follows:

> In the face of credit market developments, a zero level of the interest rate will be maintained for some time, well beyond the period of current financial market turmoil. *Some higher inflation will arise after the initial deflationary stimulus, but there will be no long-run change in the inflation rate after five years' time.* Short-term interest rate settings will be compatible with this objective and with moderating declines in real economic activity. Some declines in real activity, however, will likely be necessary as consumption, investment, and employment are altered in response to changes in financial market conditions.

The essence of optimal monetary policy during a zero-lower-bound period is managing the expectations of market participants about future policy and its implications for output and inflation, as Eggertsson and Woodford (2003) stress and LLNY remind us. So, "forward guidance" to market participants is essential during zero-lower-bound periods, but it seems that somewhat different guidance is necessary in the two settings. Just as in normal times, when the zero lower bound is not binding, it is important that the monetary authority make clear how its current choices are linked to its future actions, as well as to its desired path for nominal and real variables.

For central banks, such as the Bank of Canada, that are contemplating price-level targets, it is relevant that the optimal policy includes base drift. It would be desirable to quantify the losses from requiring that policy be expected to return to a constant price-level path within a specific period, such as five years.

10. Credibility

LLNY also remind us of an important point stressed by Adam and Billi (2006, 2007) and Nakov (2008): there are much larger output losses and a greater deflationary impulse if there is a discretionary rather than committed monetary authority. In a zero-lower-bound situation, the monetary authority can no longer use its traditional instrument interest rate policy, so that outcomes depend critically on whether it is able to bring about increases in expected inflation and expected future real activity.

For example, in their Great Recession shock displayed in their figure 5 (and reproduced in this paper), the loss of output on impact is about 10 percent under optimal policy, but it is a staggering 30 percent under discretion. The reason is that, under discretion, the current monetary authority cannot commit to follow stimulative policies in the future.

This gets to a subtle set of issues concerning credibility of monetary policymakers. In the standard analysis stemming from Kydland and Prescott (1977), a monetary authority wants private agents to believe that he *will not* produce inflation in the future (see Clarida, Galí, and Gertler 1999 and Woodford 2003 discussions of this point in the context of the New Keynesian model). In the current setting, the monetary authority wants private agents to believe that he *will* produce inflation. The common thread is that the monetary authority needs to have private agents believe that he will follow through on announced plans.

In discussing this general research topic, Vice Chairman Donald Kohn writes about Federal Reserve decisionmaking:

> We have not followed the theoretical prescription of promising to keep rates low enough for long enough to create a period of above-normal inflation. The arguments in favor of

> such a policy hinge on a clear understanding on the part of the public that the central bank will tolerate increased inflation only temporarily—say, for a few years once the economy has recovered—before returning to the original inflation target in the long term. In standard theoretical model environments, long-run inflation expectations are perfectly anchored. In reality, however, the anchoring of inflation expectations has been a hard-won achievement of monetary policy over the past few decades, and we should not take this stability for granted. Models are by their nature only a stylized representation of reality, and a policy of achieving "temporarily" higher inflation over the medium term would run the risk of altering inflation expectations beyond the horizon that is desirable. Were that to happen, the costs of bringing expectations back to their current anchored state might be quite high. (Kohn 2009)

From this perspective, it seems important to understand how a central bank should optimally design policy in a zero-lower-bound situation when it has concerns about the interplay between longer-term inflation expectations and its current policy actions.

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