Interbank Lending, Credit-Risk Premia, and Collateral*

Florian Heider and Marie Hoerova
European Central Bank

We study the functioning of secured and unsecured interbank markets in the presence of credit risk. The model generates empirical predictions that are in line with developments during the 2007–09 financial crisis. Interest rates decouple across secured and unsecured markets following an adverse shock to credit risk. The scarcity of underlying collateral may amplify the volatility of interest rates in secured markets. We use the model to discuss various policy responses to the crisis.

JEL Codes: G01, G21, E58.

1. Introduction

Interbank markets play a key role in the financial system. They are vital for banks' liquidity management. Secured, or repo, markets have been a fast-growing segment of money markets: they have doubled in size since 2002 with gross amounts outstanding of about $10 trillion in the United States and comparable amounts in the euro area just prior to the start of the crisis in August 2007. Since repo transactions are backed by collateral securities similar to those used in the central bank's refinancing operations, repo markets are a key part of the transmission of monetary policy. At the same time, the interest rate in the unsecured three-month interbank market acts

*We thank Douglas Gale, Rafael Repullo, Elu von Thadden, and seminar participants at the European Central Bank and the Federal Reserve Bank of New York (conference on "Pricing and Provision of Liquidity Insurance") for helpful comments. Dimitrios Rakitzis and Francesca Fabbri provided excellent research assistance. The views expressed do not necessarily reflect those of the European Central Bank or the Eurosystem. The authors are at the European Central Bank, Financial Research Division, Kaiserstrasse 29, D-60311 Frankfurt; E-mails: florian.heider@ecb.int and marie.hoerova@ecb.int.

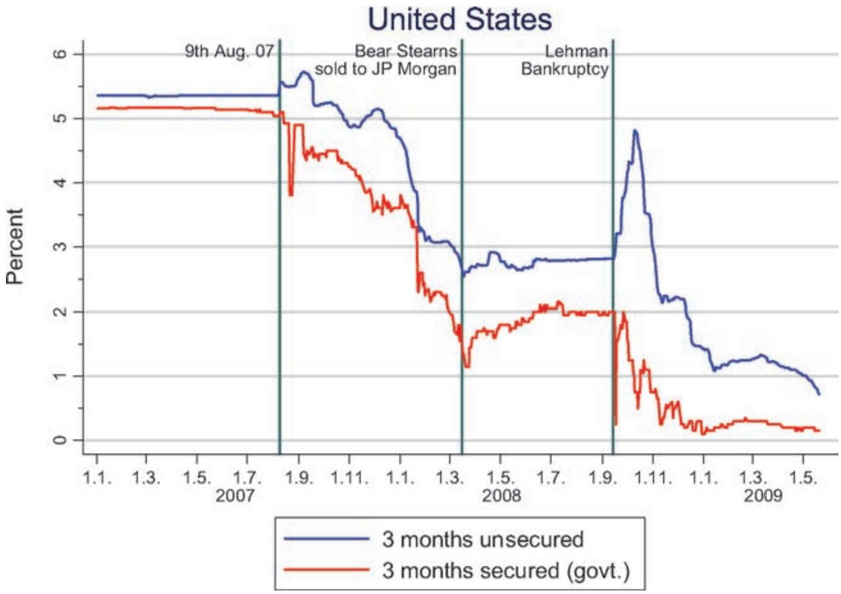
Figure 1. Decoupling of Secured and Unsecured Interbank Rates in the Euro Area

as a benchmark for pricing fixed-income securities throughout the economy.

In normal times, interbank markets function smoothly. Rates are broadly stable across secured and unsecured segments, as well as across different collateral classes. Since August 2007, however, the functioning of interbank markets has become severely impaired around the world. The tensions in the interbank markets have become a key feature of the 2007–09 crisis (see, for example, Allen and Carletti 2008, and Brunnermeier 2009).

One striking manifestation of the tensions in the interbank markets has been the decoupling of interest rates between secured and unsecured markets. Figure 1 shows the unsecured and secured (by government securities) three-month interbank rates for the euro area since January 2007. Prior to the outbreak of the crisis in August 2007, the rates were closely tied together. In August 2007, they moved in opposite directions, with the unsecured rate increasing and the secured rate decreasing. They decoupled again following the Lehman bankruptcy and, to a lesser extent, just prior to the sale of Bear Stearns.

Figure 2. Decoupling of Secured and Unsecured Interbank Rates in the United States



A second, related feature of the tensions in the interbank markets has been the difference in the severity of the disruptions in the United States and in the euro area. Figure 2 shows rates in secured and unsecured interbank markets in the United States. As in the euro area, there is a decoupling of the rates at the start of the financial crisis and a further divergence after the sale of Bear Stearns and the bankruptcy of Lehman. However, the decoupling and the volatility of the rates is more pronounced than in the euro area.

Why have secured and unsecured interbank interest rates decoupled? Why has the U.S. repo market experienced significantly more disruptions than the euro-area market? What underlying friction can explain these developments? And what policy responses are possible to tackle the tensions in interbank markets?

To examine these questions, we present a model of interbank markets with both secured and unsecured lending in the presence of credit risk. Credit risk and the accompanying possibility of default, stemming from the complexity of securitization, was at the heart of the financial crisis (see Gorton 2008, 2009, and Taylor 2009). We model the interbank market in the spirit of Bhattacharya and Gale (1987), who in turn build on Diamond and Dybvig (1983). Banks face liquidity demand of varying intensity. Some may need to realize cash quickly due to demands of customers who draw on committed lines of credit or on their demandable deposits. Since idiosyncratic liquidity shocks are noncontractible, this creates a scope for an interbank market where banks with excess liquidity trade with banks in need of liquidity.

Banks can invest in liquid assets (cash), illiquid assets (loans), and in bonds. In their portfolio choice, they face a trade-off between liquidity and return. Illiquid investments are profitable but risky.[1] Banks can obtain funding liquidity in the unsecured interbank market by issuing claims on the illiquid investment, which has limited market liquidity.[2] Due to the risk of illiquid investments, banks may become insolvent and thus unable to repay their interbank loan. This makes unsecured interbank lending risky. To compensate lenders, borrowers have to pay a premium for funds obtained in the unsecured interbank market.

To model the secured interbank market, we introduce bonds that provide a positive net return in the long run. Unlike the illiquid asset, they can also be traded for liquidity in the short term. We consider the case of safe bonds, e.g., government bonds. Since unsecured borrowing is costly due to credit risk, banks in need of liquidity will sell bonds to reduce their borrowing needs. We assume that government bonds are in fixed supply and that they are scarce enough not to crowd out the unsecured market. The risk of banks' illiquid assets will affect the price of safe government bonds since banks with a liquidity surplus must be willing to both buy the bonds offered and lend in the unsecured interbank market. In equilibrium there must not be an arbitrage opportunity between secured and unsecured lending. We use the link between secured and unsecured markets to derive a number of empirical predictions.

[1] Illiquidity as a key factor contributing to the fragility of modern financial systems is emphasized by Diamond and Rajan (2008) and Brunnermeier (2009), for example.

[2] See also Brunnermeier and Pedersen (2009), who distinguish between market liquidity and funding liquidity.

This paper is part of a growing literature analyzing the ability of interbank markets to smooth out liquidity shocks. We use the framework developed by Freixas and Holthausen (2005), who examine the scope for the integration of unsecured interbank markets when cross-country information is noisy. They show that introducing secured interbank markets reduces interest rates and improves conditions when unsecured markets are not integrated. Their introduction may, however, hinder the process of integration.

Several recent papers examine various frictions in interbank markets that can justify a policy intervention. The role of asymmetric information about credit risk is emphasized in Heider, Hoerova, and Holthausen (2009). The model generates several possible regimes in the interbank market, including one in which trading breaks down. The regimes are akin to the developments prior to and during the 2007–09 financial crisis. Imperfect competition is examined in Acharya, Gromb, and Yorulmazer (2008). If liquidity-rich banks use their market power to extract surplus from liquidity-poor banks, a central bank can provide an outside option for the latter. Freixas, Martin, and Skeie (2008) show that when multiple, Pareto-ranked equilibria exist in the interbank market, a central bank can act as a coordination device for market participants and ensure that a more efficient equilibrium is reached. Freixas and Jorge (2008) analyze the effects of interbank market imperfections for the transmission of monetary policy. Bruche and Suarez (2009) explore implications of deposit insurance and spatial separation for the ability of money markets to smooth out regional differences in savings rates. Acharya, Shin, and Yorulmazer (2009) study the effects of financial crises and their resolution on banks' choice of liquid asset holdings. In Allen, Carletti, and Gale (2009), secured interbank markets can be characterized by excessive price volatility when there is a lack of opportunities for hedging aggregate and idiosyncratic liquidity shocks. By using open-market operations, a central bank can reduce price volatility and improve welfare.[3]

[3]Aggregate shortages are also examined in Diamond and Rajan (2005) where bank failures can be contagious due to a shrinking of the pool of available liquidity. Freixas, Parigi, and Rochet (2000) analyze systemic risk and contagion in a financial network and its ability to withstand the insolvency of one bank. In Allen and

The remainder of the paper is organized as follows. In section 2, we describe the setup of the model. In section 3, we solve the benchmark case when banks can only trade in the unsecured interbank market. In section 4, we allow banks to invest in safe bonds. In section 5, we present empirical implications and relate them to the developments during the 2007–09 financial crisis. In section 6, we discuss policy responses to mitigate the tensions in interbank markets, and in section 7 we offer concluding remarks. All proofs are in the appendix.

2. The Model

The model is based on Freixas and Holthausen (2005). There are three dates—$t = 0$, 1, and 2—and a single homogeneous good that can be used for consumption and investment. There is no discounting between dates.

There is a [0,1] continuum of identical, risk-neutral, profit-maximizing banks. We assume that the banking industry is perfectly competitive. Banks manage the funds on behalf of risk-neutral households with future liquidity needs.[4] To meet the liquidity needs of households, banks offer them claims worth c_1 and c_2 that can be withdrawn at $t = 1$ and $t = 2$, e.g., demand deposits or lines of credit. We assume that $c_1 > 0$. Households do require some payout in response to their liquidity need at $t = 1$.[5] The aggregate demand for liquidity is certain: a fraction λ of households withdraws their

Gale (2000), the financial connections leading to contagion arise endogenously as a means of insurance against liquidity shocks.

[4]We do not address the question of why households use banks to manage their funds, nor why banks offer demandable debt in return. Moreover, we abstract from any risk-sharing concerns and side-step the question whether interbank markets are an optimal arrangement. There is a large literature dealing with these important normative issues, starting with Diamond and Dybvig (1983), Bhattacharya and Gale (1987), and Jacklin (1987). For recent examples, see Diamond and Rajan (2001), Allen and Gale (2004), or Farhi, Golosov, and Tsyvinski (2008).

[5]In principle, risk-neutral households are indifferent between consuming at $t = 1$ and $t = 2$. In order to have an active interbank market, we assume that some households will have a strictly positive need for early consumption, which must be satisfied by banks. For example, some households may have to pay a tax at $t = 1$.

claims at $t = 1$. The remaining fraction $1-\lambda$ withdraws at $t = 2$. At the individual bank level, however, the demand for liquidity is uncertain. A fraction π_h of banks face a high liquidity demand $\lambda_h > \lambda$ at $t = 1$, and the remaining fraction $\pi_l = 1 - \pi_h$ of banks face a low liquidity demand $\lambda_l < \lambda$. Hence, we have $\lambda = \pi_h\lambda_h + \pi_l\lambda_l$. Let the subscript $k = l, h$ denote whether a bank faces a low or a high need for liquidity at $t = 1$. Since aggregate liquidity needs are known, a bank with a high liquidity shock at $t = 1$ will have a low liquidity shock at $t = 2$: $1-\lambda_h < 1-\lambda_l$. We assume that banks' idiosyncratic liquidity shocks are not contractible. A bank's demandable liabilities cannot be contingent on whether it faces a high or a low liquidity shock at $t = 1$ and $t = 2$. This is the key friction that will give rise to an interbank market.

At $t = 0$, banks invest the funds of households either into a long-term illiquid asset (*loans*), a short-term liquid asset (*cash*), or into government bonds. We assume that each bank has one unit of the good under management at $t = 0$. Each unit invested in the liquid asset offers a return equal to one unit of the good after one period (costless storage). Each unit invested in the illiquid asset yields an uncertain payoff at $t = 2$. The investment into the illiquid asset can either succeed with probability p or fail with probability $1 - p$. If it succeeds, the bank is solvent and receives a return on the illiquid investment worth R units of the good at $t = 2$. If the investment fails, we assume that the bank is insolvent and is taken over by a deposit insurance fund. The fund assumes all the liabilities of an insolvent bank.[6] The investment into the illiquid asset does not produce any return at $t = 1$. Moreover, the illiquid asset is nontradable.

Government bonds yield a certain return equal to Y at $t = 2$. We assume that $pR > Y > 1$ so that bonds do not dominate the illiquid asset. Like the illiquid long-term investment, government bonds do not offer a return at $t = 1$. Unlike the illiquid asset, however, government bonds can be traded at $t = 1$ at a price P_1. Since we employ the term "liquidity" as the ability to produce cash flow at $t = 1$, the liquidity of government bonds is therefore *endogenous.* Government

[6]Thus, banks are protected by limited liability. Note that the deposit insurance fund only intervenes if the bank is insolvent, i.e., if the illiquid investment has failed.

bonds are in fixed supply. Let B denote the supply of government bonds to the banking sector at $t = 0$.[7]

Banks face a trade-off between liquidity and return when making their portfolio decision at $t = 0$. The short-term liquid asset allows banks to satisfy households' need for liquidity at $t = 1$. The illiquid asset is more profitable in the long run. Government bonds lie in between and are in fixed supply. Let α denote the fraction of bank assets at $t = 0$ invested in the illiquid asset, β denote the fraction invested in government bonds, and $1 - \alpha - \beta$ denote the remaining fraction invested in the liquid asset.

Since banks face different liquidity demands at $t = 1$, interbank markets can develop. Banks with low levels of withdrawals can provide liquidity to banks with high levels of withdrawals. We consider both secured and unsecured interbank markets. For ease of exposition, we model the secured market (repo agreements) as the trading of government bonds and treat $\frac{1}{P_1}$ as the repo rate.[8] The unsecured market consists of borrowing and lending amounts L_l and L_h, respectively, at an interest rate r. Given that banks can be insolvent when their illiquid investment fails, lenders in the unsecured interbank market will be exposed to credit risk. The deposit insurance fund does not cover interbank loans. However, borrowers always have to repay their interbank loan if they are solvent. Should a borrowers' counterparty be insolvent, the repayment goes to the deposit insurance fund. We denote the probability that an unsecured interbank loan is repaid by $\hat{p}$.

We assume that the interbank markets for unsecured loans and for government bonds are anonymous and competitive. Banks are price takers and are completely diversified across unsecured interbank loans. That is, a lender's expected return per unit lent in the unsecured interbank market is $p\hat{p}(1+r)$. With probability p a lender is solvent, in which case he collects the interest repayment $1 + r$ on a proportion $\hat{p}$ of the interbank loans made. The per-unit expected cost to a borrower is $p(1 + r)$.

[7]As we will show, if bonds were in unlimited supply, banks would prefer to satisfy their liquidity needs at $t = 1$ solely by trading bonds to avoid the risk premium of unsecured borrowing.

[8]In interbank repo markets, government bonds serve as collateral. This arrangement is different from an outright sale of bonds in that the original owner of the bond still collects the interest payment Y.

Figure 3. Assets and Financial Claims

	$t = 0$	$t = 1$	$t = 2$ → time
Cash	-1	1 -1	1
Loans	-1	0	p: R $1-p$: 0
Govt. Bonds	$-\frac{1}{P_0}$	P_1 $-\frac{1}{P_1}$	Y
Risky Interbank Debt		-1	$\hat{p}$: $1+r$ $1-\hat{p}$: 0
		1	$-(1+r)$

Figure 3 summarizes the payoffs of assets and financial claims. Note that the payoff shown for risky interbank debt is conditional on banks being solvent at $t = 2$.

The sequence of events is summarized in figure 4. At $t = 0$, banks invest households' funds in illiquid loans, government bonds, and cash. Government bonds are in fixed supply to the banking sector and their price at $t = 0$, P_0 must be such that (i) the market

Figure 4. The Timing of Events

$t = 0$	$t = 1$	$t = 2$ → time
Banks offer deposit contracts (c_1, c_2).	Idiosyncratic liquidity shocks are realized.	The return of the illiquid asset and the government bond realize.
Banks invest into a risky illiquid asset, a safe liquid asset, and government bonds.	Banks borrow and lend in secured and/or unsecured interbank markets. Additionally, they can reinvest into the liquid asset.	Interbank loans are repaid.
	A fraction of households withdraws and consumes c_1.	The remaining fraction of households withdraws and consumes c_2.

for government bonds at $t = 0$ clears and (ii) it is consistent with banks' optimal holding of government bonds. At $t = 1$, after receiving an idiosyncratic liquidity shock, banks manage their liquidity by borrowing or lending in the unsecured interbank market, buying or selling government bonds and possibly reinvesting into the liquid asset in order to maximize bank profits at $t = 2$, taking their portfolio allocation $(\alpha, \beta, 1-\alpha-\beta)$ and the payout to households (c_1, c_2) as given. Both the interbank market for unsecured loans and for government bonds must clear. Prices are set by a Walrasian auctioneer so that (i) decentralized trading is consistent with banks' portfolios of bonds, illiquid loans, and cash, and (ii) there is no arbitrage opportunity between government bonds and unsecured interbank loans. At $t = 2$, returns on the illiquid asset and bonds are realized, interbank loans are repaid, and solvent banks pay out all their cash flow to households.

3. Benchmark: No Government Bonds

In this section we solve the model without government bonds (i.e., $\beta = 0$). The analysis clarifies how the model works and provides a benchmark. The main text gives the outline of the arguments. The details of the proofs are in the appendix. We proceed backwards by first considering banks' liquidity management at $t = 1$.

3.1 Liquidity Management

Having received liquidity shocks, $k = l, h$, banks manage their liquidity at $t = 1$ while taking their assets $(\alpha, 1 - \alpha)$ and liabilities (c_1, c_2) as given.

A bank that faces a low level of withdrawals at $t = 1$, $k = l$, has spare liquidity. The bank can thus choose to lend an amount L_l at a rate r in the interbank market. The bank can also reinvest a fraction γ_l^1 of funds left over in the liquid asset. At $t = 1$, a type l bank maximizes $t = 2$ profits

$$\max_{\gamma_l^1, L_l} p\left[R\alpha + \gamma_l^1(1-\alpha) + \hat{p}(1+r)L_l - (1-\lambda_l)c_2\right] \tag{1}$$

subject to

$$\lambda_l c_1 + L_l + \gamma_l^1(1-\alpha) \leq (1-\alpha)$$

and feasibility constraints: $0 \leq \gamma_l^1 \leq 1$ and $L_l \geq 0$.

Conditional on being solvent (with probability p), the profits at $t = 2$ of a bank with a surplus of liquidity at $t = 1$ are the sum of the proceeds from the illiquid investment, $R\alpha$, from the reinvestment into the liquid asset, $\gamma_l^1(1-\alpha)$, and the repayments of risky interbank loans, $\hat{p}(1+r)L_l$, minus the payout to households withdrawing at $t = 2$, $(1-\lambda_l)c_2$. The budget constraint requires that the outflow of liquidity at $t = 1$ (deposit withdrawals, $\lambda_l c_1$; reinvestment into the liquid asset, $\gamma_l^1(1-\alpha)$; and interbank lending, L_l) is matched by the inflow (return on the liquid asset, $1-\alpha$).

A bank that has received a high liquidity shock, $k = h$, will be a borrower in the interbank market, solving

$$\max_{\gamma_h^1, L_h} p\left[R\alpha + \gamma_h^1(1-\alpha) - (1+r)L_h - (1-\lambda_h)c_2\right] \tag{2}$$

subject to

$$\lambda_h c_1 + \gamma_h^1(1-\alpha) \leq (1-\alpha) + L_h$$

and feasibility constraints: $0 \leq \gamma_h^1 \leq 1$ and $L_h \geq 0$.

There are two differences between the optimization problems of a lender and a borrower. First, a borrower expects having to repay $(1+r)L_h$ with probability p, while a lender expects a repayment $\hat{p}(1+r)L_l$ with probability p. A lender is exposed to credit risk. Second, interbank loans are an outflow for a lender and an inflow for a borrower.

Given that banks must provide some liquidity to households, $c_1 > 0$, the interbank market will be active as banks trade to smooth out the idiosyncratic liquidity shocks, $L_l > 0$ and $L_h > 0$.

The marginal value of (inside) liquidity at $t = 1$, $1-\alpha$, is given by the Lagrange multiplier, denoted by μ^k, on the budget constraints of the optimization problems (1) and (2).

LEMMA 1 (Marginal Value of Liquidity). *The marginal value of liquidity is* $\mu^l = p\hat{p}(1+r)$ *for a lender and* $\mu^h = p(1+r)$ *for a borrower.*

A lender values liquidity at $t = 1$ since he can lend it out at an expected return of $p\hat{p}(1 + r)$. A borrower values liquidity since it saves the cost of borrowing in the interbank market, $p(1 + r)$. The marginal value of liquidity is lower for a lender because of credit risk.

The following result describes a bank's decision to reinvest into the liquid asset.

LEMMA 2 (Reinvestment into the Liquid Asset). *A borrower does not reinvest in the liquid asset at $t = 1$: $\gamma_h^1 = 0$. A lender does not reinvest in the liquid asset if and only if $\hat{p}(1 + r) \geq 1$.*

It cannot be optimal for a bank with a shortage of liquidity to borrow in the interbank market at rate $1 + r$ and to reinvest the obtained liquidity in the liquid asset since it would yield a negative net return. The same is not true for a lender since his rate of return on lending in the interbank market is only $\hat{p}(1+r)$ due to credit risk. If a lender reinvests his liquidity instead of lending it out, then the interbank market cannot be active. Thus, we have to check whether $\hat{p}(1+r) \geq 1$ once we have obtained the interest rate in the interbank market.

Market clearing in the interbank market, $\pi_l L_l = \pi_h L_h$, yields the following:

LEMMA 3 (Interbank Market Clearing). *The amount of the liquid asset held by banks exactly balances the aggregate payout at $t = 1$:*

$$\lambda c_1 = 1 - \alpha.$$

The interbank market fully smoothes out the idiosyncratic liquidity shocks, λ_k.

3.2 Pricing Liquidity

The price of unsecured interbank loans, $1 + r$, which banks take as given when making their portfolio choice, must be consistent with an interior portfolio allocation, $0 < \alpha < 1$. The profitability of the illiquid asset implies that a bank would never want to invest everything into the liquid asset and thus $\alpha > 0$. The need for a positive payout to households at $t = 1$, $c_1 > 0$, implies that banks will not be able to invest everything into the illiquid asset, $\alpha < 1$.

An interior portfolio allocation α solves

$$\max_{0<\alpha<1} \quad \pi_l p\big[R\alpha + \hat{p}(1+r)L_l - (1-\lambda_l)c_2\big] \\ + \pi_h p\big[R\alpha - (1+r)L_h - (1-\lambda_h)c_2\big] \qquad (3)$$

subject to

$$L_l = (1-\alpha) - \lambda_l c_1 \qquad (4)$$

$$L_h = \lambda_h c_1 - (1-\alpha), \qquad (5)$$

where we have used that $\gamma_k^1 = 0$ (lemma 2).

The first-order condition requires that

$$\pi_h p(1+r) + \pi_l p\hat{p}(1+r) = \pi_h pR + \pi_l pR$$

or, equivalently,

$$(\pi_h + \pi_l\hat{p})(1+r) = R. \qquad (6)$$

The interbank interest rate r—i.e., the price of liquidity traded in the unsecured interbank market—is effectively given by a no-arbitrage condition. The right-hand side is the expected return from investing an additional unit into the illiquid asset, R. The left-hand side is the expected return from investing an additional unit into the liquid asset. With probability π_h, a bank will have a shortage of liquidity at $t = 1$, and one more unit of the liquid asset saves on borrowing in the interbank market at an expected cost of $(1 + r)$ (conditional on being solvent). With probability π_l, a bank will have excess liquidity, and one more unit of the liquid asset can be lent out at an expected return $\hat{p}(1 + r)$ (again conditional on being solvent). Note that banks' own probability of being solvent at $t = 2$, p, cancels out in (6) since it affects the expected return on the liquid and the illiquid investment symmetrically.

What is the level of credit risk? Since lenders hold a fully diversified portfolio of unsecured interbank loans, the proportion of loans that will not be repaid is given by the proportion of borrowers whose illiquid investment failed and who are thus insolvent at $t = 2$,

$$1 - \hat{p} = 1 - p. \tag{7}$$

We therefore have the following result:

PROPOSITION 1 (Pricing). *The price of liquidity at $t = 1$ is given by*

$$1 + r = \frac{R}{\delta}, \tag{8}$$

where

$$\frac{1}{\delta} \equiv \frac{1}{\pi_h + \pi_l p} > 1 \tag{9}$$

is the premium of lending in the interbank market due to banks' risky assets.

Given the price of liquidity (8), a bank with a surplus of liquidity will always want to lend it out rather than reinvest it. That is, the condition in lemma 2 is always satisfied: $p\frac{R}{\delta} > 1$ since $pR > 1$ and $\delta < 1$.

Liquidity becomes more costly when (i) asset risk increases (lower p) and (ii) a bank is more likely to become a lender (higher π_l) and thus is more likely to be subject to credit risk.

3.3 Portfolio Allocation

A bank's portfolio allocation α must be consistent with the promised payout to households, as well as market clearing and competition. We assume that banks pay out everything to households at $t = 2$. For a solvent bank that has lent in the unsecured interbank market, this means that

$$R\alpha + \hat{p}(1 + r)[(1 - \alpha) - \lambda_l c_1] - (1 - \lambda_l)c_2 = 0,$$

while for a solvent bank that has borrowed, it must be that

$$R\alpha - (1 + r)[\lambda_h c_1 - (1 - \alpha)] - (1 - \lambda_h)c_2 = 0.$$

Both types of banks must break even at $t = 2$ when solvent.[9] Note that a bank's payout to households at $t = 2$ cannot be contingent on whether it has lent or borrowed at $t = 1$. Using (i) market clearing at $t = 1$ (lemma 3), which links the proportion of the investment into the liquid asset $1 - \alpha$ to the payout c_1, (ii) the price of liquidity at $t = 1$ (equation (8)), and (iii) the link between credit and asset risk (equation (7)), we arrive at the following result:

PROPOSITION 2 (Portfolio Allocation). *Banks' portfolio allocation across the liquid and the illiquid asset satisfies*

$$\frac{\alpha}{1-\alpha} = \frac{1}{\delta} \frac{(1-\lambda_l)\pi_l + (1-\lambda_h)\pi_h p}{\lambda_l \pi_l + \lambda_h \pi_h}. \tag{10}$$

A bank chooses to hold a more liquid portfolio if it expects a higher level of withdrawals at $t = 1$ (λ_k increases). With respect to the probability of becoming a lender, π_l, and asset risk, p, there are two effects at play: the risk premium $\frac{1}{\delta}$ and the ratio between withdrawals at $t = 1$ versus $t = 2$ (the second fraction on the right-hand side of (10)). With respect to the probability of becoming a lender, both effects go in the same direction: higher π_l increases the risk premium and the relative proportion of $t = 2$ withdrawals.[10] Consequently, a higher probability of having a liquidity surplus at $t = 1$ leads to a less liquid portfolio at $t = 0$.

With respect to the risk of banks' illiquid assets, p, the two effects work in opposite directions. More asset risk increases the risk premium in the unsecured market but lowers the ratio of $t = 2$ versus $t = 1$ withdrawals. Higher asset risk means more credit risk for lenders and, consequently, fewer profits and a lower payout at $t = 2$. At the same time, lenders have more withdrawals than borrowers at $t = 2$, yet banks' withdrawable claims cannot be made contingent

[9] We also assume that the deposit insurance fund only intervenes if banks' illiquid investment fails (see footnote 6). If the investment succeeds, banks are not allowed to default on their deposits at $t = 2$ for regulatory reasons. The assumption that deposit insurance only intervenes when the illiquid investment fails is for simplicity only. The assumption is responsible for the clean link between asset risk and credit risk in the interbank market, $\hat{p} = p$.

[10] The derivative with respect to π_l of the second fraction on the right-hand side of (10) is positive if and only if $\lambda_h(1 - \lambda_l) > p\lambda_l(1 - \lambda_h)$. This always holds since $\lambda_h > \lambda_l$.

on banks' idiosyncratic liquidity shocks. To counter this imbalance at $t = 2$, a bank holds more liquid assets when asset risk is higher. This allows it to lend more and thus to increase revenue at $t = 2$ in case it received a low liquidity shock at $t = 1$. Similarly, it decreases its revenue at $t = 2$ in case it received a high liquidity shock and ends up being a borrower. The derivative of the right-hand side of equation (10) with respect to p is negative if and only if

$$(1 - \lambda_h)\pi_h^2 < (1 - \lambda_l)\pi_l^2. \tag{11}$$

A sufficient condition for more credit risk leading to less liquid investments is that banks are (weakly) more likely to have a liquidity surplus than a shortage, $\pi_l \geq \pi_h$ or $\pi_l \geq \frac{1}{2}$.

3.4 A Benchmark—No Risk

It is useful to consider the benchmark case when there is no asset risk and hence no credit risk. Substituting $p = 1$ into (10) yields the following result:

Corollary 1 (No Risk). *Without risk, $p = 1$, the interest rate in the unsecured interbank market $1 + r$ is equal to R, and the fraction invested in the illiquid asset is equal to the expected amount of withdrawals at $t = 2$: $\alpha^* = 1 - \lambda$.*

Without asset risk there is no credit risk for lenders in the unsecured interbank market. The amount invested in the liquid asset exactly covers the expected amount of withdrawals at $t = 1$. The interbank market smoothes out the problem of uneven demand for liquidity across banks at no cost. The fraction invested in the illiquid investment exactly covers the expected amount of withdrawals at $t = 2$. Without credit risk, lenders no longer lose revenue at $t = 2$.

4. Access to Government Bonds

In this section we allow banks to invest a fraction β of their portfolio into government bonds at $t = 0$ and to trade these bonds at $t = 1$. To solve the model we follow the same steps as in the previous section. The main text gives the outline of the arguments. The detailed proofs are in the appendix.

4.1 Liquidity Management

In order to manage their liquidity needs at $t = 1$, banks choose a fraction of government bond holdings to sell, β_k^S; a fraction of liquid asset holdings to be reinvested in the liquid asset, γ_k^1; a fraction of liquid asset holdings to be used to acquire more government bonds, γ_k^2; and how much to borrow/lend in the interbank market, L_k.

A bank that faces a low level of withdrawals at $t = 1$, type l, solves the following problem:

$$\max_{\beta_l^S,\gamma_l^1,\gamma_l^2,L_l} p\left[R\alpha + \left(\gamma_l^1 + \gamma_l^2\frac{Y}{P_1}\right)\left((1-\alpha-\beta) + \beta_l^S\frac{\beta}{P_0}P_1\right)\right.$$
$$\left. + (1-\beta_l^S)\frac{\beta}{P_0}Y + \hat{p}(1+r)L_l - (1-\lambda_l)c_2\right] \quad (12)$$

subject to

$$\lambda_l c_1 + L_l + \left(\gamma_l^1 + \gamma_l^2\right)\left((1-\alpha-\beta) + \beta_l^S\frac{\beta}{P_0}P_1\right)$$
$$\leq (1-\alpha-\beta) + \beta_l^S\frac{\beta}{P_0}P_1 \quad (13)$$

and feasibility constraints: $0 \leq \beta_l^S \leq 1$, $0 \leq \gamma_l^1$, $0 \leq \gamma_l^2$, $\gamma_l^1 + \gamma_l^2 \leq 1$, and $L_l \geq 0$.

A bank that has received a high liquidity shock, type h, will be a borrower in the interbank market, solving

$$\max_{\beta_h^S,\gamma_h^1,\gamma_h^2,L_h} p\left[R\alpha + \left(\gamma_h^1 + \gamma_h^2\frac{Y}{P_1}\right)\left((1-\alpha-\beta) + \beta_h^S\frac{\beta}{P_0}P_1\right)\right.$$
$$\left. + \left(1-\beta_h^S\right)\frac{\beta}{P_0}Y - (1+r)L_h - (1-\lambda_h)c_2\right] \quad (14)$$

subject to

$$\lambda_h c_1 + \left(\gamma_h^1 + \gamma_h^2\right)\left((1-\alpha-\beta) + \beta_h^S\frac{\beta}{P_0}P_1\right)$$
$$\leq (1-\alpha-\beta) + \beta_h^S\frac{\beta}{P_0}P_1 + L_h \quad (15)$$

and feasibility constraints: $0 \leq \beta_h^S \leq 1$, $0 \leq \gamma_h^1$, $0 \leq \gamma_h^2$, $\gamma_h^1 + \gamma_h^2 \leq 1$, and $L_h \geq 0$.

Access to bonds changes the liquidity management of banks as follows. Banks hold $\frac{\beta}{P_0}$ units of bonds. They can sell a fraction β_k^S of their bond holdings at price P_1. Hence, the amount of funds available at $t = 1$ is the sum of liquid asset holdings, $1 - \alpha - \beta$, and the proceeds from selling bonds, $\beta_k^S \frac{\beta}{P_0} P_1$. Banks can also acquire new bonds using γ_k^2 fraction of their liquid asset holdings.

At $t = 2$, banks earn return Y per unit of bond holdings. The return is earned on bonds bought at $t = 0$ that were not sold at $t = 1$, $(1 - \beta_k^S)\frac{\beta}{P_0}$ units, and on additional bonds bought at $t = 1$, $\frac{\gamma_k^2}{P_1}\big((1 - \alpha - \beta) + \beta_k^S \cdot \frac{\beta}{P_0} P_1\big)$ units.

Market clearing in the bond market requires that

$$\left(\pi_l \beta_l^S + \pi_h \beta_h^S\right) \frac{\beta}{P_0} P_1 = \pi_l \gamma_l^2 \left((1 - \alpha - \beta) + \beta_l^S \frac{\beta}{P_0} P_1\right) + \pi_h \gamma_h^2 \left((1 - \alpha - \beta) + \beta_h^S \frac{\beta}{P_0} P_1\right). \quad (16)$$

The left-hand side of (16) is the value of bonds sold by banks at $t = 1$, while the right-hand side is the amount available to buy them. The demand for bonds at $t = 1$ will depend on how much banks decide to hold in liquid assets at $t = 0$, $1 - \alpha - \beta$.

As before, banks need to satisfy households' demand for liquidity at $t = 1$. Access to safe government bonds will, however, reduce the amount that banks in need of liquidity must borrow unsecured. Acquiring liquidity through the sale of bonds is cheaper since the provider of liquidity (the buyer of the bond) does not need to be compensated for credit risk. To focus on the more interesting case in which the trading of bonds and unsecured interbank lending coexist, we assume that there are not enough bonds to fully cover banks' liquidity shortage at $t = 1$.

The introduction of bonds does not change the marginal value of liquidity. It is still given by lemma 1.

A bank with a shortage of liquidity at $t = 1$ will neither sell its bonds to reinvest in the liquid asset nor will it hold on to them. It will sell them in order to reduce the amount it needs to borrow in the unsecured interbank market.

Lemma 4 (Liquidity Management of a Bank with a Shortage). *A bank with a liquidity shortage will not reinvest, neither in bonds nor in the liquid asset,* $\gamma_h^1 = 0, \gamma_h^2 = 0$, *and it will sell all its bonds:* $\beta_h^S = 1$.

Since bonds are scarce and the unsecured market is active, banks with a surplus of liquidity must still find it attractive to lend unsecured. The return on bonds must not be larger than the return on unsecured lending. Since lenders need to be compensated for credit risk in unsecured lending, banks with a shortage of liquidity will sell all their bonds first and then borrow the remaining amount.

Given that banks with a liquidity shortage sell bonds and borrow in the unsecured market, banks with a liquidity surplus must buy bonds and lend unsecured.

Lemma 5 (Liquidity Management of a Bank with a Surplus). *A bank with a liquidity surplus will buy additional bonds:* $\gamma_l^1 = 0$, $\gamma_l^2 > 0$, *and* $\beta_l^S = 0$.

Using the results in lemmas 4 and 5, we can simplify (16), the market-clearing condition in the bond market:

$$\pi_h \frac{\beta}{P_0} P_1 = \pi_l \gamma_l^2 (1 - \alpha - \beta). \tag{17}$$

Market clearing in the bond market and the unsecured interbank market yields the following:

Lemma 6 (Interbank Market Clearing). *The amount of the liquid asset held by banks exactly balances the aggregate payout to households at* $t = 1$*:*

$$\lambda c_1 = 1 - \alpha - \beta.$$

As before, trading at $t = 1$ fully smoothes out the idiosyncratic liquidity shocks.

4.2 Pricing Liquidity

Since both the bond market and the unsecured loan market are open, there must not be an arbitrage opportunity between the two markets at $t = 1$:

$$\frac{Y}{P_1} = \hat{p}(1+r) \geq 1. \tag{18}$$

Buying bonds and unsecured lending must offer the same return. Moreover, the return must be weakly larger than one since otherwise banks would prefer to reinvest into the liquid asset.

As before, banks are price takers in the market for unsecured interbank loans. The price at which banks engage in decentralized trading must be consistent with an interior portfolio allocation, $0 < \alpha < 1$ and $0 < \beta < 1$. Investing everything into bonds is inconsistent with satisfying households' need for liquidity at $t = 1$. All banks would have to sell bonds and no bank would be able to buy bonds. Hence, $\beta < 1$. Since bonds are not subject to credit risk, banks with high liquidity shocks prefer to sell them rather than borrow unsecured. Since there is a positive probability of a high liquidity shock ex ante, banks want to hold some bonds, $\beta > 0$. Banks will also have to hold the liquid asset to satisfy the demand for liquidity at $t = 1$, $\alpha < 1$. Finally, the profitability of the illiquid asset implies $\alpha > 0$.

An interior portfolio allocation solves

$$\max_{0<\alpha<1, 0<\beta<1} \quad \pi_l p\left[R\alpha + \left(\frac{\pi_h}{\pi_l} + 1\right)\frac{Y}{P_0}\beta + \hat{p}(1+r)L_l - (1-\lambda_l)c_2\right] + \pi_h p\big[R\alpha - (1+r)L_h - (1-\lambda_h)c_2\big] \tag{19}$$

subject to

$$L_l = (1-\alpha-\beta) - \frac{\pi_h}{\pi_l}\frac{P_1}{P_0}\beta - \lambda_l c_1$$

and

$$L_h = \lambda_h c_1 - (1-\alpha-\beta) - \frac{P_1}{P_0}\beta,$$

where we have used the results in lemmas 4 and 5 on banks' liquidity management and market clearing in the bond market (17) in order to substitute for γ_l^2.

The first-order condition for an interior allocation of the illiquid asset, α, is

$$p\big[R - \pi_l \hat{p}(1+r) - \pi_h(1+r)\big] = 0. \tag{20}$$

As in the case without access to bonds, the proportion of interbank loans that are not repaid is given by the probability of being insolvent since lenders hold a fully diversified portfolio of loans. Using (7) to simplify (20) yields

$$1 + r = \frac{R}{\delta}. \tag{21}$$

Condition (21) is identical to condition (8). The cost of unsecured borrowing is not affected by the access to bonds.

Due to the no-arbitrage condition between bonds and unsecured loans (equation (18)), condition (21) ties down the price of bonds at $t = 1$:

$$\frac{Y}{P_1} = \frac{pR}{\delta}. \tag{22}$$

The condition immediately implies that $\frac{Y}{P_1} > 1$ since $pR > 1$ and $\delta < 1$. That is, bonds trade at a discount at $t = 1$. Lenders must be compensated for providing liquidity. If bonds did not trade at a discount, then holding the liquid asset to lend it out unsecured is not very attractive. The no-arbitrage condition would imply that $p(1 + r) = 1$ and hence $1 + r = \frac{1}{p} < \frac{R}{\delta}$, which is inconsistent with $\alpha < 1$. Banks would invest everything into the illiquid asset.

The requirement on an interior portfolio allocation for bonds ties down the price of bonds at $t = 0$.

Lemma 7. *The price of bonds at* $t = 0$ *is equal to the price at* $t = 1$*:*

$$P_0 = P_1. \tag{23}$$

Given that bonds and liquid asset holdings must coexist at $t = 0$ in order to manage liquidity shocks at $t = 1$, it is intuitive that the first-period yield on bonds equals the return on the liquid asset.

The following proposition summarizes the pricing of bonds and unsecured interbank loans:

Proposition 3 (Pricing). *The interest rate in the unsecured market is* $1 + r = \frac{R}{\delta}$*. The yield to maturity of the bond at* $t = 0$ *and at* $t = 1$ *is identical. It is given by* $\frac{Y}{P_0} = \frac{Y}{P_1} = \frac{pR}{\delta} > 1$*.*

4.3 Portfolio Allocation

As in the case without access to bonds, banks' portfolio allocation (α, β) must lead to a payout to households that is consistent with (i) market clearing at $t = 1$ and (ii) a full payout at $t = 2$ such that banks make zero profits if they are solvent and the payout is not contingent on $k = l, h$. The following proposition and its derivation are analogous to proposition 2.

PROPOSITION 4 (Portfolio Allocation). *Banks' portfolio allocation across the liquid and illiquid asset satisfies*

$$\frac{\alpha}{1-\alpha-\beta} = \frac{1}{\delta}\frac{(1-\lambda_l)\pi_l + (1-\lambda_h)\pi_h p}{\lambda_l\pi_l + \lambda_h\pi_h} - \frac{BY}{pR}\frac{1}{1-\alpha-\beta}\frac{(1-\lambda_l)-(1-\lambda_h)p}{\lambda_h-\lambda_l}. \tag{24}$$

The fraction invested in bonds is given by market clearing at $t = 0$*:* $\beta = BP_0 = \delta\frac{BY}{pR}$.

The left-hand side of (24) is as in (10), except that we have to subtract the bond holdings β to obtain the investment in the liquid asset. The first term on the right-hand side is the same as in the case without bonds. The second term captures the effect of having access to government bonds. Note that if $\beta = 0$ and $B = 0$, then equation (24) reduces to equation (10). Banks hold a more liquid portfolio ceteris paribus when they have access to bonds. Bonds provide banks with a safe return at $t = 2$ so that banks need to invest less into the illiquid investment in order to satisfy withdrawals at $t = 2$. At the same time, trading bonds is a valuable alternative to the unsecured interbank market.

The size of the banking sector relative to the amount of bonds available matters. The effect of bonds on the investment in liquid and illiquid assets is stronger when the ratio of the value of bonds to the expected value of banks' productive assets, $\frac{BY}{pR}$, is larger. Finally, banks' bond holdings are proportional to the relative size of bonds to banks' productive assets, $\frac{BY}{pR}$, and the constant of proportionality is given by the risk discount factor in the unsecured market, δ.

The effect of bonds on ex ante liquidity holdings is stronger when there is more credit risk (lower p), there are more withdrawals at banks with a liquidity surplus (higher λ_l), and there are fewer withdrawals at banks with a liquidity shortage (lower λ_h).

Suppose that banks are equally likely to have a liquidity shortage or a liquidity surplus, $\pi_l = \pi_h = \frac{1}{2}$. Then more credit risk increases the first term on the right-hand side (see condition (11)). It also increases the second term on the right-hand side, making the overall impact of more credit risk on banks' portfolio choice ambiguous.

5. Empirical Implications

Looking at figures 1 and 2, it seems that repo markets secured by government bonds in the United States and in the euro area followed different dynamics between August 2007 and May 2009. Below, we discuss empirical predictions of the model that may help explain the developments.

5.1 Decoupling of Secured and Unsecured Rates

Government bonds become relatively more valuable if the credit-risk problem becomes more severe, i.e., as p decreases. To see this, note that the following no-arbitrage conditions hold ex ante (see proposition 3):

$$\frac{R}{\delta} = 1 + r \text{ and } \frac{pR}{\delta} = \frac{Y}{P_1}.$$

Higher credit risk increases the credit-risk premium $\frac{1}{\delta}$, leading to a higher interest on unsecured lending $1+r$. A higher interest on unsecured lending decreases the price of bonds ceteris paribus; see equation (18). But there is a second, countervailing effect since the yield on *safe* bonds must be equal to the expected return on *risky* interbank loans. Overall, the second effect dominates since the credit-risk premium $\frac{1}{\delta}$ does not increase one for one with changes in p. Ex ante, a bank is uncertain whether it will be a lender, and thus exposed to credit risk, or not. In sum, following a shock to credit risk, unsecured rates and rates secured by government bonds move in opposite directions.

Changes in the perception of credit risk can explain why secured and unsecured rates decoupled with the onset of the financial crisis in August 2007, around the sale of Bear Stearns and following the Lehman bankruptcy. In the summer of 2007, the discovery of subprime mortgages in the portfolios of banks and bank-sponsored conduits led to a marketwide reassessment of credit risk. The sale of Bear Stearns caused a temporary turbulence, while the failure of Lehman led to a dramatic revision of expected default probabilities.

5.2 Spillovers and Credit Risk Levels

The potential for spillover effects from the unsecured to the secured market increases as the level of credit risk increases. To see this, note that

$$R = \frac{Y}{P_1}\left[\pi_l + \frac{1}{p}\pi_h\right]$$

must hold ex ante (see proposition 3), which implies that

$$-\frac{\frac{\partial P_1}{\partial p}}{\frac{P_1}{p}} = \frac{\pi_h}{\delta} < 1$$

for all $p > 0$.

It follows that the elasticity of the price of government bonds to credit risk is the lowest for $p = 1$ (no credit risk) since the elasticity is decreasing in p. This is consistent with the fact that the decoupling between the secured and unsecured rates was most pronounced in the aftermath of the Lehman failure, when the perceived level of credit risk in the banking sector was very high.

5.3 Relative Scarcity of Collateral

How does the scarcity of the underlying collateral affect the dynamics of repo rates when credit risk increases? Our model implies that the sensitivity of the price of government bonds to credit risk is lower in a country with a larger supply of government bonds. In other words, we have that, ex post,[11]

[11] Ex ante, the supply of bonds B has no impact on the price of bonds (see proposition 3).

$$\frac{\partial^2 P_1}{\partial B \partial p} > 0.$$

To see this, note that equation (24) and the fact that $\frac{pR}{\delta} = \frac{Y}{P_1}$ imply that

$$\frac{R\alpha}{1-\alpha-\beta} = \frac{1}{p}\frac{Y}{P_1}\frac{(1-\lambda_l)+(1-\lambda_h)\frac{\pi_h}{\pi_l}p}{\lambda_l+\lambda_h\frac{\pi_h}{\pi_l}} - \frac{1}{p}\frac{BY}{1-\alpha-\beta}\frac{(1-\lambda_l)-(1-\lambda_h)p}{\lambda_h-\lambda_l}. \quad (25)$$

Applying the implicit function theorem to the equation above, we have that

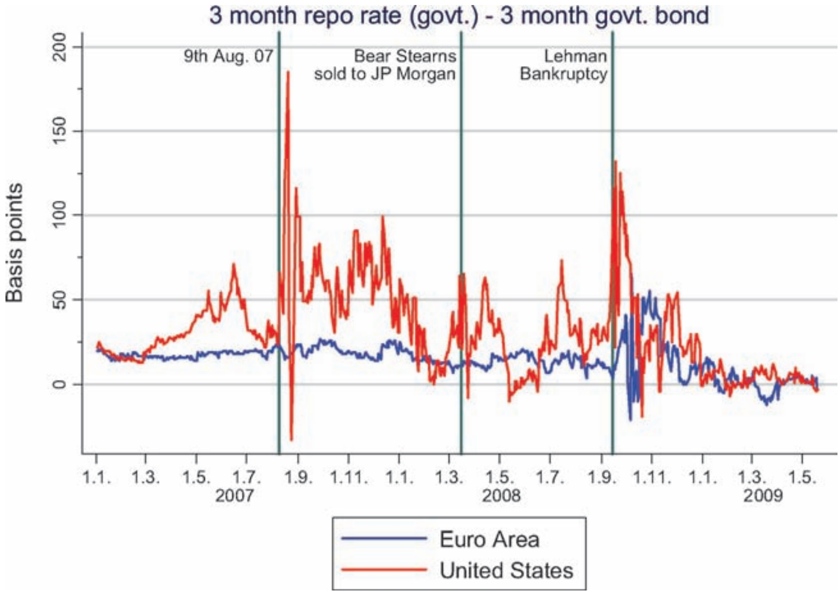
$$\frac{dP_1}{dB} = -P_1^2\frac{1}{1-\alpha-\beta}\frac{(1-\lambda_l)-(1-\lambda_h)p}{\lambda_h-\lambda_l} \times \frac{\pi_l\lambda_l+\pi_h\lambda_h}{\pi_l(1-\lambda_l)+\pi_h(1-\lambda_h)p} < 0. \quad (26)$$

Hence, if there is an unexpected shock to the amount of government bonds available in the banking sector—i.e., B decreases (say, due to a high demand for these securities outside the banking sector at the time when banks need to cope with liquidity shocks)—then the price of bonds P_1 must increase. Taking the derivative of (26) with respect to p, we get the desired result, since $\frac{\partial P_1}{\partial p} < 0$ and

$$\frac{\partial}{\partial p}\left(\frac{(1-\lambda_l)-(1-\lambda_h)p}{\pi_l(1-\lambda_l)+\pi_h(1-\lambda_h)p}\right) = \frac{-(1-\lambda_h)(1-\lambda_l)}{(\pi_l(1-\lambda_l)+\pi_h(1-\lambda_h)p)^2} < 0.$$

With the onset of the crisis in August 2007, repo rates in the United States became much more volatile than in the euro area. We document this in figure 5, which plots the spread between the three-month repo rate secured by government bonds and the yield on the three-month government bond. Hördahl and King (2008) argue that the higher volatility in the United States can be partly explained by the increased "safe haven" demand for U.S. Treasury securities, which made Treasuries relatively scarce. The Federal Reserve responded by introducing measures that increased the supply of high-quality collateral for private repo markets. We discuss policy responses further in section 6.

Figure 5. Tensions in Treasury-Backed Repo Markets, Euro Area and United States



5.4 Aggregate Liquidity Shocks

If there is an unanticipated shock to the relative proportion of banks with high and low liquidity demands, $\frac{\pi_h}{\pi_l}$, then liquidity becomes more scarce and even the price of government bonds declines. To show that this is the case, we apply the implicit function theorem to (25). It is straightforward to show that the sign of the derivative of P_1 with respect to $\frac{\pi_h}{\pi_l}$ is determined by the sign of the following expression:

$$\frac{\partial}{\partial \frac{\pi_h}{\pi_l}} \left(\frac{(1-\lambda_l) + (1-\lambda_h)\frac{\pi_h}{\pi_l} p}{\lambda_l + \lambda_h \frac{\pi_h}{\pi_l}} \right) = \frac{p\lambda_l(1-\lambda_h) - \lambda_h(1-\lambda_l)}{\left(\lambda_l + \lambda_h \frac{\pi_h}{\pi_l}\right)^2}.$$

As long as

$$p < \frac{\lambda_h(1-\lambda_l)}{\lambda_l(1-\lambda_h)},$$

this derivative is negative. Note that for $\lambda_h > \lambda_l$, which is what we assume, $\frac{\lambda_h(1-\lambda_l)}{\lambda_l(1-\lambda_h)} > 1$ and thus the inequality above always holds. It follows that P_1 declines after an unexpected increase in the aggregate demand for liquidity, $\frac{\pi_h}{\pi_l}$.

Following the same steps, it follows that the unsecured rate $1+r$ must increase if the aggregate demand for liquidity increases. Consequently, both secured and unsecured rates would move in the same direction after an unexpected aggregate liquidity shock; i.e., the decoupling of rates cannot be explained by such a shock.

6. Policy Implications

Unsecured markets are particularly vulnerable to changes in the perceived creditworthiness of counterparties. In repo transactions, such concerns are mitigated to some extent by the presence of collateral. Yet, our model illustrates how tensions in the unsecured market spill over to the market secured by collateral of the highest quality. Moreover, the volatility of repo rates can be exacerbated by structural characteristics such as the scarcity of securities that are used as collateral.

Central banks are particularly concerned with the well functioning of interbank markets because it is an important element in the transmission of monetary policy, and because persisting tensions may affect the financing conditions faced by nonfinancial corporations and households. In many countries, central banks have reacted to the observed tensions by introducing measures to support the interbank market, trying to avoid marketwide liquidity problems turning into solvency problems for individual institutions. The goal of this section is to examine a number of policy responses implemented since August 2007 that aimed at reducing tensions in interbank markets.

Specifically, we examine how the range of collateral accepted by a central bank affects the liquidity conditions of banks and how central banks can help alleviate tensions associated with the scarcity of high-quality collateral. In line with the predictions of the model, we present evidence that these measures can be effective in reducing tensions in secured markets. At the same time, they are not designed to resolve the underlying problems in the unsecured segment and the associated spillovers, if those are driven by credit-risk concerns.

6.1 Collateral Accepted by the Central Bank

Central banks provide liquidity to the banking sector against eligible collateral. The range of acceptable collateral varies across countries. Since the onset of the crisis, however, central banks have generally lowered the minimum credit rating and increased the quantity of lending they provide. For example, the Federal Reserve expanded its collateral list for repo operations in March, May, and September 2008, in response to market tensions. Moreover, it established the Term Auction Facility (TAF) in December 2007. The TAF provides term credit through periodic auctions to a broader range of counterparties and against a broader range of collateral than open-market operations. The Federal Reserve stressed that "this facility could help ensure that liquidity provisions can be disseminated efficiently even when the unsecured interbank markets are under stress."[12] The European Central Bank (ECB) headed into the crisis with the broadest list of eligible collateral among its peers, including nonmarketable securities and commercial loans. As a result, the ECB made no changes until mid-October 2008, when it expanded the eligible collateral significantly and lowered the minimum credit rating from A– to BBB– as the crisis intensified.

What are the implications of a wider range of collateral accepted in a central bank's operations according to our model? First, allowing securities other than Treasuries can reduce the volatility of the repo rates backed by Treasuries, as it reduces the pressure on acquiring Treasury securities and the limits imposed by their fixed supply. Moreover, we showed that if there is an unexpected aggregate liquidity shock, funding pressures can appear in all interbank market segments. By providing liquidity, a central bank can counter the effects of aggregate shocks and ensure that financial institutions do not sell their assets, including Treasuries, at distressed prices.

How effective were changes to the collateral framework of central banks during the crisis? McAndrews, Sarkar, and Wang (2008) provide evidence that the introduction of the TAF was associated with

[12]See Board of Governors of the Federal Reserve System (2007).

downward shifts of the LIBOR by reducing the liquidity risk premium. Christensen, Lopez, and Rudebusch (2009) analyze the role of the TAF in reducing the spreads between term LIBOR rates and the yield on Treasuries of corresponding maturity. They construct a counterfactual path and conclude that in the absence of the TAF, the LIBOR would have been higher. On the other hand, Taylor and Williams (2009) argue that the TAF had no significant impact on interest rate spreads, as it did not address the fundamental problem of credit risk on banks' balance sheets.

6.2 Upgrading Collateral

If concerns about the creditworthiness of counterparties make it expensive to borrow in the unsecured market, financial institutions try to obtain more funds in the secured market. However, we show that if the underlying collateral is scarce, repo market rates will be volatile. Measures aimed at increasing the supply of collateral can thus improve the allocation of liquidity in interbank markets.

For example, the Federal Reserve introduced the Term Securities Lending Facility (TSLF) in March 2008. It lends Treasury securities to dealers, taking less liquid securities, including agency debt securities and mortgage-backed securities (MBS), as collateral. The Treasury securities are allocated to dealers via auctions.[13] The primary dealers then use those Treasury securities to obtain financing in private repo markets. The TSLF thus increases the ability of dealers to obtain financing and decreases their need to sell assets into illiquid markets. The direct benefits that can be expected from the TSLF are, first, an increase in the supply of Treasury collateral in the private repo market and, second, a reduction of the supply of less liquid collateral.

[13]The TSLF is divided into two schedules: schedule 1 TSLF operations (i.e., auctions for Treasury and agency securities) are separated from schedule 2 TSLF operations (i.e., schedule 1 plus other investment-grade collateral). Schedule 2 collateral originally included schedule 1 collateral plus AAA/Aaa-rated non-agency residential MBS, commercial MBS, and agency collateralized mortgage obligations (CMOs). Schedule 2 collateral was expanded to include AAA/Aaa-rated asset-backed securities starting with the May 8, 2008 auction and all investment-grade debt securities starting with the September 17, 2008 auction.

The TSLF is closely related to the Primary Dealer Credit Facility (PDCF), which is also available to primary dealers. A key difference is that the PDCF is a standing facility whereas the TSLF is an auction facility. As a standing facility, the PDCF offers the advantage of availability on a continuous basis. It also accepts a broader class of securities as collateral. Whereas the TAF (discussed in the previous section) is only available to depository institutions, the TSLF is available to primary dealers. Both programs address the tensions in interbank markets via different market participants.

Fleming, Hrung, and Keane (2009) provide evidence of the impact of the introduction of the TSLF on repo spreads between Treasury collateral and lower-quality collateral. They document that the introduction of the TSLF was associated with an increase in repo rates relative to the federal funds rate. This is consistent with the predictions of our model that reducing the scarcity of high-quality collateral should result in higher Treasury repo rates. Moreover, the introduction of the TSLF narrowed financing spreads during spring 2008, particularly after the first auction. Much of the narrowing seems to have come from an increase in Treasury rates rather than a decrease of the rates for non-Treasury collateral.

7. Conclusion

Despite the presence of collateral, the disruptions in the unsecured interbank market during the 2007–09 financial crisis have also affected secured markets. This paper presents a model of secured and unsecured interbank lending in the presence of credit risk. Credit-risk premia in the unsecured market will affect the price of riskless bonds when they are used to manage banks' liquidity shocks.

Going forward, our analysis points to a number of issues for further research. First, the size of the banking sector relative to the amount of collateral matters. We saw that the presence of bonds reduces the amount banks have to borrow in unsecured markets. The positive effect of bonds is stronger when the ratio of banks' balance sheets to the value of bonds is larger. Hence, the interplay between the relative size of banking, financial markets, and the economy deserves further attention.

Second, our analysis abstracted from risk-sharing concerns. Banks were simply maximizing the total amount of demandable liabilities. Still, we obtain a credit-risk premium in unsecured interbank markets. Introducing risk aversion of banks' customers is beyond the scope of this paper and constitutes a fruitful avenue for further research. With respect to the spillover of credit risk across interbank markets, we anticipate that risk aversion can add an additional premium that would exacerbate the tensions that we identified.

Third, we assumed that the various shocks in our model are uncorrelated. The financial crisis has made it painfully clear that, in reality, the risk embedded in banks' illiquid assets, their liquidity needs, and shocks to collateral values are interlinked. The challenge will therefore be to model and analyze the joint distribution of the risks in banks' balance sheets, especially "at the tail." Banks' risk management practices have to take into account the forces affecting different collateral classes and the market's response in times of stress when liquidity and high-quality collateral are scarce.

Appendix

Proof of Lemma 1

The interbank market is active so that the constraints $L_k \geq 0$ are slack. Let μ^k be the Lagrange multiplier on the budget constraint. The first-order condition for a lender with respect to L_l is

$$p\hat{p}(1+r) - \mu^l = 0, \qquad (27)$$

while the first-order condition for a borrower with respect to L_h is

$$-p(1+r) + \mu^h = 0. \qquad (28)$$

Proof of Lemma 2

Let μ_1^k be the Lagrange multipliers on $0 \leq \gamma_k^1$. The constraint $\gamma_k^1 \leq 1$ cannot be binding since otherwise all available funds at $t = 1$ are reinvested and nothing can be paid or lent out. The first-order condition for a type k bank with respect to γ_k^1 is

$$(1-\alpha)(p - \mu^k) + \mu_1^k = 0. \qquad (29)$$

Substituting $\mu^h = p(1+r)$ (lemma 1) yields

$$(1-\alpha)(-pr) = -\mu_1^h < 0, \tag{30}$$

since the left-hand side is negative. It cannot be zero since $\alpha = 1$ cannot be optimal. A type h bank would have to finance its entire need for liquidity by borrowing in the interbank market at a rate $r > 0$, whereas it could just store some liquidity without cost using the short-term asset. Since $-\mu_1^h < 0$, we have $\gamma_h^1 = 0$.

Consider now the case of a lender. Substituting $\mu^l = p\hat{p}(1+r)$ (lemma 1) into (29) yields

$$(1-\alpha)p(1-\hat{p}(1+r)) = -\mu_1^l.$$

Again, $\alpha = 1$ cannot be optimal. A type l bank cannot invest everything into the illiquid asset and still lend in the interbank market. Hence, $\gamma_l^1 = 0$ if and only if $\hat{p}(1+r) \geq 1$ (we assume that a type l bank does not reinvest into the liquid asset when the condition holds as an equality).

Proof of Lemma 3

Using the binding budget constraints from the optimization problems (1) and (2) (lemma 1) to substitute for L_l and L_h in the market-clearing condition $\pi_l L_l = \pi_h L_h$ and using $\gamma_k^1 = 0$ (lemma 2) gives the result.

Proof of Lemma 4

The first-order condition of a borrower with respect to reinvesting into the liquid asset at $t = 1$, γ_h^1, is

$$\left((1-\alpha-\beta) + \beta_h^S \frac{\beta}{P_0} P_1\right)(p - \mu^h) + \mu_1^h = 0,$$

where μ^h is the marginal value of liquidity for a borrower (given by lemma 1) and μ_1^h is the multiplier on the feasibility constraint $\gamma_h^1 \geq 0$. Note that $(1-\alpha-\beta) + \beta_k^S \frac{\beta}{P_0} P_1 > 0$ since we are considering interior portfolio allocations, $1-\alpha-\beta > 0$. Since $\mu^h = p(1+r) > p$, we have that $\mu_1^h > 0$ and thus $\gamma_h^1 = 0$. As in the case without bonds, a borrower does not reinvest into the liquid asset.

From the first-order condition of a lender with respect to bond purchases at $t = 1$, γ_l^2, we have that

$$\left((1-\alpha-\beta) + \beta_l^S \frac{\beta}{P_0} P_1\right)(p\frac{Y}{P_1} - \mu^l) + \mu_2^l = 0, \tag{31}$$

where μ^l is the marginal value of liquidity for a lender (given by lemma 1) and μ_2^l is the multiplier on the feasibility constraint $\gamma_l^2 \geq 0$. Note that the feasibility constraint $\gamma_l^1 + \gamma_l^2 \leq 1$ must be automatically satisfied since otherwise all available funds at $t = 1$ are reinvested and nothing can be paid or lent out. Since $\mu^l = p\hat{p}(1+r)$, the first-order condition holds if

$$\frac{Y}{P_1} \leq \hat{p}(1+r). \tag{32}$$

The yield on the bond at $t = 1$ must be less than or equal to the expected return of unsecured interbank lending (given that the unsecured interbank market is open).

The first-order condition of a borrower with respect to bond purchases at $t = 1$, γ_h^2, is

$$\left((1-\alpha-\beta) + \beta_h^S \frac{\beta}{P_0} P_1\right)(p\frac{Y}{P_1} - \mu^h) + \mu_2^h = 0,$$

where μ_2^h is the multiplier on the feasibility constraint $\gamma_h^2 \geq 0$. Due to condition (32), we have that $\mu_2^h > 0$ and hence $\gamma_h^2 = 0$. A borrower does not reinvest using bonds either.

The first-order condition of a borrower with respect to bond sales at $t = 1$, β_h^S, is

$$\frac{\beta}{P_0}\left[p\big(P_1\gamma_h^1 + Y(\gamma_h^2 - 1)\big) - \mu^h P_1\big(\gamma_h^1 + \gamma_h^2 - 1\big)\right] + \mu_3^h - \mu_4^h = 0,$$

where μ_3^h and μ_4^h are the Lagrange multipliers on $0 \leq \beta_h^S \leq 1$. Using $\gamma_h^1 = 0$, $\gamma_h^2 = 0$, and $\mu^h = p(1+r)$, we have

$$p\frac{\beta}{P_0}\big[-Y + (1+r)P_1\big] + \mu_3^h - \mu_4^h = 0.$$

Due to condition (32), the term in squared brackets is positive. For the condition to hold, it must be that $\mu_4^h > 0$, and hence $\beta_h^S = 1$. The borrower sells all his bonds at $t = 1$.

Proof of Lemma 5

Market clearing for bonds at $t = 1$ requires that

$$(\pi_l \beta^S + \pi_h)\frac{\beta}{P_0}P_1 = \pi_l \gamma_l^2 \left((1-\alpha-\beta) + \beta_l^S \frac{\beta}{P_0}P_1\right),$$

where we have used $\beta_h^S = 1$ and $\gamma_h^2 = 0$. Market clearing therefore requires that $\gamma_l^2 > 0$. Since borrowers sell bonds, lenders must buy them.

Given that $\gamma_l^2 > 0$, and hence $\mu_2^l = 0$, the lender's first-order condition with respect to bond purchases (31) requires that

$$\frac{Y}{P_1} = \hat{p}(1+r). \tag{33}$$

The yield on safe bonds must be equal to the expected return on risky interbank loans, as both markets are open.

The first-order condition of a lender with respect to bond sales at $t = 1$, β_l^S, is

$$\frac{\beta}{P_0}\left[p\big(P_1\gamma_l^1 + Y(\gamma_l^2 - 1)\big) - \mu^l P_1\big(\gamma_l^1 + \gamma_l^2 - 1\big)\right] + \mu_3^l - \mu_4^l = 0,$$

where μ_3^l and μ_4^l are the Lagrange multipliers on $0 \leq \beta_l^S \leq 1$. Using (33), the condition becomes

$$p\frac{\beta}{P_0}\big((P_1 - Y)\gamma_l^1\big) + \mu_3^l - \mu_4^l = 0. \tag{34}$$

It cannot be that $P_1 > Y$ since lenders would not want to buy any bonds at $t = 1$. When $P_1 < Y$, then $\mu_3^l > 0$ and hence $\beta_l^S = 0$. If $P_1 = Y$, then we can let $\beta_l^S = 0$ without loss of generality. To see this, plug $P_1 = Y$ and $\hat{p}(1+r) = 1$ (see condition (33)) into the lender's problem at $t = 1$ (equations (12) and (13)):

$$p\left[R\alpha + (1-\alpha-\beta) + \frac{\beta}{P_0}Y - \lambda_l c_1 - (1-\lambda_l)c_2\right], \tag{35}$$

where we substituted the budget constraint into the objective function using L_l. The objective function is independent of β_l^S.

The first-order condition of a lender with respect to reinvesting into the liquid asset at $t = 1$, γ_l^1, is

$$(1 - \alpha - \beta)p\left(1 - \frac{Y}{P_1}\right) + \mu_1^l = 0,$$

where we used $\beta_l^S = 0$, the lender's marginal value of liquidity $\mu^l = p\hat{p}(1 + r)$, and (33). We have ruled out that $P_1 > Y$. When $P_1 = Y$, the lender's problem is independent of γ_l^1 (see (35)) and we can set $\gamma_l^1 = 0$ without loss of generality. When $P_1 < Y$, then $\mu_1^l > 0$ and hence $\gamma_l^1 = 0$.

Proof of Lemma 6

As in the proof of lemma 3, the extra element is the presence of γ_l^2, the amount of bonds bought by banks with a liquidity surplus. But we can use the condition on market clearing in the bond market (17) to solve for γ_l^2.

Proof of Lemma 7

The first-order condition with respect to the allocation into the government bond, β, requires that

$$p\left[\frac{Y}{P_0} + \hat{p}(1 + r)\left(-\pi_l - \pi_h\frac{P_1}{P_0}\right) - (1 + r)\pi_h\left(1 - \frac{P_1}{P_0}\right)\right] = 0.$$

Solving the first-order condition for P_0 and using (7) yields

$$P_0 = \frac{1}{p\pi_l + \pi_h}\left(\frac{Y}{1 + r} + \pi_h(1 - p)P_1\right).$$

Using (18) to substitute for $\frac{Y}{1+r}$ results in $P_0 = P_1\frac{(1-\pi_h)p+\pi_h}{p\pi_l+\pi_h}$, which gives the desired result since $1 - \pi_h = \pi_l$.

Proof of Proposition 4

We require that

$$R\alpha + \gamma_l^2 \frac{Y}{P_1}(1-\alpha-\beta) + p(1+r)\left[\left(1-\gamma_l^2\right)(1-\alpha-\beta) - \lambda_l c_1\right] + \frac{\beta}{P_0}Y - (1-\lambda_l)c_2 = 0$$

and that

$$R\alpha - (1+r)\left[\lambda_h c_1 - (1-\alpha-\beta) - \frac{\beta}{P_0}P_1\right] - (1-\lambda_h)c_2 = 0,$$

where we have used the results from lemmas 1, 4, and 5, and equation (7). The amount of bonds purchased, γ_l^2, is given by market clearing in the bond market (equation (16) or, after simplification, (17)).

Using the result for c_1 from lemma 6, using one condition above to solve for c_2 and substituting back into the other condition yields

$$\frac{R}{1+r}\,\frac{\alpha}{1-\alpha-\beta} = \frac{(1-\lambda_l)\pi_l + (1-\lambda_h)\pi_h p}{\lambda_l \pi_l + \lambda_h \pi_h} - \frac{BP_1}{1-\alpha-\beta}\,\frac{(1-\lambda_l) - (1-\lambda_h)p}{\lambda_h - \lambda_l}. \tag{36}$$

Combining (36) with the result in proposition 3 gives the desired result.

References

Acharya, V. V., D. Gromb, and T. Yorulmazer. 2008. "Imperfect Competition in the Interbank Market for Liquidity as a Rationale for Central Banking." Working Paper, London Business School.

Acharya, V., H. Shin, and T. Yorulmazer. 2009. "Crisis Resolution and Bank Liquidity." Working Paper, New York University Stern School of Business.

Allen, F. and E. Carletti. 2008. "The Role of Liquidity in Financial Crises." Paper presented at the Federal Reserve Bank of Kansas City Symposium on Maintaining Stability in a Changing Financial System, Jackson Hole, Wyoming, August 21–23.

Allen, F., E. Carletti, and D. Gale. 2009. "Interbank Market Liquidity and Central Bank Intervention." *Journal of Monetary Economics* 56 (5): 639–52.

Allen, F., and D. Gale. 2000. "Financial Contagion." *Journal of Political Economy* 108 (1): 1–33.

———. 2004. "Financial Intermediaries and Markets." *Econometrica* 72 (4): 1023–61.

Bhattacharya, S., and D. Gale. 1987. "Preference Shocks, Liquidity, and Central Bank Policy." In *New Approaches to Monetary Economics*, ed. W. A. Barnett and K. J. Singleton, 69–88. Cambridge: Cambridge University Press.

Board of Governors of the Federal Reserve System. 2007. "Federal Reserve and Other Central Banks Announce Measures Designed to Address Elevated Pressures in Short-Term Funding Markets." Press Release, December 12.

Bruche, M., and J. Suarez. 2009. "The Macroeconomics of Money Market Freezes." CEPR Discussion Paper No. 7304.

Brunnermeier, M. K. 2009. "Deciphering the Liquidity and Credit Crunch 2007–2008." *Journal of Economic Perspectives* 23 (1): 77–100.

Brunnermeier, M. K., and L. H. Pedersen. 2009. "Market Liquidity and Funding Liquidity." *Review of Financial Studies* 22 (6): 2201–38.

Christensen, J., J. Lopez, and G. Rudebusch. 2009. "Do Central Bank Liquidity Operations Affect Interbank Lending Rates?" Mimeo, Federal Reserve Bank of San Francisco.

Diamond, D. W., and P. H. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91 (3): 401–19.

Diamond, D. W., and R. G. Rajan. 2001. "Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking." *Journal of Political Economy* 109 (2): 287–327.

———. 2005. "Liquidity Shortages and Banking Crises." *Journal of Finance* 60 (2): 615–47.

———. 2008. "The Credit Crisis: Conjectures about Causes and Remedies." Working Paper, University of Chicago Booth School of Business.

Farhi, E., M. Golosov, and A. Tsyvinski. 2008. "A Theory of Liquidity and Regulation of Financial Intermediation." *Review of Economic Studies* 76 (3): 973–92.

Fleming, M. J., W. B. Hrung, and F. M. Keane. 2009. "The Term Securities Lending Facility: Origin, Design, and Effects." *Current Issues in Economics and Finance* (Federal Reserve Bank of New York) 15 (2).

Freixas, X., and C. Holthausen. 2005. "Interbank Market Integration under Asymmetric Information." *Review of Financial Studies* 18 (2): 459–90.

Freixas, X., and J. Jorge. 2008. "The Role of Interbank Markets in Monetary Policy: A Model with Rationing." *Journal of Money, Credit, and Banking* 40 (6): 1151–76.

Freixas, X., A. Martin, and D. R. Skeie. 2008. "Bank Liquidity, Interbank Markets, and Monetary Policy." Working Paper.

Freixas, X., B. M. Parigi, and J. C. Rochet. 2000. "Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank." *Journal of Money, Credit, and Banking* 32 (3): 611–38.

Gorton, G. 2008. "The Panic of 2007." Paper presented at the Federal Reserve Bank of Kansas City Symposium on Maintaining Stability in a Changing Financial System, Jackson Hole, Wyoming, August 21–23.

———. 2009. "Information, Liquidity, and the (Ongoing) Panic of 2007." *American Economic Review: Papers and Proceedings* 99 (2): 567–72.

Heider, F., M. Hoerova, and C. Holthausen. 2009. "Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk." ECB Working Paper.

Hördahl, P., and M. R. King. 2008. "Developments in Repo Markets During the Financial Turmoil." *BIS Quarterly Review* (December).

Jacklin, C. 1987. "Demand Deposits, Trading Restrictions, and Risk Sharing." In *Contractual Arrangements for Intertemporal Trade*, ed. E. C. Prescott and N. Wallace, 26–47. Minneapolis: University of Minnesota Press.

McAndrews, J., A. Sarkar, and Z. Wang. 2008. "The Effect of the Term Auction Facility on the London Inter-Bank Offered Rate." Federal Reserve Bank of New York Staff Report No. 335.

Taylor, J. B. 2009. "The Financial Crisis and the Policy Responses: An Empirical Analysis of What Went Wrong." NBER Working Paper No. 14631.

Taylor, J. B., and J. C. Williams. 2009. "A Black Swan in the Money Market." *American Economic Journal: Macroeconomics* 1 (1): 58–83.