Firm-Specific Capital and Welfare*

Tommy Sveen[a] and Lutz Weinke[b,c]
[a]Monetary Policy Department, Norges Bank
[b]Department of Economics, Duke University
[c]Institute for Advanced Studies, Vienna

What are the consequences for monetary policy design implied by the fact that price setting and investment typically take place simultaneously at the firm level? To address this question we analyze simple (constrained) optimal interest rate rules in the context of a dynamic New Keynesian model featuring firm-specific capital accumulation as well as sticky prices and wages à la Calvo. We make the case for Taylor-type rules. They are remarkably robust in the sense that their welfare implications do appear to hinge neither on the specific assumptions regarding capital accumulation that are used in their derivation nor on the particular definition of natural output that is used to construct the output gap.

JEL Codes: E22, E31, E52.

1. Introduction

How does *firm-specific* capital accumulation affect the desirability of alternative arrangements for the conduct of monetary policy? We address this question employing a New Keynesian (NK) framework, i.e., a dynamic stochastic general equilibrium model featuring nominal rigidities combined with monopolistic competition. Specifically,

*Thanks to seminar participants at the 2006 Annual Congress of the EEA, Duke University, HEC Montréal, Humboldt-Universität zu Berlin, Ludwig-Maximilians-Universität München, Magyar Nemzeti Bank, Norges Bank, Universität Bonn, Université du Luxembourg, Universität Trier, Universität zu Köln, and Universität Würzburg. Special thanks to Ida Wolden Bache, Jordi Galí, Øistein Røisland, and Stephanie Schmitt-Grohé, as well as to the editors Carl Walsh and Michael Woodford and two anonymous referees. The usual disclaimer applies. The views expressed in this paper are those of the authors and should not be attributed to Norges Bank. E-mails: tommy.sveen@norges-bank.no and weinkel@econ.duke.edu.

we consider an economic environment with sticky prices and wages à la Calvo (1983). Our model is therefore similar to the one developed in Erceg, Henderson, and Levin (2000) except for the fact that we allow for capital accumulation.[1] The welfare criterion is derived from the average of household utility functions, along the lines of Rotemberg and Woodford (1998).

What is the relevance of our analysis? Edge (2003) shows how the work by Rotemberg and Woodford (1998) can be extended to conduct a welfare analysis in the context of an NK model where capital accumulation is endogenous. She assumes, however, that firms have access to a rental market for capital,[2] which is not an innocuous simplification in an NK model, as analyzed in Sveen and Weinke (2005, 2007) and Woodford (2005).[3] In the present paper we show how a welfare analysis can be conducted in the context of an NK model featuring firm-specific capital accumulation (FS for short). Moreover, we explain how and why the conclusions regarding the desirability of monetary policy change if a rental market for capital (RM for short) is assumed instead. The latter analysis is of considerable interest given the widespread use of the rental-market assumption in the context of welfare-based evaluations of monetary policy.[4]

We obtain three results. First, we analyze interest rate rules according to which the central bank adjusts the nominal interest rate only in response to changes in price and wage inflation. In that context we find that optimized interest rate rules prescribe putting relatively more weight on price inflation than on wage inflation under FS, whereas the opposite is true under RM. Moreover, using the optimized interest rate rule associated with RM in the FS specification implies a large welfare loss, as we discuss. Interestingly, the

[1]Erceg, Henderson, and Levin (2000) assume that the aggregate capital stock is constant and that there exists a rental market for capital.

[2]Another difference between our work and Edge's is that she assumes frictionless investment, whereas we follow Woodford (2005) in assuming a convex adjustment cost at the firm level.

[3]Schmitt-Grohé and Uribe (2007b) argue that both the rental-market assumption and the assumption of firm-specific capital are somewhat extreme. However, the work by Altig et al. (2005) and Eichenbaum and Fisher (2007) suggests that the assumption of firm-specific capital is appealing on empirical grounds.

[4]See, e.g., Levin et al. (2006) and Schmitt-Grohé and Uribe (2007b).

additional endogenous price stickiness implied by the presence of firm-specific capital (and the lack thereof under RM) is identified as the main reason behind the difference in implications for optimal monetary policy design. Sveen and Weinke (2005) show that this is the *only* difference between the two models if attention is restricted to a first-order approximation to equilibrium dynamics. In the present paper we therefore find that our price stickiness metric is also useful from a normative point of view. This is surprising because our welfare criterion, a second-order approximation to the unconditional expectation of the household's utility, is *not* identical in the two models if the price stickiness is increased in RM in such a way that FS and RM are identical, up to the first order. Let us relate our first result to the existing literature. Schmitt-Grohé and Uribe (2007a) demonstrate in the context of a rental-market model that the relative weight attached to price and wage inflation in an optimized interest rate rule depends crucially on which nominal variable is stickier.[5] Our analysis shows that the difference in policy implications between FS and RM can be understood in an analogous way.

We also analyze Taylor-type rules, i.e., interest rate rules prescribing that the central bank reacts to price inflation and to the output gap. Our second result is that those interest rate rules are remarkably robust in the following sense. If the optimized rule implied by RM is used in FS, then the resulting welfare loss is small compared with the outcome under the optimized rule associated with that model.[6] Consequently, the central bank does not need to take a stand on which specification of capital accumulation is the empirically more plausible one if it uses a Taylor-type rule.

But how should the output gap be defined? So far there is no consensus in the literature on the answer to that question. Neiss

[5]Schmitt-Grohé and Uribe (2006) make the case for price stability as the central goal of optimal monetary policy. They show that desirable outcomes can be implemented by a combination of passive monetary and active fiscal policy. In the present paper we focus exclusively on optimal monetary policy.

[6]In related work Levin, Wieland, and Williams (1999) and Levin and Williams (2003) argue that simple Taylor-type rules optimized for one model perform well under alternative models. Our work extends their analysis to a setting that takes into account how the microfounded welfare criterion depends on the model that is used.

and Nelson (2003) and Woodford (2003, ch. 5) propose two alternative definitions. Our third result is that the difference between these two competing definitions matters very little for the resulting welfare implications, and we explain why this is so.

The remainder of the paper is organized as follows. The model is outlined in section 2. We present the welfare criterion in section 3. Our results are shown and interpreted in section 4. Section 5 concludes.

2. The Model

The model we use to analyze the implications of firm-specific capital accumulation for monetary policy design is an NK framework with complete financial markets.

2.1 Households

Households maximize expected discounted utility

$$E_t \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, N_{t+k}(h)),$$

where β is the subjective discount factor. The subscript t is generally used to indicate that a variable is dated as of that period and E_t denotes an expectation operator that is conditional on information available through time t. Hours worked by household h are given by $N_t(h)$ and C_t is a Dixit-Stiglitz consumption aggregate. Specifically,

$$C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where ε is the elasticity of substitution between different varieties of goods $C_t(i)$. The associated price index is defined as $P_t \equiv (\int_0^1 P_t(i)^{1-\varepsilon} di)^{\frac{1}{1-\varepsilon}}$ and optimal allocation of any spending on the available goods implies that consumption expenditure can be written as $P_t C_t$. Household h's period utility is given by the following function:

$$U(C_t, N_t(h)) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\phi}}{1+\phi}, \tag{2}$$

where parameter σ denotes the household's relative risk aversion and parameter ϕ can be interpreted as the inverse of the Frisch aggregate labor-supply elasticity. Each household is assumed to be the monopolistically competitive supplier of its differentiated type of labor, $N_t(h)$. As in Erceg, Henderson, and Levin (2000), we assume staggered wage setting à la Calvo (1983); i.e., each household faces a constant and exogenous probability, θ_w, of getting to reoptimize its wage in any given period. Our assumptions of separable preferences combined with complete financial markets imply that the heterogeneity across households in their hours worked does not translate into consumption heterogeneity. This is reflected in our notation. Optimizing behavior on the part of firms implies that demand for type h labor, $N_t^d(h)$, is given by

$$N_t^d(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\varepsilon_N} N_t^d, \tag{3}$$

where $W_t(h)$ denotes the nominal wage posted by household h and ε_N gives the elasticity of substitution between different types of labor. Finally, W_t and N_t^d denote, respectively, the aggregate nominal wage and aggregate labor demand. They are defined as the corresponding aggregate prices and quantities for goods.

Under standard assumptions, the relevant budget constraint prescribes that the present value of all expenditures cannot be greater than the value of a household's initial assets and the present value of its income. The latter derives from wage payments and profits resulting from ownership of firms net of taxes.[7] We assume that there are only lump-sum taxes and the only role of the government is to levy these taxes to finance subsidies in goods and factor markets which render the steady state of our model Pareto optimal. This assumption in turn is needed to compute our welfare criterion up to the second order using a first-order approximation to the equilibrium dynamics, as we are going to see.

For future reference, let us note two implications of households' optimizing behavior. First, we obtain a stochastic discount factor for random nominal payments, $Q_{t,t+1}$, from a standard intertemporal optimality condition,

[7] For details, see Woodford (2003, ch. 2).

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)=Q_{t,t+1}. \tag{4}$$

The stochastic discount factor is linked to the gross nominal interest rate, R_t, by the relationship $E_t\{Q_{t,t+1}\}=R_t^{-1}$, which holds in equilibrium. Second, under our assumptions the first-order condition for wage setting reads

$$E_t^w\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k\,N_{t+k}^d(h)\,C_{t+k}^{\sigma}\left[\frac{W_t(h)}{P_{t+k}}-MRS_{t+k}(h)\right]\right\}=0, \tag{5}$$

where $MRS_t(h)\equiv N_t(h)^{\phi}\ C_t^{\sigma}$ is the marginal rate of substitution of consumption for leisure of household h. Moreover, E_t^w is meant to indicate an expectation that is conditional on time t information, but integrating only over those future states in which the household has not reset its wage since period t.

2.2 Firms

There is a continuum of firms, and each of them is the monopolistically competitive producer of a differentiated good. Each firm i has access to a Cobb-Douglas technology,

$$Y_t(i)=X_tK_t(i)^{\alpha}N_t(i)^{1-\alpha}, \tag{6}$$

where parameter α measures the capital share in the production function. Aggregate technology is given by X_t, and $K_t(i)$ and $N_t(i)$ denote, respectively, firm i's capital stock and labor input used in its production $Y_t(i)$. Technology shocks are assumed to be the only source of aggregate uncertainty.[8] This is another modeling choice that is guided by Erceg, Henderson, and Levin (2000).[9] Specifically, we consider a stationary $AR(1)$ process for the log of technology,

$$x_t=\rho_a x_{t-1}+\varepsilon_t, \tag{7}$$

[8]Of course, the extent to which technology shocks are an important source behind the observable business-cycle fluctuations is the topic of an ongoing debate. See, e.g., Galí and Rabanal (2005).

[9]Strictly speaking, Erceg, Henderson, and Levin (2000) do not only assume technology shocks, but they restrict their welfare analysis to this kind of shock.

where parameter $\rho_a \in (0,1)$ and ε_t is assumed to be i.i.d. with mean zero. Firms face three additional restrictions. First, we assume Calvo (1983) pricing; i.e., each firm faces a constant and exogenous probability, θ, of getting to reoptimize its price in any given period. Second, we follow Woodford (2005) in assuming that investment at the firm level is restricted in the following way:

$$I_t(i) = \Gamma\left(\frac{K_{t+1}(i)}{K_t(i)}\right) K_t(i). \tag{8}$$

In the last equation, $I_t(i)$ denotes the amount of the composite good[10] necessary to change firm i's capital stock from $K_t(i)$ to $K_{t+1}(i)$ one period later. Moreover, function $\Gamma(\cdot)$ is assumed to satisfy the following: $\Gamma(1) = \delta$, $\Gamma'(1) = 1$, and $\Gamma''(1) = \epsilon_\psi$. Parameter δ denotes the depreciation rate and $\epsilon_\psi > 0$ measures the convex capital adjustment cost in a log-linear approximation to the equilibrium dynamics. Third, cost minimization by firms and households implies that demand for each individual good i can be written as follows:

$$Y_t^d(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t^d, \tag{9}$$

where $Y_t^d \equiv C_t + I_t$ denotes aggregate demand and $I_t \equiv \int_0^1 I_t(i)di$ denotes aggregate investment demand. Given those constraints, each firm i is assumed to maximize its market value:

$$\max \sum_{k=0}^{\infty} E_t\{Q_{t,t+k}\left[Y_{t+k}^d(i)P_{t+k}(i) - W_{t+k}N_{t+k}(i) - P_{t+k}I_{t+k}(i)\right]\}.$$

For future reference, let us mention two implications of optimizing behavior at the firm level. First, firm i's first-order condition for capital accumulation reads

$$\Gamma_t'(\cdot)P_t = E_t\left\{Q_{t,t+1}P_{t+1}\left[MS_{t+1}(i) - \Gamma_{t+1}(\cdot) + \Gamma_{t+1}'(\cdot)\frac{K_{t+2}(i)}{K_{t+1}(i)}\right]\right\}, \tag{10}$$

[10] We assume that the elasticity of substitution is the same as in the consumption aggregate.

where $MS_t(i) \equiv \frac{W_t}{P_t}\frac{MPK_t(i)}{MPL_t(i)}$ is the real marginal return to firm i's capital. The latter results from savings in labor costs. Second, let us note that under our assumptions firm i's first-order condition for price setting reads

$$\sum_{k=0}^{\infty} \theta^k E_t^p \{Q_{t,t+k} Y_{t+k}^d(i) \left[P_t^*(i) - P_{t+k} MC_{t+k}^n(i)\right]\} = 0, \qquad (11)$$

where $MC_t(i) \equiv \frac{W_t/P_t}{MPL_t(i)}$ measures the real marginal cost, $MPL_t(i)$ is the marginal product of labor of firm i, and E_t^p is meant to indicate an expectation that is conditional on time t information, but integrating only over those future states in which the firm has not reoptimized its price since period t.

2.3 Market Clearing

Clearing of the labor market requires for each type of labor h

$$N_t(h) = N_t^d(h). \qquad (12)$$

Likewise, market clearing for each variety i requires at each point in time

$$Y_t(i) = C_t^d(i) + I_t^d(i), \qquad (13)$$

where $C_t^d(i)$ is consumption demand for good i, while $I_t^d(i)$ denotes investment demand for that good. To close the model, we need to specify monetary policy. We will come back to that point.

2.4 Some Linearized Equilibrium Conditions

The starting point of our welfare analysis is a linear approximation to the equilibrium dynamics around a steady state with zero inflation. Since the details of the linearization have been developed elsewhere,[11] we just briefly mention the resulting equilibrium conditions. Unless specified otherwise, a lowercase letter denotes the log-deviation of the original variable from its steady-state value. Let us

[11] See, e.g., Erceg, Henderson, and Levin (2000), Sveen and Weinke (2005), and Woodford (2005).

already note that the linearized equilibrium conditions are identical for FS and RM, except for the respective inflation equations.

The consumption Euler equation reads

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1} - \rho), \qquad (14)$$

where $\rho \equiv -\log \beta$ is the time discount rate. We have also used the notation $r_t \equiv \log(R_t)$ for the nominal interest rate and $\pi_t \equiv \log(\frac{P_t}{P_{t-1}})$ for inflation. The law of motion of capital is obtained from averaging and aggregating optimizing investment decisions on the part of firms. This implies

$$\Delta k_{t+1} = \beta E_t \Delta k_{t+2} + \frac{1}{\epsilon_\psi}[(1-\beta(1-\delta))E_t ms_{t+1} - (r_t - E_t \pi_{t+1} - \rho)], \qquad (15)$$

where $K_t \equiv \int_0^1 K_t(i)di$ denotes aggregate capital, and $MS_t \equiv \int_0^1 MS_t(i)di$ measures the average real marginal return to capital. Aggregate production is pinned down by aggregate labor, capital, and technology:

$$y_t = x_t + \alpha k_t + (1-\alpha)n_t. \qquad (16)$$

The wage-inflation equation results from averaging and aggregating optimal wage-setting decisions on the part of households. It takes the following simple form:

$$\omega_t = \beta E_t \omega_{t+1} + \lambda_\omega (mrs_t - rw_t), \qquad (17)$$

where $\lambda_\omega \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w} \frac{1}{1+\eta\varepsilon_N}$. Moreover, $\omega_t \equiv \log(\frac{W_t}{W_{t-1}})$ denotes nominal wage inflation, $MRS_t \equiv \int_0^1 MRS_t(h)dh$ gives the average marginal rate of substitution of consumption for leisure, and $RW_t \equiv \int_0^1 \frac{W_t(h)}{P_t}dh$ denotes the average real wage.

The price-inflation equation associated with FS takes the familiar form

$$\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t, \qquad (18)$$

where $MC_t \equiv \int_0^1 \frac{MC_t^n(i)}{P_t}di$ denotes the average real marginal cost. In RM, parameter λ takes its standard value $\frac{(1-\beta\theta)(1-\theta)}{\theta}$. Importantly, the determination of the value for parameter λ changes if FS is assumed instead. In that case, its value is computed numerically, as discussed in Woodford (2005). His method posits linearized rules for price setting and for investment:

$$\widehat{p}_t^*(i) = \widehat{p}_t^* - \tau_1 \widehat{k}_t(i), \tag{19}$$

$$\widehat{k}_{t+1}(i) = \tau_2 \widehat{k}_t(i) + \tau_3 \widehat{p}_t(i), \tag{20}$$

where τ_1, τ_2, and τ_3 are parameters that are determined by the method of undetermined coefficients. In stating the decision rules, we have also used the definitions $\widehat{p}_t^*(i) \equiv \log(\frac{P_t^*(i)}{P_t})$, $\widehat{p}_t(i) \equiv \log(\frac{P_t(i)}{P_t})$, $\widehat{p}_t^* \equiv \log(\frac{P_t^*}{P_t})$, and $\widehat{k}_t(i) \equiv \log(\frac{K_t(i)}{K_t})$. For our purposes in the present paper, these rules turn out to be of crucial importance, for they allow us to compute the welfare-relevant second moments of the cross-sectional distributions of prices and capital holdings.

The goods market-clearing equation reflects our assumption that there are subsidies offsetting the distortions associated with monopolistic competition in goods and labor markets. This implies that the steady state of our model is Pareto efficient. Specifically, we have

$$y_t = \zeta c_t + \frac{1-\zeta}{\delta}[k_{t+1} - (1-\delta)k_t], \tag{21}$$

where $\zeta \equiv \frac{\rho+\delta(1-\alpha)}{\rho+\delta}$ denotes the steady-state consumption to output ratio.

The frequency of our model is quarterly. Unless stated otherwise, we assign the following values to the model parameters. We assume $\alpha = 0.36$ for the capital share. Our choice for the value of the risk-aversion parameter σ is 2. The elasticity of substitution between goods, ε, is set to 11, and we assume $\varepsilon_N = 6$ for the elasticity of substitution between different types of labor. Our baseline value for the Calvo parameter for price setting, θ_p, is 0.75, and we assume the same value for its wage-setting counterpart θ_w. The rate of capital depreciation, δ, is assumed to be equal to 0.025, and we set $\epsilon_\psi = 3$ for the capital adjustment cost. These parameter values are justified in Sveen and Weinke (2007) and the references therein. Finally, the coefficient of autocorrelation in the process of technology, ρ_a, is assumed to take the value 0.95, as in Erceg, Henderson, and Levin (2000).

3. Welfare

Let the policymaker's period welfare function be the unweighted average of households' period utility,

$$\mathcal{W}_t \equiv U(C_t) + \int_0^1 V(N_t(h))dh = U(C_t) + E_h\{V(N_t(h))\}. \quad (22)$$

In what follows, period welfare is expressed as a fraction of steady-state Pareto optimal consumption; i.e., we consider $\frac{\mathcal{W}_t - \overline{\mathcal{W}}}{\overline{U}_C \overline{C}}$, where a bar indicates the steady-state value of the original variable and U_C is the marginal utility of consumption. Based on the method in Rotemberg and Woodford (1998), we compute a second-order approximation to period welfare.[12] Our welfare criterion is the unconditional expectation of period welfare, which we write in the way proposed by Svensson (2000):[13]

$$\widetilde{\mathcal{W}} \equiv \lim_{\beta \to 1} E_t \left\{ (1-\beta) \sum_{k=0}^{\infty} \beta^k \left(\frac{\mathcal{W}_{t+k} - \overline{\mathcal{W}}}{\overline{U}_C \overline{C}} \right) \right\}.$$

In appendix 1 we derive the following expression for that welfare criterion in the context of FS:

$$\begin{aligned} \widetilde{\mathcal{W}}^{FS} \simeq\ & \Omega_1 E\{y_t^2\} + \Omega_2 E\{c_t^2\} + \Omega_3 E\{i_t^2\} + \Omega_4 E\{(\Delta k_{t+1})^2\} \\ & + \Omega_5 E\{n_t^2\} + \Omega_6 E\{\lambda_t\} + \Omega_1^{FS} E\{\Delta_t\} \\ & + \Omega_2^{FS} E\{\kappa_t\} + \Omega_3^{FS} E\{\psi_t\}, \end{aligned} \quad (23)$$

where the symbol $\simeq$ is meant to indicate that an approximation is accurate up to the second order. The operator E denotes the unconditional expectation. We have also used the following definitions: $\Delta_t \equiv Var_i \widehat{p}_t(i)$, $\kappa_t \equiv Var_i k_t(i)$, $\psi_t \equiv Cov_i(\widehat{p}_t(i), k_t(i))$, and $\lambda_t \equiv Var_h \widehat{w}_t(h)$, with $\widehat{W_t}(h) \equiv \frac{W_t(h)}{W_t}$ and Var_i and Cov_i denoting the cross-sectional variance and covariance operators. Parameters

[12] The proof that the method of Rotemberg and Woodford (1998) can be applied to the problem at hand carries over from Edge's (2003) work to our work because the relevant steady-state properties of the two models are identical.

[13] We scale the discounted sum by $(1-\beta)$ to keep the limit finite as the discount factor goes to unity (from below).

Ω_1 to Ω_6 and Ω_1^{FS} to Ω_3^{FS} are functions of the structural parameters. The latter group of parameters is specific to FS, whereas the former one is common between FS and RM. Both sets of parameters are defined in appendix 1. The main complication that we have to face is to calculate the cross-sectional variances of prices and capital holdings at each point in time as well as their covariance. In order to overcome that difficulty, we make one key observation: Woodford's (2005) linearized rules for price setting and for investment can be used to compute the relevant second moments with the accuracy that is needed for our second-order approximation to welfare.

It is useful to note two properties of equation (23). First, for economically relevant calibrations of our model, the resulting coefficient values in the welfare criterion do not all have the same sign. Second, the variables entering (23) must respect the linearized equilibrium conditions described in the previous section. This is important because these two aspects of (23) are precisely the reason why our welfare criterion does not imply that increasing the variability of those variables that enter with a positive coefficient would lead to an increase in welfare. For instance, increasing the variability of output implies that the associated use of factor inputs would also need to become more variable, and this in turn would reduce welfare.

For the rental-market case, our welfare criterion reads

$$\widetilde{\mathcal{W}}^{RM} \simeq \Omega_1 E\{y_t^2\} + \Omega_2 E\{c_t^2\} + \Omega_3 E\{i_t^2\} + \Omega_4 E\{(\Delta k_{t+1})^2\} + \Omega_5 E\{n_t^2\} + \Omega_6 E\{\lambda_t\} + \Omega^{RM} E\{\Delta_t\}, \tag{24}$$

as shown in appendix 2, where we also define parameter Ω^{RM}. Compared with FS, the analysis is greatly simplified by the fact that the capital labor ratio is constant across firms, as discussed in Edge (2003). In the next section we will use our model to analyze the desirability of alternative arrangements for the conduct of monetary policy. This involves a numerical evaluation of the welfare criterion. Before we turn to this, let us make a final remark. It is not necessary for our purposes to rewrite the welfare criterion in the ways proposed by Edge (2003). In order to state the welfare criterion in terms of suitably defined gap terms, she uses the linearized equilibrium conditions to substitute out some of the variables in the welfare measure. This implies cross-terms among the remaining variables, which are absent in our formulation.

4. Results

We consider two prominent families of monetary policy rules and analyze constrained optimal rules; i.e., we restrict attention to a particular subset of possible parameter values that parameterize the rule.[14] Our main objective is to explain how and why the associated constrained optimal values of the policy parameters change in each case depending on whether or not a rental market for capital is assumed. It is useful to note that rational-expectations equilibrium is locally unique (i.e., determinate) under the constrained optimal policies.

4.1 The Welfare Consequences of Responding to Price and Wage Inflation

We start by considering interest rate rules of the form

$$r_t = \rho + \tau_r(r_{t-1} - \rho) + \tau_s[\tau_\omega \omega_t + (1 - \tau_\omega)\pi_t], \tag{25}$$

where parameter τ_s measures the overall responsiveness of the nominal interest rate to changes in inflation, whereas τ_ω is the relative weight put on wage inflation. The weight on price inflation is therefore given by $(1 - \tau_\omega)$. Finally, parameter τ_r denotes the interest rate smoothing coefficient. Only positive parameter values are considered and, moreover, we require parameter τ_ω to be less than or equal to one.[15]

We compare the optimized interest rate rules under FS and RM. In each case we report the optimized coefficients entering the interest

[14] A different approach to welfare-based evaluation of monetary policy is to consider optimal monetary policy. There are, however, different possible and plausible definitions of optimality that have been proposed in the literature. One key distinction is the one between discretion versus commitment, and two important variants of the latter concept are Ramsey optimal policy versus timeless perspective.

[15] Our computational strategy is as follows. First, we define a function that takes the policy parameters as an input and returns the associated value of our welfare criterion. We use DYNARE (http://www.cepremap.cnrs.fr/dynare/) to construct that function. In a second step, we find the optimal parameter configuration among the values we consider. Thanks to Larry Christiano for providing us with Matlab code, which we have used in the computation of λ.

Table 1. Price- and Wage-Inflation Rule

Parameter	FS	RM
τ_r	0.972	0.824
τ_s	2.390	5.852
τ_w	0.415	0.709
Welfare	−9.363	−8.169

rate rule as well as the associated welfare. We follow Erceg, Henderson, and Levin (2000) and isolate the cost of nominal rigidities; i.e., we compute welfare for an economy with flexible prices and flexible wages and subtract the resulting expression from our welfare measure. As they do, we divide this difference by the productivity innovation variance.[16] The results are shown in table 1.

Regardless of whether FS or RM is used, the implied optimized rule prescribes to adjust the nominal interest rate in response to changes in both wage inflation and price inflation. That is intuitive: both kinds of inflation are costly in welfare terms since we model two nominal rigidities. Interestingly, the optimized rule prescribes to react relatively more to price inflation in FS, whereas the opposite holds true in RM. Our intuition is as follows. We observe two things. First, Sveen and Weinke (2005) show that price stickiness can be used to measure the difference between RM and FS, if attention is restricted to a first-order approximation to the equilibrium dynamics: the feature of firm-specific capital implies that price setters internalize the consequences of their price-setting decisions for the marginal cost they face. That makes them more reluctant to change their prices in FS than under RM.[17] Specifically, we show in

[16]Let us give a concrete example for the interpretation of the welfare numbers in our tables. Suppose the productivity innovation variance is 0.01^2. Then, the number -10 for welfare would mean that the representative household would be willing to give up $10 \times 0.01^2 \times 100 = 0.1$ percentage point of steady-state (Pareto optimal) consumption in order to avoid the business-cycle cost associated with the presence of the nominal rigidities in our model.

[17]Similar intuitions have originally been developed by Galí, Gertler, and López-Salido (2001) and Sbordone (2002) in the context of models where capital is assumed to be a constant factor. For an early model featuring differences in the marginal cost across firms, see Woodford (1998).

our 2005 paper that a value of about 0.9 is needed in RM in order to obtain equivalence with FS if the value 0.75 is assigned to the price-stickiness parameter in the latter case and all the remaining parameters are held constant at conventional values. Put differently, the rental-market assumption turns off the *endogenous* price stickiness that is implied by the alternative specification with firm-specific capital. It is an important corollary of our result that an upward-biased estimate of the price-stickiness parameter is obtained if the econometrician looks at the macro data through the lens of RM (if the data-generating process is better described by FS).[18] Second, it is a well-understood property of many New Keynesian models that the central bank achieves the most desirable welfare outcome if it cares relatively more about the nominal variable that is relatively stickier.[19] Combining these two observations, the previous finding seems intuitive. Since the rental-market assumption eliminates the endogenous part of the price stickiness, the central bank should care relatively more about wage inflation in that model. The reason is endogenous wage stickiness. That feature is common to FS and RM: in both models households internalize the consequences of their wage-setting decisions for the marginal disutility of labor they face. On the other hand, if firm-specific capital is taken into account, then the implied endogenous price stickiness is strong enough to make it worthwhile for the central bank to care relatively more about price inflation.

So far our intuition relies on a finding—namely, our price-stickiness metric—that has been obtained in the context of a first-order approximation to the equilibrium dynamics. This intuition could easily be misleading for our purposes here. The reason is that the second-order approximation to the average of households' expected utility, our welfare criterion, is not equivalent in both models if we just change the price stickiness in such a way that the two models would be identical up to the first order. We therefore challenge our intuition by conducting the following experiment, the results of which are shown in table 2.

[18]Based on that observation, Eichenbaum and Fisher (2007) provide some empirical evidence for the plausibility of FS.

[19]See, e.g., Aoki (2001) and Benigno (2003).

Table 2. Robustness I: Rules from the RM Model Used in FS

Parameter	RM Rule with $\theta = 0.75$	RM Rule with $\theta = 0.8947$
τ_r	0.824	1.003
τ_s	5.852	1.095
τ_w	0.709	0.360
Welfare	−12.678	−9.472

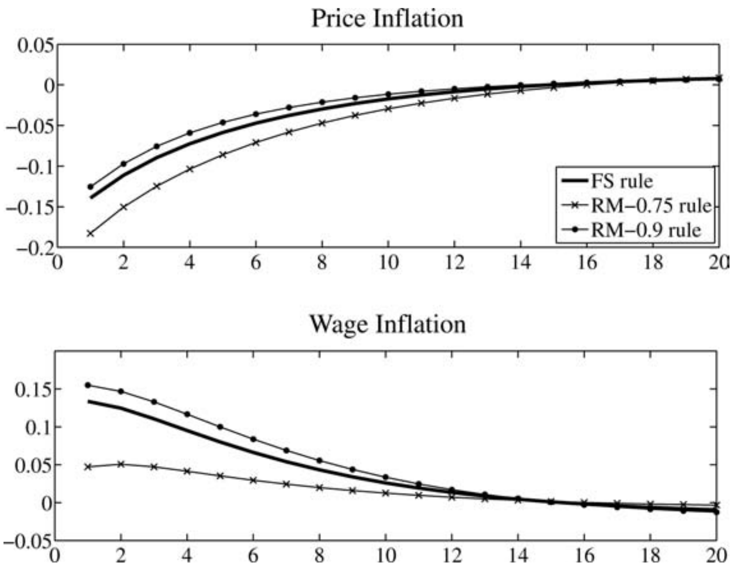
We compute welfare in FS as implied by the optimized policy rule in RM under the baseline calibration. The resulting decrease in welfare is 29.1 percent with respect to the outcome under the optimized rule for FS. Now we compute constrained optimal policy in RM for a price-stickiness parameter equal to 0.8947, in which case RM and FS are identical, up to the first order. The implied optimized rule looks similar to the one associated with FS under the baseline calibration. Specifically, the rule prescribes to react relatively more to price inflation than to wage inflation. Moreover, the decrease in welfare which obtains if that rule is used in FS is just 3.6 percent, which we regard as being negligible. The last result suggests that our price-stickiness metric is useful from a welfare point of view.[20]

To further illustrate the macroeconomic consequences of the three different monetary policy rules, we construct impulse responses to a one-standard-deviation shock to productivity for price inflation and wage inflation. They are shown in figure 1. Under the baseline calibration, the optimal simple rule for FS implies that price inflation is stabilized relatively more than is the case if the optimized rule for RM is used instead. However, if the price-stickiness parameter is set to 0.8947 in RM, then the implied optimized rule delivers an outcome in FS that is similar to the one under the optimized rule for that model.

Next we consider the welfare implications of interest rate rules prescribing that the central bank adjusts the nominal interest rate

[20]In principle, whether or not the price-stickiness metric is useful to tell the difference in welfare implications between FS and RM could depend on the specification of monetary policy. For all the policies we consider, however, our metric turns out to be useful.

Figure 1. Impulse Responses to a Technology Shock with Different Wage-Inflation Rules



not only in response to nominal variables but also as a function of a measure of real economic activity.

4.2 The Welfare Consequences of Taylor-Type Rules

We now turn to the welfare implications of Taylor-type rules,

$$r_t = \rho + \tau_r(r_{t-1} - \rho) + \tau_\pi \pi_t + \tau_y y_t^{gap}, \tag{26}$$

where parameters τ_π and τ_y measure the responsiveness of the nominal interest rate to the respective changes in the rate of price inflation and the output gap. All parameters are assumed to be non-negative. The output gap, y_t^{gap}, is generally defined as the difference between the equilibrium output in an economy with frictions and natural output, i.e., the equilibrium output that would obtain in the absence of nominal frictions. In the context of a model featuring endogenous capital accumulation, Woodford (2003, ch. 5) proposes to refine the notion of natural output in the following way. He uses the equilibrium output that would obtain if the nominal rigidities were absent

Table 3. Taylor-Type Rule with Woodford Output Gap

Parameter	FS	RM
τ_r	1.019	1.641
τ_π	0	0.2618
τ_y	0.096	0.331
Welfare	−9.471	−8.288

and expected to be absent in the future but taking as given the capital stock resulting from optimizing investment behavior in the past in an environment *with* the nominal rigidities present. Woodford argues that this measure of natural output is more closely related to equilibrium determination than the alternative measure which has been used by Neiss and Nelson (2003). Under their definition, natural output is the equilibrium output that would obtain if nominal rigidities were not only currently absent and expected to be absent in the future but had also been absent *in the past*. Indeed, intuitively, the Neiss and Nelson definition of natural output appears to be a bit artificial. We find, however, that from a practical point of view it does not matter for the design of constrained optimal interest rate rules which concept of natural output is used to compute the output gap. We will come back to this point. Before that, let us consider some welfare implications of Taylor-type rules using Woodford's definition of the output gap.[21] The results are shown in table 3.

The optimal rule implied by FS prescribes not to respond to changes in price inflation. On the other hand, under RM, we find that the central bank should react to both price inflation and the output gap. In both models, the optimal interest rate rule features superinertia; i.e., the interest rate smoothing coefficient is larger

[21] Our computational strategy to calculate natural output under Woodford's definition is straightforward. First, we calculate the parameters of the linear function mapping aggregate capital and technology into equilibrium aggregate output in an environment *without* any nominal frictions present. Second, we take the equilibrium value of aggregate capital as implied by FS (or by RM when we study that case), combine it with the level of technology, and compute Woodford's natural output, invoking the above mapping.

Table 4. Robustness II: Rules from the RM Model Used in FS

Parameter	RM Rule with $\theta = 0.75$	RM Rule with $\theta = 0.8947$
τ_r	1.641	1.034
τ_π	0.262	0.059
τ_y	0.331	0.030
Welfare	−9.839	−9.619

than one. Our finding that a strong response to changes in the output gap is desirable on welfare grounds is reminiscent of a result by Erceg, Henderson, and Levin (2000). They show that stabilizing the output gap completely achieves an outcome that is close to the one that is optimal in the context of their model. We have already mentioned the fact that our model is more complicated than theirs since we allow for endogenous capital accumulation. However, the intuition behind their result also seems to be at work in our model: by targeting the output gap, the central bank makes sure that fluctuations in wages and prices take place in such a way that the relatively stickier variable moves less.[22]

Our next result is that the loss is negligible if welfare in FS is computed using the optimized rule implied by RM. We therefore argue that Taylor-type rules are very robust. The results are shown in table 4.

As the last table also indicates, welfare associated with using the rule implied by RM in FS can be further increased if the price stickiness is adjusted in RM in such a way that both models would be identical up to the first order. Once again, our price-stickiness metric turns out to be useful. The policy implications of RM are surprisingly accurate if an upward-biased estimate of the price-stickiness parameter (of the kind that the econometrician actually obtains if she looks at the data through the lens of that model) is used in the analysis. Somewhat surprisingly, however, the optimal relative weight attached to the output gap in RM becomes *smaller* (and

[22]Interestingly, the optimal responsiveness to the output gap is zero if we allow for an output-gap response of the nominal interest rate in addition to the responses to price inflation and wage inflation that we had analyzed before.

Table 5. Taylor-Type Rule with Neiss and Nelson Output Gap

Parameter	FS	RM
τ_r	1.033	2.019
τ_π	0	0.133
τ_y	0.090	0.344
Welfare	−9.416	−8.392

hence less in line with the corresponding value implied by FS) if the price stickiness is increased. That feature appears, however, to be specific to Woodford's definition of natural output, as we are going to see next. Finally, we analyze Taylor-type rules using Neiss and Nelson's (2003) definition of the output gap. The results are reported in table 5.

Overall, optimized rules implied by FS and RM are very similar to the ones obtained before under Woodford's definition of the output gap. In particular, we find again that under RM the optimized rule prescribes to react to both inflation and the output gap, whereas the optimized rule associated with FS features no response to inflation. We also confirm our previous finding that Taylor-type rules are very robust. If the optimized rule implied by RM is used under FS, then the resulting welfare loss is negligible and, moreover, the loss can be further reduced if the price-stickiness parameter is adjusted in RM according to our metric. The results are shown in table 6.

There is only one (small) difference with respect to the previous analysis of Taylor-type rules featuring an output gap à la Woodford. Under the Neiss and Nelson definition, the resulting interest

Table 6. Robustness III: Rules from the RM Model Used in FS

Parameter	RM Rule with $\theta = 0.75$	RM Rule with $\theta = 0.8947$
τ_r	2.019	1.023
τ_π	0.133	0
τ_y	0.344	0.053
Welfare	−9.981	−9.442

rate rules become more similar between FS and RM if we adjust the price stickiness in RM as prescribed by our metric.[23]

Our intuition for why the particular definition of the output gap that is used in the analysis of optimal monetary policy matters so little is simple. The capital stock does not change much at business-cycle frequencies, and the difference between the change in capital implied by a model with and without nominal rigidities present is even less important. In fact, the unconditional correlation between both output-gap measures is 0.9988 in FS under our baseline calibration. This remarkably high correlation between the two variables might, of course, be altered in the presence of additional sources of uncertainty. This caveat notwithstanding, we conclude that the practical relevance of the difference in the two output-gap measures seems to be relatively limited.

Regardless of the definition of the output gap, Taylor-type rules appear to be very robust. The output gap is, of course, not directly observable. However, our results stress the importance of constructing (theory-consistent) observable measures of that variable.

So far, our analysis has been restricted to the welfare comparison of interest rate rules that would be (constrained) optimal under alternative assumptions for capital accumulation. A different question regards the extent to which the welfare properties of a given rule change if we vary its policy parameters. We turn to this next.

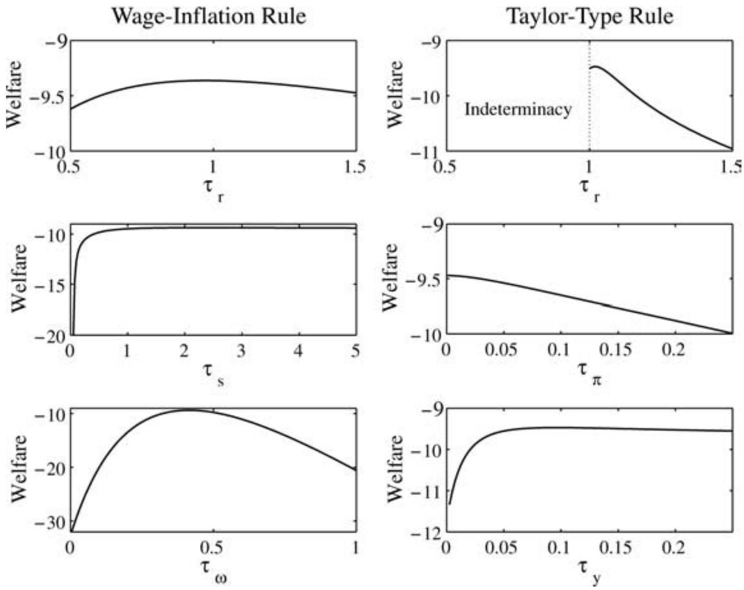
4.3 Sensitivity Analysis

Let us reconsider the Taylor-type rule stated in equation (26). In what follows we restrict attention to the Woodford definition of the output gap.[24] The left-hand panel of figure 2 shows the results of an analogous sensitivity analysis for the case of the wage- and price-inflation interest rate rule in equation (25). Interestingly, we observe that welfare is very sensitive to changes in the parameter that measures the relative importance attached to price and wage inflation in the rule. Also, in this sense, these rules are not robust.

[23]The finding that, if anything, small details of the optimized interest rate rules change depending on which measure of the output gap is used is also confirmed by further robustness checks that we have conducted for alternative interest rate rules.

[24]Additional results are available upon request.

Figure 2. Sensitivity Analysis



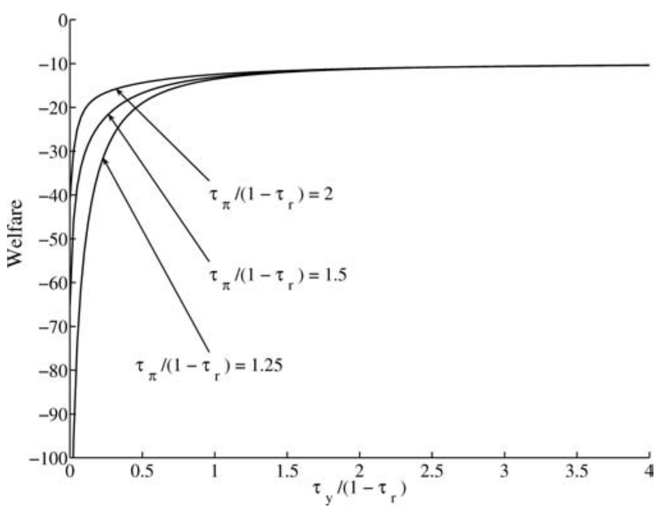
The right-hand panel of figure 2 shows by how much welfare decreases if the policymaker deviates from the optimal rule. Specifically, the figure illustrates the welfare associated with a change in one policy parameter at a time while holding the remaining parameters constant at their (constrained) optimal values. The figure also shows the respective indeterminacy regions.

From an economic point of view, our sensitivity analysis with respect to the output-gap response in the rule deserves special attention. It is shown that small deviations from the optimal rule result in tiny welfare losses. Only to the extent that the central bank attaches very little importance to the output gap in setting the nominal interest rate does the resulting rule imply a large welfare loss with respect to the optimal policy. Also, in this sense, Taylor-type interest rate rules turn out to be robust.

We have already mentioned the fact that constrained optimal Taylor-type rules feature superinertia in both FS and RM. That property does not seem to be realistic. In fact, in estimated central bank reaction functions, the interest rate smoothing coefficient is typically significantly positive but smaller than one (see, e.g., Woodford 2003, ch. 1). It is therefore of interest to examine whether our previous conclusion regarding the robustness of Taylor-type rules remains valid if we restrict attention to an empirically plausible value of the interest rate smoothing coefficient. We set $\tau_r = 0.7$, which is in line with empirical estimates (see, e.g., Galí and Rabanal 2005), and analyze the implications for welfare associated with alternative values for the remaining parameters in equation (26) (including the constrained optimal ones for that case). The results are shown in figure 3.

The last figure makes clear that Taylor-type rules are indeed robust in the sense that we have emphasized before. To the extent that the central bank reacts strongly enough to changes in the output gap, welfare is close to the (constrained) optimal one. Sveen and Weinke (2005, 2007) have made the case for adjusting the nominal interest rate in response to changes in real economic activity based on the argument that the resulting interest rate rules do generally imply determinacy. In the present paper we sharpen our earlier conclusion on welfare grounds. Also, from a welfare point of view, we find that a central bank should react by sufficiently much in response to changes in real economic activity, as measured by the output gap. The last result extends some recent work by Schmitt-Grohé and Uribe (2007b). They show in the context of a New Keynesian

Figure 3. Sensitivity Analysis for Inertial Taylor-Type Rule



model featuring endogenous capital accumulation that large welfare losses are generally implied by policy rules prescribing that the central bank adjusts the nominal interest rate in response to changes in output. We find that a different conclusion obtains if the central bank uses the output gap instead. The welfare properties of an interest rate rule therefore depend crucially on the measure of real economic activity used in its definition.

5. Conclusion

The present paper makes progress in explaining the welfare consequences of firm-specific capital accumulation. We analyze (constrained) optimal interest rate rules prescribing that the nominal interest rate is set as a function of a small number of macroeconomic variables. Our results suggest that Taylor-type interest rate rules are desirable from a welfare point of view.

Appendix 1. Welfare with Firm-Specific Capital

Throughout the appendix, we use the notation and the definitions that are introduced in the text. We approximate our utility-based welfare criterion up to the second order. In what follows, we make frequent use of two rules:

$$\frac{A_t - \overline{A}}{\overline{A}} \simeq a_t + \frac{1}{2} a_t^2, \tag{27}$$

where $a_t \equiv \ln(\frac{A_t}{\overline{A}})$. Moreover, if $A_t = \left(\int_0^1 A_t(i)^\gamma di\right)^{\frac{1}{\gamma}}$ then

$$a_t \simeq E_i a_t(i) + \frac{1}{2}\gamma \, Var_i a_t(i). \tag{28}$$

As we have already mentioned in the text, the policymaker's period welfare function reads

$$\mathcal{W}_t \equiv U(C_t) - \int_0^1 V(N_t(h))dh = U(C_t) - E_h\{V(N_t(h))\}. \tag{29}$$

Now we compute a second-order Taylor expansion of period welfare:

$$\mathcal{W}_t \simeq \overline{\mathcal{W}} + \overline{U}_C \overline{C} \left(c_t + \frac{1}{2} c_t^2 \right) - \overline{V}_N \overline{N} E_h \left\{ n_t(h) + \frac{1}{2} n_t(h)^2 \right\}$$
$$+ \frac{1}{2} \overline{U}_{CC} \overline{C}^2 c_t^2 - \frac{1}{2} \overline{V}_{NN} \overline{N}^2 E_h \{ n_t(h)^2 \}. \tag{30}$$

Next we show how the linear terms in consumption and employment in the last expression can be approximated up to the second order. We start by analyzing the consumption portion of welfare. To this end, we invoke the resource constraint.

The Consumption Portion of Welfare

The resource constraint reads

$$Y_t = C_t + I_t. \tag{31}$$

The following relationship holds true:

$$c_t + \frac{1}{2} c_t^2 \simeq \frac{1}{\zeta} \left(y_t + \frac{1}{2} y_t^2 \right) - \frac{1-\zeta}{\zeta} \left(i_t + \frac{1}{2} i_t^2 \right). \tag{32}$$

Next we analyze the investment portion of the resource constraint. Our starting point is the log-deviation of investment at the firm level:

$$i_t \simeq E_i\{i_t(i)\} + \frac{1}{2} Var_i\{i_t(i)\}. \tag{33}$$

Given our specification of the capital adjustment cost, we have

$$i_t(i) \simeq \frac{1}{\delta} \left[k_{t+1}(i) - (1-\delta) k_t(i) \right.$$
$$\left. + \frac{1}{2} \left(\varepsilon_\psi - \frac{1-\delta}{\delta} \right) (k_{t+1}(i) - k_t(i))^2 \right],$$
$$Var_i\{i_t(i)\} \simeq \frac{1}{\delta^2} [\kappa_{t+1} + (1-\delta)^2 \kappa_t - 2(1-\delta) Cov_i\{k_{t+1}(i), k_t(i)\}].$$

Using the investment rule, we derive a second-order approximation to the covariance term in the last relationship:

$$Cov_i\{k_{t+1}(i), k_t(i)\} \simeq \tau_2 Var_i\{k_t(i)\} + \tau_3 Cov_i\{\widetilde{p}_t(i), k_t(i)\}.$$

We also note that

$$E_i\{k_t(i)\} \simeq k_t - \frac{1}{2}\kappa_t.$$

Combining the last five results, we obtain

$$\begin{aligned} i_t \simeq \frac{1}{\delta}\Bigg[&\left(k_{t+1} - \frac{1}{2}\kappa_{t+1}\right) - (1-\delta)\left(k_t - \frac{1}{2}\kappa_t\right) \\ &+ \frac{1}{2}\left(\varepsilon_\psi - \frac{1-\delta}{\delta}\right) E_i\{[k_{t+1}(i) - k_t(i)]^2\}\Bigg] \\ &+ \frac{1}{2}\frac{1}{\delta^2}\kappa_{t+1} + \frac{1}{2}\left(\frac{1-\delta}{\delta}\right)^2 \kappa_t - \frac{1-\delta}{\delta^2}[\tau_2\omega_t + \tau_3\psi_t]. \end{aligned}$$

Let us now consider the term $E_i\{[k_{t+1}(i) - k_t(i)]^2\}$ in the last expression.

$$E_i\{[k_{t+1}(i) - k_t(i)]^2\} \simeq \kappa_{t+1} + k_{t+1}^2 + \kappa_t + k_t^2 - 2E_i\{k_{t+1}(i)k_t(i)\},$$

and $E_i\{k_{t+1}(i)k_t(i)\}$ is approximated with the desired accuracy by

$$E_i\{k_{t+1}(i)k_t(i)\} \simeq k_{t+1}k_t + \tau_2\kappa_t + \tau_3\psi_t.$$

We therefore have

$$\begin{aligned} i_t \simeq \frac{1}{\delta}k_{t+1} - \frac{1-\delta}{\delta}k_t + \frac{1}{2}\frac{1}{\delta}\left(\varepsilon_\psi - \frac{1-\delta}{\delta}\right)(k_{t+1} - k_t)^2 \\ + \frac{1}{2}\frac{\varepsilon_\psi}{\delta}\kappa_{t+1} + \frac{1}{2}\frac{\varepsilon_\psi}{\delta}(1 - 2\tau_2)\kappa_t - \frac{\tau_3\varepsilon_\psi}{\delta}\psi_t. \end{aligned} \tag{34}$$

Next we analyze the labor portion of welfare.

The Labor Portion of Welfare

Aggregate labor supply is given by $N_t \equiv \left(\int_0^1 N_t(i)^{\frac{\zeta-1}{\zeta}} di\right)^{\frac{\zeta}{\zeta-1}}$. Using the second rule, we can write

$$n_t \simeq E_h n_t(h) + \frac{1}{2}\frac{\zeta-1}{\zeta} Var_h n_t(h). \tag{35}$$

Aggregate labor demand reads

$$L_t \equiv \int_0^1 L_t(i)di = \int_0^1 \left(\frac{Y_t(i)}{X_t K_t(i)^\alpha}\right)^{\frac{1}{1-\alpha}} di = B_t^{\frac{1}{1-\alpha}},$$

where $B_t \equiv (\int_0^1 B_t(i)^{\frac{1}{1-\alpha}} di)^{1-\alpha}$ and $B_t(i) \equiv \frac{Y_t(i)}{X_t K_t(i)^\alpha}$. Clearing of the labor market implies that $N_t = L_t$. We can therefore write

$$n_t = \frac{1}{1-\alpha} b_t.$$

Invoking the second rule, we obtain

$$b_t \simeq E_i b_t(i) + \frac{1}{2}\frac{1}{1-\alpha} Var_i b_t(i),$$

and we also note that

$$b_t(i) = y_t(i) - x_t - \alpha k_t(i).$$

The last result implies

$$\begin{aligned} E_i b_t(i) &= E_i y_t(i) - x_t - \alpha E_i k_t(i), \\ Var_i b_t(i) &= Var_i y_t(i) + \alpha^2 \kappa_t - 2\alpha Cov_i(y_t(i), k_t(i)). \end{aligned}$$

We therefore obtain

$$\begin{aligned} b_t \simeq\ & E_i y_t(i) - x_t - \alpha E_i k_t(i) \\ & + \frac{1}{2}\frac{1}{1-\alpha}[Var_i y_t(i) + \alpha^2 \kappa_t - 2\alpha Cov_i(y_t(i), k_t(i))]. \end{aligned}$$

From the definitions of K_t and Y_t, it follows that

$$\begin{aligned} E_i k_t(i) &\simeq k_t - \frac{1}{2}\kappa_t, \\ E_i y_t(i) &\simeq y_t - \frac{1}{2}\frac{\varepsilon - 1}{\varepsilon} Var_i y_t(i). \end{aligned}$$

Combining the last three results, we arrive at

$$\begin{aligned} b_t \simeq\ & y_t - x_t - \alpha k_t + \frac{1}{2}\frac{\alpha}{1-\alpha}\kappa_t + \frac{1}{2}\frac{1}{\varepsilon}\left(\frac{1-\alpha+\alpha\varepsilon}{1-\alpha}\right) Var_i y_t(i) \\ & - \frac{\alpha}{1-\alpha} Cov_i(y_t(i), k_t(i)). \end{aligned}$$

We therefore obtain

$$\begin{aligned} E_h n_t(h) \simeq\ & \frac{1}{1-\alpha}(y_t - x_t - \alpha k_t) + \frac{1}{2}\frac{\alpha}{(1-\alpha)^2}\kappa_t \\ & + \frac{1}{2}\frac{1}{\varepsilon}\frac{1-\alpha+\alpha\varepsilon}{(1-\alpha)^2} Var_i y_t(i) \\ & - \frac{\alpha}{(1-\alpha)^2} Cov_i(y_t(i), k_t(i)) - \frac{1}{2}\frac{\varepsilon_N - 1}{\varepsilon_N} Var_h n_t(h). \end{aligned}$$

Using the demand functions for goods and labor services, we obtain

$$\begin{aligned} Var_i y_t(i) &\simeq \varepsilon^2 \Delta_t, \\ Cov_i(y_t(i), k_t(i)) &\simeq -\varepsilon \psi_t, \\ Var_h n_t(h) &\simeq \varepsilon_N^2 \lambda_t. \end{aligned}$$

We therefore have

$$\begin{aligned} E_h n_t(h) \simeq\ & \frac{1}{1-\alpha}(y_t - x_t - \alpha k_t) + \frac{1}{2}\frac{1-\alpha+\alpha\varepsilon}{(1-\alpha)^2}\varepsilon\Delta_t \\ & + \frac{1}{2}\frac{\alpha}{(1-\alpha)^2}\kappa_t + \frac{\alpha\varepsilon}{(1-\alpha)^2}\psi_t - \frac{1}{2}(\varepsilon_N - 1)\varepsilon_N \lambda_t. \end{aligned} \tag{36}$$

Finally, we note that $E_h\{n_t(h)^2\}$ can be written as

$$E_h\{n_t(h)^2\} \simeq \varepsilon_N^2 \lambda_t + n_t^2. \tag{37}$$

The Welfare Function

The Pareto optimality of the steady state implies $\overline{V}_N \overline{N} = U_C \overline{C}\frac{1-\alpha}{\zeta}$. Combining that result with equations (30), (32), (34), (36), and (37), we arrive at the following approximation to period welfare:

$$\begin{aligned}
\frac{\mathcal{W}_t - \overline{\mathcal{W}}}{\overline{U}_C \overline{C}} \simeq\ & \frac{1}{\zeta} x_t - \frac{1}{\zeta} \frac{\alpha}{\rho+\delta}[k_{t+1} - (1+\rho)k_t] + \frac{1}{2}\frac{1}{\zeta} y_t^2 - \frac{\sigma}{2} c_t^2 \\
& - \frac{1}{2}\frac{1-\zeta}{\zeta} i_t^2 - \frac{1}{2}\frac{1-\zeta}{\zeta}\frac{1}{\delta}\left(\varepsilon_\psi - \frac{1-\delta}{\delta}\right)(k_{t+1} - k_t)^2 \\
& - \frac{1}{2}\frac{1-\alpha}{\zeta}(1+\phi) n_t^2 - \frac{1}{2}\frac{\varepsilon}{\zeta}\frac{1-\alpha+\alpha\varepsilon}{1-\alpha}\Delta_t \\
& - \frac{1}{2}\frac{(1-\alpha)\varepsilon_N}{\zeta}(1+\varepsilon_N \phi)\lambda_t - \frac{1}{2}\frac{1-\zeta}{\zeta}\frac{\varepsilon_\psi}{\delta}\kappa_{t+1} \\
& - \frac{1}{2}\frac{1}{\zeta}\left[\frac{(1-\zeta)\varepsilon_\psi}{\delta}(1-2\tau_2) + \frac{\alpha}{1-\alpha}\right]\kappa_t \\
& - \frac{1}{\zeta}\left[\frac{\alpha\varepsilon}{1-\alpha} - \frac{(1-\zeta)\tau_3\varepsilon_\psi}{\delta}\right]\psi_t. \qquad (38)
\end{aligned}$$

The first linear term in the last expression is proportional to the level of aggregate (log) technology, x_t, which is exogenous. The remaining linear terms are proportional to current and next period's aggregate capital, k_t and k_{t+1}. Next we compute $\frac{1-\beta}{\overline{U}_C \overline{C}} E_t\{\sum_{k=0}^{\infty} \beta^k (\mathcal{W}_{t+k} - \overline{\mathcal{W}})\}$, which allows us to invoke a result by Edge (2003). As in her model, the terms in aggregate capital cancel except for the initial one. Following the lead of Svensson (2000), we consider the limit for $\beta \to 1$. This allows us to abstract from initial conditions.[25] This gives the expression stated in the text:

$$\begin{aligned}
\widetilde{\mathcal{W}}^{FS} \simeq\ & \Omega_1 E\{y_t^2\} + \Omega_2 E\{c_t^2\} + \Omega_3 E\{i_t^2\} + \Omega_4 E\{(\Delta k_{t+1})^2\} \\
& + \Omega_5 E\{n_t^2\} + \Omega_6 E\{\lambda_t\} + \Omega_1^{FS} E\{\Delta_t\} \\
& + \Omega_2^{FS} E\{\kappa_t\} + \Omega_3^{FS} E\{\psi_t\}, \qquad (39)
\end{aligned}$$

with

$$\begin{aligned}
& \Omega_1 \equiv \frac{1}{2}\frac{1}{\zeta},\ \Omega_2 \equiv -\frac{\sigma}{2}, \Omega_3 \equiv -\frac{1}{2}\frac{1-\zeta}{\zeta}, \\
& \Omega_4 \equiv -\frac{1}{2}\frac{1}{\delta}\frac{1-\zeta}{\zeta}\left(\varepsilon_\psi - \frac{1-\delta}{\delta}\right),
\end{aligned}$$

[25] For a formal proof, see Dennis (2007).

$$\Omega_5 \equiv -\frac{1}{2}\frac{1-\alpha}{\zeta}(1+\phi), \Omega_6 \equiv -\frac{1}{2}\frac{(1-\alpha)\varepsilon_N}{\zeta}(1+\phi\varepsilon_N),$$
$$\Omega_1^{FS} \equiv -\frac{1}{2}\frac{\varepsilon}{\zeta}\frac{1-\alpha+\alpha\varepsilon}{1-\alpha},$$
$$\Omega_2^{FS} \equiv -\frac{1}{2}\frac{1}{\zeta}\left\{\frac{2(1-\zeta)\varepsilon_\psi(1-\tau_2)}{\delta} + \frac{\alpha}{1-\alpha} - \frac{2}{\delta}\frac{1-\zeta}{\delta}\right\},$$
$$\Omega_3^{FS} \equiv -\frac{1}{\zeta}\left(\frac{\alpha\varepsilon}{1-\alpha} - \frac{(1-\zeta)\tau_3\varepsilon_\psi}{\delta}\right).$$

Next we derive recursive formulations for the cross-sectional variances of wages, prices, and capital holdings, as well as the welfare-relevant covariance of prices and capital holdings. As in Erceg, Henderson, and Levin (2000), the cross-sectional variance of wages, λ_t, can be written in the following way:

$$\lambda_t \simeq \theta_w \lambda_{t-1} + \frac{\theta_w}{1-\theta_w}(\omega_t)^2. \tag{40}$$

We define $\widetilde{p}_t \equiv E_i \ln P_t(i)$ and write the price-dispersion term, Δ_t, in the way proposed by Woodford (2003, ch. 6):

$$\begin{aligned}\Delta_t &= E_i[(\ln P_t(i) - \widetilde{p}_{t-1})^2] - (\widetilde{p}_t - \widetilde{p}_{t-1})^2,\\ &= \theta_p E_i[(\ln P_{t-1}(i) - \widetilde{p}_{t-1})^2]\\ &\quad + (1-\theta_p)E_i\left[\left(\ln P_t^*(i) - \widetilde{p}_{t-1}\right)^2\right] - (\widetilde{p}_t - \widetilde{p}_{t-1})^2.\end{aligned}$$

After invoking the price-setting rule stated in equation (19), we obtain the following recursive formulation for Δ_t:

$$\begin{aligned}\Delta_t &\simeq \theta_p\Delta_{t-1} + (1-\theta_p)E_i\left[\left(\widetilde{p}_t^* - \tau_1\widehat{k}_t(i) - \widetilde{p}_{t-1}\right)^2\right] - (\widetilde{p}_t - \widetilde{p}_{t-1})^2\\ &\simeq \theta_p\Delta_{t-1} + (1-\theta_p)E_i\left[\left(\frac{1}{1-\theta_p}(\widetilde{p}_t - \widetilde{p}_{t-1}) - \tau_1\widehat{k}_t(i)\right)^2\right]\\ &\quad - (\widetilde{p}_t - \widetilde{p}_{t-1})^2\\ &\simeq \theta_p\Delta_{t-1} + (1-\theta_p)\tau_1^2\kappa_t + \frac{\theta_p}{1-\theta_p}\pi_t^2.\end{aligned} \tag{41}$$

The investment rule stated in equation (20) implies directly that a recursive formulation for the cross-sectional variance of capital holdings, κ_t, is of the form

$$\kappa_t \simeq \tau_2^2 \kappa_{t-1} + \tau_3^2 \Delta_{t-1}. \tag{42}$$

Finally, the cross-sectional covariance of prices and capital holdings, ψ_t, can also be expressed in a recursive way:

$$\begin{aligned}
\psi_t \simeq\ & E_i[\widehat{k}_t(i)\widehat{p}_t(i)], \\
\simeq\ & \theta_p E_i[\widehat{k}_t(i)(\ln P_{t-1}(i) - \ln P_t)] + (1-\theta_p)E_i\left[\widehat{k}_t(i)\widehat{p}_t^*(i)\right], \\
\simeq\ & \theta_p E_i[\left(\tau_2\widehat{k}_{t-1}(i) + \tau_3\widehat{p}_{t-1}(i)\right)\widehat{p}_{t-1}(i)] \\
& + (1-\theta_p)E_i\left[\widehat{k}_t(i)\left(\widehat{p}_t^* - \tau_1\widehat{k}_t(i)\right)\right], \\
\simeq\ & \theta_p\tau_2\psi_{t-1} + \theta_p\tau_3\Delta_{t-1} - \tau_1(1-\theta_p)\kappa_t.
\end{aligned} \tag{43}$$

Appendix 2. Welfare with Rental Market

In the rental-market case, the analysis is simplified by that fact that the capital labor ratio is constant across firms, as discussed in Edge (2003). The resulting welfare criterion reads

$$\begin{aligned}
\widetilde{\mathcal{W}}^{RM} \simeq\ & \Omega_1 E\left\{y_t^2\right\} + \Omega_2 E\left\{c_t^2\right\} + \Omega_3 E\left\{i_t^2\right\} + \Omega_4 E\{(\Delta k_{t+1})^2\} \\
& + \Omega_5 E\left\{n_t^2\right\} + \Omega_6 E\{\lambda_t\} + \Omega^{RM} E\{\Delta_t\},
\end{aligned} \tag{44}$$

with $\Omega^{RM} \equiv -\frac{1}{2}\frac{\varepsilon}{\zeta}$. Finally, the variance terms can be written as

$$\Delta_t = \theta_p \Delta_{t-1} + \frac{\theta_p}{1-\theta_p}\pi_t^2, \tag{45}$$

$$\lambda_t = \theta_w \lambda_{t-1} + \frac{\theta_w}{1-\theta_w}(\Delta w_t)^2, \tag{46}$$

as discussed in Erceg, Henderson, and Levin (2000) and Rotemberg and Woodford (1999).

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