Incomplete Interest Rate Pass-Through and Optimal Monetary Policy*

Teruyoshi Kobayashi
Department of Economics, Chukyo University

Many recent empirical studies have reported that the pass-through from money-market rates to retail lending rates is far from complete in the euro area. This paper formally shows that when only a fraction of all the loan rates is adjusted in response to a shift in the policy rate, fluctuations in the average loan rate lead to welfare costs. Accordingly, the central bank is required to stabilize the rate of change in the average loan rate in addition to inflation and output. It turns out that the requirement for loan rate stabilization justifies, to some extent, the idea of policy rate smoothing in the face of a productivity shock and/or a preference shock. However, a drastic policy reaction is needed in response to a shock that directly shifts retail loan rates, such as an unexpected shift in the loan rate premium.

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1. Introduction

Many empirical studies have shown that in the majority of industrialized countries, a cost channel plays an important role in the

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transmission of monetary policy.\footnote{See, for example, Barth and Ramey (2001), Angeloni, Kashyap, and Mojon (2003), Christiano, Eichenbaum, and Evans (2005), Chowdhury, Hoffmann, and Schabert (2006), and Ravenna and Walsh (2006).} Along with this, many authors have attempted to incorporate a cost channel in formal models of monetary policy. For example, Christiano, Eichenbaum, and Evans (2005) introduce a cost channel into the New Keynesian framework in accounting for the actual dynamics of inflation and output in the United States, while Ravenna and Walsh (2006) explore optimal monetary policy in the presence of a cost channel.

However, a huge number of recent studies have also reported that, especially in the euro area, shifts in money-market rates, including the policy rate, are not completely passed through to retail lending rates.\footnote{Some recent studies, to name a few, are Mojon (2000), Weth (2002), Angeloni, Kashyap, and Mojon (2003), Gambacorta (2004), de Bondt, Mojon, and Valla (2005), Kok Sørensen and Werner (2006), and Gropp, Kok Sørensen, and Licht- enberger (2007). A brief review of the literature on interest rate pass-through is provided in the next section.} Naturally, since loan rates are determined by commercial banks, to what extent shifts in money-market rates affect loan rates and thereby the behavior of firms depends on how commercial banks react to the shifts in the money-market rates. If not all of the commercial banks promptly respond to a change in the money-market rates, then a policy shift will not affect the whole economy equally.\footnote{Possible explanations for the existence of loan rate stickiness have been continuously discussed in the literature. Some of those explanations are introduced in the next section.}

Given this situation, it is natural to ask whether or not the presence of loan rate sluggishness alters the desirable monetary policy compared with the case in which a shift in the policy rate is immediately followed by changes in retail lending rates. Nevertheless, to the best of my knowledge, little attention has been paid to such a normative issue since the main purpose of the previous studies was to estimate the degree of pass-through.

The principal aim of this paper is to formally explore optimal monetary policy in an economy with imperfect interest rate pass-through, where retail lending rates are allowed to differ across regions. Following Christiano and Eichenbaum (1992), Christiano, Eichenbaum, and Evans (2005), and Ravenna and Walsh (2006),
it is assumed in our model that the marginal cost of each production firm depends on a borrowing rate, since the owner of each firm needs to borrow funds from a commercial bank in order to compensate for wage bills that have to be paid in advance. A novel feature of our model is that there is only one commercial bank in each region, and each commercial bank does business only in the region where it is located. Since loan markets are assumed to be geographically segmented, each firm owner can borrow funds only from the corresponding regional bank. In this environment, retail loan rates are not necessarily the same across firms. The commercial banks’ problem for loan rate determination is specified as Calvo-type pricing.

It is shown that the approximated utility function takes a form similar to the objective function that frequently appears in the literature on “interest rate smoothing.” An important difference, however, is that the central bank is now required to stabilize the rate of change in the average loan rate, not the rate of change in the policy rate. The necessity for the stabilization of the average loan rate can be understood by analogy with the requirement for inflation stabilization, which has been widely discussed within the standard Calvo-type staggered-price model. Under staggered pricing, the rate of inflation should be stabilized because price dispersion would otherwise take place. Under staggered loan rates, changes in the average loan rate must be dampened because loan rate dispersion would otherwise take place. Since loan rate dispersion inevitably causes price dispersion through the cost channel, it consequently leads to an inefficient dispersion in hours worked.

It turns out that the introduction of a loan rate stabilization term in the central bank’s loss function causes the optimal policy rate to become more inertial in the face of a productivity shock and a preference shock. This implies that the optimal policy based on a loss function with a loan rate stabilization term is quite consistent with that based on the conventionally used loss function that involves a policy rate stabilization term. Yet, this smoothing effect appears to be limited quantitatively.

On the other hand, the presence of a loan rate stabilization term requires a drastic policy response in the face of an exogenous shock that directly shifts retail loan rates, such as an unexpected change in the loan rate premium. For example, an immediate reduction in the
policy rate is needed in response to a positive loan premium shock since it can partially offset the rise in loan rates. This is in stark contrast to the policy suggested by conventional policy rate smoothing. The case of a loan premium shock is an example for which it is crucial for the central bank to clearly distinguish between policy rate smoothing and loan rate smoothing.

The rest of the paper is organized as follows. The next section briefly reviews recent empirical studies on interest rate pass-through. Section 3 presents a baseline model, and section 4 summarizes the equilibrium dynamics of the economy. Section 5 derives a utility-based objective function of the central bank, and optimal monetary policy is explored in section 6. Section 7 concludes the paper.

2. A Review of Recent Studies on Interest Rate Pass-Through

Over the past decade, a huge number of empirical studies have been conducted in an attempt to estimate the degree of interest rate pass-through in the euro area. In the literature, the terminology “interest rate pass-through” generally has two meanings: loan rate pass-through and deposit rate pass-through. In this paper, we focus on the former since the general equilibrium model described below treats only the case of loan rate stickiness. Although it is said that deposit rates are also sticky in the euro area, constructing a formal general equilibrium model that includes loan rate stickiness is a reasonable first step to a richer model that could also take into account the sluggishness in deposit rates. This section briefly reviews recent studies on loan rate pass-through in the euro area.4

Although recent studies on loan rate pass-through differ in terms of the estimation methods and the data used, a certain amount of broad consensus has been established. First, at the euro-area aggregated level, the policy rate is only partially passed through to retail loan rates in the short run, while the estimates of the degree of

4de Bondt, Mojon, and Valla (2005) and Kok Sørensen and Werner (2006) also provide a survey of the literature on the empirical study of interest rate pass-through, including deposit rate pass-through.
pass-through differ among researchers. For example, according to table 1 of de Bondt, Mojon, and Valla (2005), the estimated degree of short-run (i.e., monthly) pass-through of changes in the market interest rates to the loan rate on short-term loans to firms varies from .25 (Sander and Kleimeier 2002; Hofmann 2003) to .76 (Heinemann and Schüler 2002). Gropp, Kok Sørensen, and Lichtenberger (2007) argued that interest rate pass-through in the euro area is incomplete even after controlling for differences in bank soundness, credit risk, and the slope of the yield curve. On the other hand, there is no general consensus about whether the long-run interest rate pass-through is perfect or not.\(^5\)

Second, although the degree of interest rate pass-through significantly differs across countries, the extent of heterogeneity has been reduced since the introduction of the euro (de Bondt 2002; Toolsema, Sturm, and de Haan 2001; Sander and Kleimeier 2004). At this point, it also seems to be widely admitted that the speed of loan rate adjustment has, to some extent, been improved (de Bondt 2002; de Bondt, Mojon, and Valla 2005).

While there is little doubt about the existence of sluggishness in loan rates, there is still much debate as to why it exists and why the extent of pass-through differs across countries. For instance, Gropp, Kok Sørensen, and Lichtenberger (2007) insisted that the competitiveness of the financial market is a key to understanding the degree of pass-through. They showed that a larger degree of loan rate pass-through would be attained as financial markets become more competitive. Schwarzbauer (2006) pointed out that differences in financial structure, measured by the ratio of bank deposits to GDP and the ratio of market capitalization to GDP, have a significant influence on the heterogeneity among euro-area countries in the speed of pass-through. de Bondt, Mojon, and Valla (2005) argued that retail bank rates are not completely responsive to money-market rates since bank rates are tied to long-term market interest rates even in the case of short-term bank rates. From a different point

\(^5\)For instance, Mojon (2000), Heinemann and Schüler (2002), Hofmann (2003), and Sander and Kleimeier (2004) reported that the long-run pass-through of market rates to interest rates on short-term loans to firms is complete. On the other hand, Donnay and De Gryse (2001) and Toolsema, Sturm, and de Haan (2001) argued that the loan rate pass-through is incomplete even in the long run.
of view, Kleimeier and Sander (2006) emphasized the role of monetary policymaking by central banks as a determinant of the degree of pass-through. They argued that better-anticipated policy changes tend to result in a quicker response of retail interest rates.\(^6\)

In the theoretical model presented in the next section, we consider a situation where financial markets are segmented and thus each regional bank has a monopolistic power. While the well-known Calvo-type staggered pricing is applied to banks’ loan rate settings, it turns out that the degree of pass-through depends largely on the central bank’s policy rate setting. Moreover, a newly charged loan rate can be interpreted as a weighted average of short- and long-term market rates, where the size of each weight is dependent on the degree of stickiness. Thus, although our way of introducing loan rate stickiness into the general equilibrium model is fairly simple, the model’s implications for the relationship between loan rates and the policy rate seem quite consistent with what some of the previous studies have suggested.

3. The Model

The economy consists of a representative household, intermediate-goods firms, final-goods firms, financial intermediary, and the central bank. The representative household consumes a variety of final consumption goods while supplying labor service in the intermediate-goods sector. Each intermediate-goods firm produces a differentiated intermediate good and sells it to final-goods firms. Following Christiano and Eichenbaum (1992), Christiano, Eichenbaum, and Evans (2005), and Ravenna and Walsh (2006), we consider a situation in which the owner of each intermediate-goods firm has to pay wages in advance to workers at the beginning of each period. The owner thereby needs to borrow funds from a commercial bank since they cannot receive revenue until the end of the period. Final-goods firms produce differentiated consumption goods by using a composite of intermediate goods.

\(^6\)For a more concrete discussion about the source of imperfect pass-through, see Gropp, Kok Sørensen, and Lichtenberger (2007). As for the heterogeneity in the degree of pass-through, see Kok Sørensen and Werner (2006).
3.1 Households

The one-period utility function of a representative household is given as

\[ U_t = u(C_t; \xi_t) - \int_0^1 v(L_t(i))di \]

\[ = \frac{(\xi_tC_t)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{L_t(i)^{1+\omega}}{1+\omega}di, \]

where \( C_t \equiv \left[ \int_0^1 C_t(j)^{\frac{\sigma-1}{\sigma}}dj \right]^{\frac{1}{\sigma-1}}, \) and \( C_t(j) \) and \( L_t(i) \) are the consumption of differentiated good \( j \) and hours worked at intermediate-goods firm in region \( i \), respectively. Henceforth, index \( i \) is used to denote a specific region as well as the variety of intermediate goods. Since there is only one intermediate-goods firm in each region, this usage is innocuous. \( \xi_t \) represents a preference shock with mean unity, and \( \theta(>1) \) denotes the elasticity of substitution between the variety of goods. It can be shown that the optimization of the allocation of consumption goods yields the aggregate price index

\[ P_t \equiv \left[ \int_0^1 P_t(j)^{1-\theta}dj \right]^{\frac{1}{1-\theta}} \]

Assume that the household is required to use cash in purchasing consumption goods. At the beginning of period \( t \), the amount of cash available for the purchase of consumption goods is \( M_{t-1} + \int_0^1 W_t(i)L_t(i)di - \int_0^1 D_t(i)di \), where \( M_{t-1} \) is the nominal balance held from period \( t-1 \) to \( t \), and \( \int_0^1 W_t(i)L_t(i)di \) represents the total wage income paid in advance by intermediate-goods firms. The household also makes a one-period deposit \( D_t(i) \) in commercial bank \( i \), the interest on which \( (R_t) \) is paid at the end of the period. It is assumed that the household has deposits in all of the commercial banks. Accordingly, the following cash-in-advance constraint must be satisfied at the beginning of period \( t \):\(^7\)

\[ \int_0^1 P_t(j)C_t(j)dj \leq M_{t-1} + \int_0^1 W_t(i)L_t(i)di - \int_0^1 D_t(i)di. \]

\(^7\)With this specification, it is implicitly assumed that financial markets open before the goods market.
The household’s budget constraint is given by

\[ M_t = M_{t-1} + \int_0^1 W_t(i)L_t(i)di - \int_0^1 D_t(i)di - \int_0^1 P_t(j)C_t(j)dj + R_t \int_0^1 D_t(i)di + \Pi_t - T_t, \]

where \( \Pi_t \) denotes the sum of profits transferred from firms and commercial banks, and \( T_t \) is a lump-sum tax.

The demand for good \( j \) is expressed as

\[ C_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\theta} C_t. \]  

(1)

The budget constraint can then be rewritten as

\[ M_t = M_{t-1} + \int_0^1 W_t(i)L_t(i)di - \int_0^1 D_t(i)di - P_tC_t + R_t \int_0^1 D_t(i)di + \Pi_t - T_t. \]

In an equilibrium with a positive interest rate, the following equality must hold:

\[ P_tC_t = M_{t-1} + \int_0^1 W_t(i)L_t(i)di - \int_0^1 D_t(i)di. \]  

(2)

This implies that the amount of total consumption expenditure is equal to cash holdings as long as there is an opportunity cost of holding cash. Then, the budget constraint leads to \( M_t = R_t \int_0^1 D_t(i)di + \Pi_t - T_t \). Eliminating the money term from equation (2) yields an alternative expression of the budget constraint:

\[ P_tC_t = R_{t-1} \int_0^1 D_{t-1}(i)di + \int_0^1 W_t(i)L_t(i)di - \int_0^1 D_t(i)di + \Pi_{t-1} - T_{t-1}. \]
The first-order conditions for the household’s optimization problem are

\[
\frac{\xi_t^{1-\sigma} C_t^{-\sigma}}{P_t} = \beta R_t E_t \left[ \frac{\xi_{t+1}^{1-\sigma} C_{t+1}^{-\sigma}}{P_{t+1}} \right],
\]

(3)

\[
\frac{W_t(i)}{P_t} = \frac{L_t(i)^\omega}{\xi_t^{1-\sigma} C_t^{-\sigma}},
\]

(4)

where \( \beta \) and \( E_t \) are the subjective discount factor and the expectations operator conditional on information in period \( t \), respectively.

3.2 Intermediate-Goods Firms

Intermediate-goods firm \( i \in (0, 1) \) produces a differentiated intermediate good, \( Z_t(i) \), by using the labor force of type \( i \) as the sole input. The production function is simply given by

\[
Z_t(i) = A_t L_t(i),
\]

(5)

where \( A_t \) is a countrywide productivity shock with mean unity. The owners of intermediate-goods firms must pay wage bills before goods markets open. Specifically, the owner of firm \( i \) borrows funds, \( W_t(i)L_t(i) \), from commercial bank \( i \) at the beginning of period \( t \) at a gross nominal interest rate \( R_t^i \). At the end of the period, intermediate-goods firm \( i \) must repay \( R_t^i W_t(i)L_t(i) \) to bank \( i \), so that the nominal marginal cost for firm \( i \) leads to \( MC_t(i) = R_t^i W_t(i)/A_t \). Here, it is assumed that firm \( i \) can borrow funds only from the regional bank \( i \) since loan markets are geographically segmented. This assumption prohibits arbitrages, and thereby lending rates are allowed to differ across regional banks. Although such a situation might overly emphasize the role of the financial market’s segmentation, a number of studies have found evidence of lending rate dispersion across intranational and international regions that cannot be explained by differences in riskiness.\(^8\)

It is assumed for simplicity that intermediate-goods firms are able to set prices flexibly. The price of $Z_t(i)$ will then be given by

$$P_t^z(i) = \frac{\theta_z}{(\theta_z - 1)(1 + \tau^m)} \frac{R_t^i W_t(i)}{A_t},$$  (6)

where $\tau^m$ is a subsidy rate imposed by the government in such a way that $\theta_z \bar{R}/[(\theta_z - 1)(1 + \tau^m)] = 1$. It should be noted that since intermediate-goods firms borrow funds, the borrowing rates become an additional production cost. Thus, a rise in borrowing rates has a direct effect of increasing intermediate-goods prices.\(^9\) Note also that since borrowing rates are allowed to differ across firms, it would become a source of price dispersion.

### 3.3 Final-Goods Firms

Each final-goods firm uses a composite of intermediate goods as the input for production. The production function is given by

$$Y_t(j) = \left[ \int_0^1 Z_t^j(i) \frac{\theta_z - 1}{\theta_z} \, di \right]^{\frac{\theta_z}{\theta_z - 1}}, \theta_z > 1,$$

where $Y_t(j)$ and $Z_t^j(i)$ represent a differentiated consumption good and the firm $j$’s demand for individual intermediate good $i$, respectively. Optimization regarding the allocation of inputs yields the price index $P_t^z \equiv \left[ \int_0^1 P_t^z(i)^{1-\theta_z} \, di \right]^{\frac{1}{1-\theta_z}}$. Accordingly, the firm $j$’s demand for intermediate good $i$ is expressed as follows:

\(^9\) $\tau^m$ eliminates the distortions stemming both from monopolistic power and a positive steady-state interest rate ($\bar{R}$). Here, a positive steady-state interest rate is distortionary since the marginal cost would no longer be equal to $v'/w'$.\(^9\)
\[ Z_t^j(i) = \left( \frac{P_t^z(i)}{P_t^z} \right)^{-\theta} Y_t(j). \]

Since \( Z_t(i) = \int_0^1 Z_t^j(i) dj \) must hold in equilibrium, the demand function leads to

\[ Z_t(i) = \left( \frac{P_t^z(i)}{P_t^z} \right)^{-\theta} \int_0^1 Y_t(j) dj = \left( \frac{P_t^z(i)}{P_t^z} \right)^{-\theta} Y_t V_t^y, \quad (7) \]

where \( Y_t \equiv \left[ \int_0^1 Y_t(j) \theta \frac{1}{\theta - 1} dj \right]^{\frac{\theta}{\theta - 1}} \) and \( V_t^y \equiv \int_0^1 (Y_t(j)/Y_t) dj \). Note that \( V_t^y \) becomes larger than unity if \( Y_t \neq Y_t(j) \) for some \( j \).

It is assumed that final-goods firms are unable to adjust prices freely. Following Calvo (1983), we consider a situation in which a fraction \( 1 - \phi \) of firms can change their prices, while the remaining fraction \( \phi \) cannot. The price-setting problem of final-goods firms leads to

\[
\max_{\tilde{P}_t} E_t \sum_{s=0}^{\infty} \phi^s \Gamma_{t,t+s} \left[ (1 + \tau^f) \tilde{P}_t - P_{t+s}^z \right] \left( \frac{\tilde{P}_t}{P_{t+s}} \right)^{-\theta} C_{t+s}, \quad (8)
\]

where \( \tilde{P}_t \) is the price of final goods set by firms that can adjust prices in period \( t \), and \( \Gamma_{t,t+s} \equiv \beta^s \frac{\theta}{\phi (\theta - 1)} \). Denotes the stochastic discount factor up to period \( t + s \). \( \tau^f \) represents a subsidy rate, where \( \tau^f = 1/(\theta - 1) \).

Log-linearizing the resultant first-order condition leads to

\[ \pi_t = \beta E_t \pi_{t+1} + \lambda_F (p_t^z - p_t), \quad (9) \]

\footnote{Note that firms that can adjust prices in the same period set an identical price. Although different intermediate-goods firms may set different prices, marginal costs for final-goods firms are identical since the allocations of intermediate inputs are the same.}
where \( \lambda_F \equiv (1 - \phi)(1 - \beta \phi) / \phi \) and \( \pi_t \equiv p_t - p_{t-1} \). Henceforth, for an arbitrary variable \( X_t, x_t \equiv \log(X_t / \bar{X}) \), where \( \bar{X} \) denotes the steady-state value. Equation (9) is a version of the New Keynesian Phillips curve that has been used in numerous recent studies. Note that the term \( p_t^z - p_t \) is equivalent to the real marginal cost of producing a final good, which is common across firms. Evidently, \( p_t^z - p_t \) becomes zero if final-goods prices are fully flexible.

3.4 Financial Intermediary

Intermediate-goods firm \( i \) needs to borrow funds from commercial bank \( i \) at the start of each period in order to compensate for wage bills that must be paid in advance. At the beginning of period \( t \), commercial bank \( i \) receives deposit \( D_t(i) \) and money injection \( M_t - M_{t-1} \equiv \Delta M_t \) from the household and the central bank, respectively. The former becomes the liability of the commercial bank, while the latter corresponds to its net worth. On the other hand, commercial bank \( i \) lends funds, \( W_t(i)L_t(i) \), to intermediate-goods firm \( i \). Therefore, the following equality must hold in equilibrium:

\[
D_t(i) + \Delta M_t = W_t(i)L_t(i), \forall i \in (0, 1).
\] (10)

The left-hand side and right-hand side can also be interpreted as representing the supply and the demand for funds, respectively. At the end of the period, commercial bank \( i \) repays its principle plus interest, \( R_t(W_t(i)L_t(i) - \Delta M_t) \), to the household. The household also indirectly receives the money injection from the central bank through the profit transfer from commercial banks.

As is shown in appendix 1, firm \( i \)'s demand for funds can be expressed as

\[
W_t(i)L_t(i) = (R_t^i)^{-\frac{1+\omega}{1+\omega \theta z}} \Lambda_t \equiv \Psi(R_t^i; \Lambda_t),
\]

where \( \Lambda_t \) is a function of aggregate variables that individual firms and commercial banks take as given. Obviously, firm \( i \)'s demand for funds, \( \Psi(R_t^i; \Lambda_t) \), decreases in \( R_t^i \) since an increase in \( R_t^i \) raises the marginal costs and thereby reduces its production.

Now let us specify the profit-maximization problem of commercial banks. It is assumed here that in each period, each commercial bank can adjust its loan rate with probability \( 1 - q \). The probability
of adjustment is independent of the time between adjustments. The
problem for commercial bank $i$ is then given by

$$\max_{R_t} E_t \sum_{s=0}^{\infty} q^s \Gamma_{t,t+s} \left[ (1 + \tau^b) R_t^i \Psi(R_t^i; \Lambda_{t+s}) - R_{t+s} \Psi(R_t^i; \Lambda_{t+s}) \right],$$

(11)

where $\tau^b$ represents a subsidy rate such that $(1 + \omega) \theta_z / [(\theta_z - 1)(1 + \tau^b)] = 1$. The commercial bank in region $i$ takes into account the effect of a change in $R_t^i$ on $W_t(i)L_t(i)$, while taking as given $P_t$, $P_t^z$, $Y_t$, $C_t$, $V_t^y$, $\Delta M_t$, and $R_t$. The second term in the square bracket is according to the equilibrium condition (10), which implies that, given the value of $\Delta M_t$, a change in $W_t(i)L_t(i)$ must be followed by the same amount of change in $D_t(i)$.

It can be shown that the first-order condition for this problem is given by

$$E_t \sum_{s=0}^{\infty} (q\beta)^s C_t \frac{\epsilon_t^{1-\sigma} \Lambda_{t+s}}{P_t^s} (R_t^i - R_{t+s}) = 0.$$  

Log-linearizing this condition yields

$$r_t^i = \tilde{r}_t = (1 - q\beta) E_t \sum_{s=0}^{\infty} (q\beta)^s r_{t+s}.$$  

(12)

This optimality condition implies that all the commercial banks that adjust in the same period impose an identical loan rate, $\tilde{r}_t$. It should be pointed out that the newly adjusted loan rates depend largely on the expectations of future policy rate as well as the current policy rate. The weight on the current policy rate is only $1 - q\beta$, while the weights on future policy rates sum up to $q\beta$. This is the well-known forward-looking property stemming from staggered pricing. If one interprets the banks’ problem as price determination under conventional Calvo pricing, the value of $q$ is simply considered as representing the degree of stickiness. From a different point of view, however, the newly adjusted loan rates expressed as (12) could be regarded as an outcome of a long-term contract, where commercial banks lend funds by charging a fixed interest rate with the proviso that there is a possibility of revaluation with probability $1 - q$. In
In this case, the length of maturity is expressed as a random variable that has a geometric distribution with parameter $1 - q$. In fact, as is shown below, there is a close relation between the newly adjusted loan rates and long-term “market” interest rates.

In order to obtain model-consistent long-term interest rates, suppose for the moment that the length of maturity is known with certainty. The representative commercial bank’s problem for the determination of an $n$-period loan rate will be given by

$$\max_{R_{n,t}} E_t \sum_{s=0}^{n-1} \Gamma_{t,t+s}[(1 + \tau^b)R_{n,t} \Psi(R_{n,t}; \Lambda_{t+s}) - R_{t+s} \Psi(R_{n,t}; \Lambda_{t+s})].$$

The first-order condition is

$$E_t \sum_{s=0}^{n-1} \beta^s \frac{C_{t+s}^{-\sigma} \sigma_{t+s}^{1-\sigma} \Lambda_{t+s}}{P_{t+s}} (R_{n,t} - R_{t+s}) = 0.$$  

It follows that

$$r_{n,t} = \left( \sum_{s=0}^{n-1} \beta^s \right)^{-1} E_t \sum_{s=0}^{n-1} \beta^s r_{t+s}. \quad (13)$$

While this is an expression of loan rates of maturity- $n$, this can also be interpreted as the $n$-period market interest rates since the bank will set $r_{n,t}$ in such a way that the expected return equals the expected cost as long as there are neither adjustment costs nor default risk.$^{12}$ Because the bank faces no uncertainty in regard to the length of periods between adjustments, $r_{n,t}$ must be an efficient estimate of the per-period cost of funds from period $t$ to $t + n$. Unsurprisingly, this endogenously derived relation, (13), takes a form known as the expectations theory of the term structure. Here, the consumer’s subjective discount factor, $\beta$, is used as the discount factor on expected future short-term rates.

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$^{11}$Index $i$ is now dropped for brevity since the following hypothetical problem is common to all banks.

$^{12}$In order to consider a competitive equilibrium of market interest rates, distortion stemming from the monopolistic market power is removed by government subsidies.
Figure 1. Weights on Long-Term Interest Rates

Using expression (13), we can present the following proposition.

**Proposition 1.** If the $n$-period market interest rate is written as (13), then the newly adjusted loan rates, $\tilde{r}_t$, can be expressed as

$$\tilde{r}_t = (1 - q\beta)(1 - q)(r_t + \delta_1 r_{2,t} + \delta_2 r_{3,t} + \ldots),$$

where $\delta_k = q^k (1 - \beta^{k+1})^{1 - \beta}$ for $k \geq 0$, and $\sum_{k=0}^{\infty} \delta_k = ((1 - q\beta)(1 - q))^{-1}$. Moreover, $\delta_{k+1} < \delta_k$ holds for all $k \geq 0$ if and only if $q < (1 + \beta)^{-1}$.

**Proof.** See appendix 2.

This proposition states that the newly adjusted loan rate can be expressed as a weighted average of long-term market interest rates of various maturities. It turns out that the weights on long-term rates are largely dependent on the probability of revaluation. Figure 1 illustrates examples of $\delta$s. As is clear from the figure, the weights on short-term rates decrease with larger $q$. This reflects the fact that the currently adjusted loan rates will be expected to live for longer periods as the revaluation probability becomes lower.

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13Interestingly, if one interprets $\delta$ as the time-varying discount factor, it takes a form of hyperbolic discounting, where the discount rate itself decreases as the maturity increases.
4. Equilibrium Dynamics

Before proceeding, let us summarize the key equilibrium relations in preparation for succeeding analyses. Appendix 3 shows that the real marginal cost of final-goods firms, \( p_z - p_t \), can be expressed as
\[
p_z - p_t = r_l + (\sigma + \omega)x_t,
\]
where \( r_l \) and \( x_t \) denote an average loan rate and an output gap, respectively. The New Keynesian Phillips curve (NKPC) can thus be written as
\[
\pi_t = \beta E_t \pi_{t+1} + \lambda_F (\sigma + \omega)x_t + \lambda_F r_l^t. \tag{14}
\]

As was pointed out by Ravenna and Walsh (2006), the difference between the standard NKPC and the NKPC with the cost-channel effect lies in the presence of an additional interest rate term. Yet, our expression differs from theirs in that the interest term in (14) is expressed by the average loan rate, not by the policy rate. Since our model incorporates profit-maximization behavior of commercial banks, retail loan rates are distinguished from the policy instrument in an endogenous manner.\(^{14}\) It turns out that the average loan rate, \( r_l^t \), becomes a determinant of inflation because a rise in the average loan rate leads to a higher marginal cost for final-goods’ production.

An obvious outcome of this modification is that as long as \( q > 0 \), the cost-channel effect is weakened compared with the case of perfect pass-through. This is not only because only a fraction \((1 - q)\) of commercial banks reset their loan rates each period, but also because a newly charged loan rate differs from the policy rate in that period. Since the correlation between the policy rate and the marginal cost of intermediate-goods firms becomes weaker as \( q \) increases, the influence of a policy shift on final-goods prices will be reduced accordingly.\(^{15}\)

\(^{14}\)Chowdhury, Hoffmann, and Schabert (2006) also make distinctions between a money-market rate and a lending rate in a model similar to ours, but their distinction depends fully on the assumption that there exists a proportional relationship between the two interest rates.

\(^{15}\)Recently, Tillmann (2007) estimated the NKPC of the form (14) using the data for the United States, the United Kingdom, and the euro area. He showed that inflation dynamics can be better explained if the short-term rate that appeared in the Ravenna-Walsh NKPC is replaced with lending rates.
The standard aggregate demand equation can be obtained by log-linearizing the Euler equation (3):

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1} - rr^n_t),$$  \hspace{1cm} (15)

where $rr^n_t \equiv \sigma((1+\omega)/(\sigma+\omega))E_t \Delta a_{t+1} + \omega((\sigma-1)/(\sigma+\omega))E_t \Delta \hat{\xi}_{t+1}$ denotes the natural rate of real interest, where $\hat{\xi}_t \equiv \log(\xi_t)$.

Now we turn our attention to the determination of the average loan rate. By the nature of commercial banks’ loan rate setting, the average loan rate is given by

$$r^l_t = qr^l_{t-1} + (1-q)\tilde{r}_t.$$  

The current average loan rate can be expressed as a weighted average of the newly adjusted loan rate and the previous average loan rate. Eliminating $\tilde{r}_t$ from (12) yields

$$\Delta r^l_t = \beta E_t \Delta r^l_{t+1} + \lambda_B (r_t - r^l_t),$$  \hspace{1cm} (16)

where $\Delta r^l_t \equiv r^l_t - r^l_{t-1}$ and $\lambda_B \equiv (1-q)(1-q\beta)/q$. Equation (16) says that a shift in the average loan rate will be caused by a discrepancy between the policy rate and the average loan rate as well as a change in the expectation of future loan rate. This equation can also be written as

$$r^l_t = \frac{\beta}{1+\beta+\lambda_B} E_t r^l_{t+1} + \frac{\lambda_B}{1+\beta+\lambda_B} r^l_t + \frac{1}{1+\beta+\lambda_B} r^l_{t-1}.$$  

Intuitively, the average loan rate is expressed as a weighted average of the expected loan rate, the current policy rate, and the previous loan rate.\textsuperscript{16} It states that the relative weights on the expected loan rate and the previous loan rate increase as the sluggishness of loan rates deteriorates. Conversely, the current loan rate approaches the current policy rate as $q$ goes to zero.

In an environment where the central bank controls $r_t$, equations (14), (15), and (16) and a policy rule describe the behavior of $\pi$, $x$, $r^l$, and $r$. We next explore the central bank’s optimal policy rate setting in the following sections.

\textsuperscript{16}After I finished writing this paper, I found that Teranishi (2008) also obtained similar results in a different setting. We arrived at the similar results completely independently of each other.
5. Social Welfare

This section attempts to obtain a welfare-based objective function for monetary policy by approximating the household’s utility function up to a second order. Appendix 4 shows that the one-period utility function can be approximated as

\[ U_t = -\frac{\bar{L}^{1+\omega}}{2}(\sigma + \omega) \left( x_t^2 + \left( \frac{\theta}{\sigma + \omega} \right) \text{var}_t p_t(j) \right) \]
\[ + \left[ \frac{\theta z}{(1 + \omega \theta)(\sigma + \omega)} \right] \text{var}_t r^i_t \} + \text{t.i.p.}, \]  

(17)

where an upper bar means that the variable denotes the corresponding steady-state value, and t.i.p. represents terms that are independent of policy, including terms higher than or equal to third order. A notable feature of equation (17) is the presence of the variance of loan rates. This result is quite intuitive given that the determination of loan rates is specified as Calvo-type pricing. Equation (17) reveals that the variance of lending rates reduces social welfare in the same manner as the variance of final-goods prices does.

Woodford (2001, 22–23) shows that the present discounted value of the variance of prices can be expressed in terms of inflation squared. That is,

\[ \sum_{s=0}^{\infty} \beta^s \text{var}_t p_t+s(j) = \lambda_F^{-1} \sum_{s=0}^{\infty} \beta^s \pi_{t+s}^2. \]

It is straightforward to apply this result to rewriting the present discounted value of the variance of lending rates. It follows that

\[ \sum_{s=0}^{\infty} \beta^s \text{var}_t r^i_t+s = \lambda_B^{-1} \sum_{s=0}^{\infty} \beta^s \left( \Delta r^i_{t+s} \right)^2. \]

It turns out that the present discounted value of the variance of lending rates can be expressed in terms of a change in the average loan rate.
Consequently, the social welfare function can be rewritten as

\[
E_t \sum_{s=0}^{\infty} \beta^s U_{t+s} = -\frac{L^{1+\omega}}{2} (\sigma + \omega) E_t \sum_{s=0}^{\infty} \beta^s \left\{ x^2_{t+s} + \psi_{\pi} \pi^2_{t+s} \right\} + \psi_r \left( \Delta r^l_{t+s} \right)^2 \right\} + \text{i.p.,} \tag{18}
\]

where \( \psi_{\pi} \equiv \theta / [\lambda F(\sigma + \omega)] \) and \( \psi_r \equiv \theta_z / [\lambda B(1 + \omega \theta_z)(\sigma + \omega)] \) represent the relative weights on inflation and the rate of change in the average loan rate, respectively. Equation (18) states that fluctuations in the average loan rate will reduce social welfare when commercial banks adjust loan rates only infrequently. This finding is closely parallel to a well-known result obtained under staggered goods prices. Under staggered goods prices, the rate of inflation enters into the welfare function because a nonzero inflation gives rise to price dispersion. Under staggered loan rate contracts, the rate of change in the average loan rate enters into the welfare function because changes in the average loan rate inevitably entail loan rate dispersion.

It might also be noted that equation (18) closely resembles a conventional loss function that has been frequently employed in the recent literature on monetary policy for the purpose of capturing actual central banks’ interest rate smoothing (i.e., policy rate smoothing) behavior. Specifically, in many previous studies it has been assumed that a monetary authority tries to minimize a loss function of the form\(^ {17} \)

\[
\text{Loss}_t^c = x^2_t + \lambda \pi^2_t + \nu (\Delta r_t)^2.
\]

This expression essentially differs from ours in that the third term is expressed in terms of the policy instrument rather than the average loan rate. Here, the relation between \( \Delta r_t \) and \( \Delta r^l_t \) can be written from proposition 1 as

\[
\Delta r^l_t = (1 - q \beta)(1 - q)^2 [\Delta r_t + \delta_1 \Delta r_{2,t} + \ldots \\
+ q(\Delta r_{t-1} + \delta_1 \Delta r_{2,t-1} + \ldots) + \ldots].
\]

\(^ {17} \)See, for example, Rudebusch and Svensson (1999), Rudebusch (2002a, 2002b), Levin and Williams (2003), and Ellingsen and Söderström (2004). See Sack and Wieland (2000) and Rudebusch (2006) for a survey of studies on interest rate smoothing.
Thus, $\Delta r_t$ constitutes only a fraction $(1 - q \beta)(1 - q)^2$ of $\Delta r_t^l$. The rest of the components of $\Delta r_t^l$ are expressed by the past policy shifts and the current and past changes in long-term rates. Notice that equation (18) and the conventional loss function never coincide since the loan rate smoothing term will disappear in the limiting case of $q = 0$, where $r_t^l = r_t$ holds. Nevertheless, the desirability of policy rate smoothing might be retained in that it contributes to the stabilization of loan rates through the stabilization of long-term rates. A further discussion about the relationship between loan rate stabilization and the central bank’s policy rate smoothing will be given in the next section.

6. Monetary Policy in the Presence of Loan Rate Stickiness

This section attempts to explore desirable monetary policy in the presence of incomplete interest rate pass-through, focusing on the question of how the desirable path of the policy rate will be modified once loan rate stickiness is taken into account. Provided that the central bank tries to maximize social welfare function (18), the presence of loan rate stickiness affects inflation and output through two channels. On one hand, the presence of loan rate stickiness mitigates the cost-channel effect of a policy shift on inflation. On the other hand, the central bank has to put some weight on loan rate stabilization in the face of loan rate stickiness. It is shown below that the former effect tends to reduce the desirability of policy rate smoothing since there is less need for the central bank to pay attention to the undesirable effect that a policy shift has on inflation. In contrast, the latter effect increases the desirability of policy rate smoothing since the stabilization of the policy rate leads, at least to some extent, to loan rate stability. These two aspects are thoroughly examined in the succeeding subsections.

In the following, we consider two alternative policy regimes: standard Taylor rule and commitment under a timeless perspective. In addition, we also investigate optimal policy in the face of a loan premium shock, which directly alters the markup in loan rate pricing. It is shown that the role of the loan rate stability term depends largely on the underlying nature of shocks.
### 6.1 Baseline Parameters

The baseline parameters used in the analysis are as follows: $\beta = .99$, $\sigma = 1.5$, $\omega = 1$ (Ravenna and Walsh 2006), and $\theta = 7.88$ (Rotemberg and Woodford 1997). We set the elasticity of substitution for intermediate goods at the value equal to $\theta$, thus $\theta_z = 7.88$. Following Galí and Monacelli (2005), we specify the process of productivity shock as $a_t = .66a_{t-1} + \zeta^a_t$, where the standard deviation of $\zeta^a_t$ is set at .007. As for the preference shock, we specify the process as $\xi_t = .5\xi_{t-1} + \zeta^\xi_t$, where the standard deviation of $\zeta^\xi_t$ is set at .005.\(^{18}\) The degree of price stickiness, $\phi$, is chosen such that the slope of the Phillips curve is equal to .58, the value reported by Lubik and Schorfheide (2004). It follows that $\phi = .623 \left( (1 - \phi)(1 - \beta\phi)(\sigma + \omega)/\phi = .58 \right)$, which leads to $\psi_\pi = 13.582$.

As mentioned in section 2, recent studies reported different estimates of the degree of loan rate pass-through at the euro-area aggregated level. Here, three alternative values are considered: $q_{L}$, $q_{M}$, and $q_{H}$. According to table 1 of de Bondt, Mojon, and Valla (2005), the lowest value of the estimated degree of loan rate pass-through for short-term loans to enterprises is .25 (Sander and Kleimeier 2002; Hofmann 2003), while the largest one is .76 (Heinemann and Schüeler 2002). Since these estimates are obtained from monthly data, we have to convert them to their quarterly counterparts. For example, in the case of the largest degree of pass-through, $q_{L}$ is set such that $1 - q_{L} = .76 + (1 - .76).76 + (1 - .76)^2.76$, which leads to $q_{L} = .014$. Likewise, $q_{H}$ is set at .422. Finally, $q_{M}$ is set at .177, the average of all the estimates reported by thirteen studies cited in table 1 of de Bondt, Mojon, and Valla (2005). This implies that the relative weight on the loan rate, $\psi_r$, is .445, .092, and .005 if $q = q_{H}$, $q_{M}$, and $q_{L}$, respectively.

### 6.2 Policy Rate Smoothing and the Degree of Interest Rate Pass-Through

Before investigating optimal policy, it should be pointed out that the degree of interest rate pass-through is heavily dependent on the preference shock.

\(^{18}\)The essential results shown below will never change in the absence of the preference shock.
policy rate behavior. The impact of a policy shift on retail loan rates can vary not only with the frequency of loan rate adjustments but also with the expectation of future policies. It is useful to gain a better understanding of the relation between policy rate smoothness and the degree of interest rate pass-through.

Suppose for exposition that the policy rule is expressed as

$$ r_{t+1} = \rho r_t + \eta_{t+1}, $$

where $\rho \in [0,1)$ describes the degree of policy rate inertia. $\eta_{t+1}$ is a white noise, which represents an unpredictable component of the policy rate. Then, the average loan rate is given as

$$ r_l^t = \frac{(1 - q)(1 - q\beta)}{1 - \rho q\beta} r_t + q r_{l,t-1}. $$

In this case, the degree of instantaneous interest rate pass-through can be expressed as

$$ \frac{\partial r_l^t}{\partial r_t} = \frac{(1 - q)(1 - q\beta)}{1 - \rho q\beta}. $$

This implies that the impact of a policy shift on the current average loan rate will become larger as the degree of policy inertia increases.

More generally, it can be shown that the impact of a current policy shift on the $s$-period-ahead average loan rate leads to

$$ \frac{\partial r_{l,t+s}}{\partial r_t} = \frac{(1 - q)(1 - q\beta)(\rho^{s+1} - q^{s+1})}{(1 - \rho q\beta)(\rho - q)}. $$

Accordingly, the degree of cumulative interest rate pass-through can be given as

$$ \sum_{s=0}^{\infty} \frac{\partial r_{l,t+s}}{\partial r_t} = \frac{1 - q\beta}{(1 - \rho q\beta)(1 - \rho)}. $$

Notice that under an inertial policy rule, current policy rate has an impact on future average loan rate not only through the persistent dynamics in the average loan rate but also through the policy rate dynamics itself. Since commercial banks’ loan rate determination is made in a forward-looking manner, a policy shift can have a larger impact on loan rates as the shift becomes more persistent.
Now, let us reexamine the above implications by using a more general policy rule. We employ the following standard Taylor rule:

\[ r_t = \rho r_{t-1} + (1 - \rho)(rr_t^* + \phi_\pi \pi_t + (\phi_x/4)x_t), \tag{19} \]

where \( \phi_\pi \) and \( \phi_x \) are set at 1.5 and .5, respectively. \( \rho \) is set at .9 under an “inertial policy,” while \( \rho = 0 \) under a “non-inertial policy.” Figure 2 illustrates impulse responses to a one-standard-deviation productivity shock.\(^{19}\) The figure shows that there is an appreciable difference between the two cases in the reaction of the average loan rate to the policy rate. Under the inertial policy rule, the paths of \( r_t \) and \( r^*_t \) are shown to be very close. Specifically, the spread between the two paths on impact is only .02 percent, where the instantaneous interest rate pass-through turns out to be 84.2 percent. Under the non-inertial policy rule, on the other hand, the initial spread amounts to .34 percent, where the instantaneous interest rate pass-through is 77.3 percent. Thus, the property that a lagged policy rate

\(^{19}\)Henceforth, interest rate responses are illustrated in annual rate.
term plays a key role as a determinant of the degree of interest rate pass-through still holds under the standard Taylor rule.

6.2.1 Some Intuitions into the Desirability of Policy Rate Smoothing

In order to obtain some intuitions into the relationship between loan rate smoothing and policy rate smoothing, let us first express the current loan rate solely in terms of the policy rate.

\[ r_t^l = (1 - q) \tilde{r}_t + q r_{t-1}^l \]

\[ = (1 - q)(1 - q\beta)[r_t + \beta q E_t r_{t+1} + (\beta q)^2 E_t r_{t+2} + \ldots] \]

\[ + q(1-q)(1-q\beta)[r_{t-1} + \beta q E_{t-1} r_t + (\beta q)^2 E_{t-1} r_{t+1} + \ldots] + \ldots. \]

It follows that

\[ \Delta r_t^l = (1 - q)(1 - q\beta)[\Delta r_t + \beta q (E_t \Delta r_{t+1} + r_t - E_{t-1} r_t) + \ldots] \]

\[ + q(1-q)(1-q\beta)[\Delta r_{t-1} + \beta q (E_{t-1} \Delta r_t + r_t - E_{t-1} r_{t-1}) \]

\[ - E_{t-2} r_{t-1} + \ldots] + \ldots. \]

This expression shows that the growth rate of the current average loan rate is determined not only by the current and the past policy rate increments but also by the expectations of future increments and policy surprises. This reveals two important implications for loan rate stabilization. First, the presence of increment terms implies that the policy rate should be continuously smoothed. This is simply because any policy rate changes inevitably give rise to a shift in the newly adjusted loan rates. The average loan rate becomes more stable as the policy rate in any given period becomes closer to the previous period’s level. This necessarily requires the policy rate to be inertial or history dependent.

Second, the “surprise” terms, \( r_t - E_{t-1} r_t, r_{t-1} - E_{t-2} r_{t-1}, \) and so on, state that the central bank should avoid causing a policy surprise, for a revision of commercial banks’ policy rate expectations will entail a shift in the newly adjusted loan rates. This is quite natural in that the commercial banks’ loan rate determination is based on the expectation of future policy rates conditional on information.
available at that time.\textsuperscript{20} It should be noted that not only expectation errors in the current period but also expectation errors made in the past cause a change in the current average loan rate. The reason for this is as follows: suppose that the policy rate has not been changed since \( m \) periods ago, and the last policy shift had not been anticipated at that time. In the current period, given that the policy rate is still expected to be constant in the future, loan rates between the ages of 1 and \( m \) need not be changed even if they have a chance of adjustment, because the last policy shift is already incorporated. In contrast, loan rates that have not been adjusted for the past \( m \) periods need to be readjusted in the current period since they have not yet incorporated the unexpected policy shift that occurred \( m \) periods ago. Since a certain fraction of all the loan rates is necessarily over the age of \( m \), their readjustments inevitably occur and cause a shift in the current average loan rate.

It is evident that once the central bank changes the policy rate, the resultant loan rate readjustments will persist forever. These loan rate readjustments will never end, even if the policy rate is (and is expected to be) kept unchanged from then on. However, such persistent effects would be alleviated if the policy shift was correctly anticipated in advance. Of course, even if a policy shift is incorporated in advance, a revision of expectation necessarily occurs at least to some extent (unless the entire policy rate path was fully incorporated at the initial period). Nevertheless, the extent of an expectation revision can be made smaller as the timing of incorporation becomes earlier since the corresponding adjustments of loan rates will be dispersed over some periods. In this sense, it could be said that the forecastability of future policy rates becomes another key to loan rate stability. Policy rate smoothing will contribute to loan rate stability by revealing some information regarding future policy rates.\textsuperscript{21}

\textsuperscript{20}In fact, Svensson (2003) notes that the central bank should minimize the surprise in the policy rate. He proposed the (ad hoc) central bank’s loss function of the form \( L_t = \text{Var}(x_t) + \lambda \text{Var}(\pi_t) + \nu \text{Var}(E_{t-1}r_t - r_t) \).

\textsuperscript{21}The necessity of the central bank’s communicability is stressed by Kleimeier and Sander (2006). They argue that the impact of policy rate shifts on retail lending rates tends to be large in countries in which the central bank communicates well with the public. See also Woodford (2005) for a discussion of central bank communication.
6.3 The Role of the Loan Rate Stabilization Term

In this section, we address the issue of how the presence of a loan rate stabilization term, \( \psi_r(\Delta r_l)^2 \), affects the desirable policy. To this end, we especially focus on its relation with conventional policy rate smoothing or policy inertia. In investigating policy inertia under various degrees of loan rate stickiness, we have to explicitly distinguish between policy inertia and intrinsic inertia. To isolate policy inertia, we need to know to what extent current policymaking depends on the previous policy decision. Thus, we measure here the desirable degree of policy inertia by the size of the coefficient on the lagged policy rate in the case of a simple rule, and by the size of the relative weight on the policy rate smoothing term in the case of commitment.

6.3.1 Simple Rule

As a policy rule, we again employ (19). Here, optimal combinations of \((\rho, \phi_{\pi}, \phi_x)\) are searched for under alternative values of \(q\), \(\psi_\pi\), and \(\psi_r\). Since we want to know the effects of introducing the loan rate stabilization term on the optimal value of \(\rho\), the relative weight on the loan rate, \(\psi_r\), is set either at the endogenously determined value or at zero.

Optimal combinations of \((\rho, \phi_{\pi}, \phi_x)\) are given in table 1.\(^{22}\) It shows that, given the values of \(q\) and \(\psi_\pi\), the optimal value of \(\rho\)

<table>
<thead>
<tr>
<th>((\psi_\pi, \psi_r))</th>
<th>((13.58, \psi_r))</th>
<th>((13.58, 0))</th>
<th>((4, \psi_r))</th>
<th>((4, 0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q = q_L)</td>
<td>(.35, 3, 0)</td>
<td>(.35, 3, 0)</td>
<td>(.35, 1, 0)</td>
<td>(.35, 1, 0)</td>
</tr>
<tr>
<td>Relative Loss(^a)</td>
<td>1.225</td>
<td>1.225</td>
<td>1.004</td>
<td>1.004</td>
</tr>
<tr>
<td>(q = q_M)</td>
<td>(.35, 3, 0)</td>
<td>(.35, 3, 0)</td>
<td>(.4, 1.65, 0)</td>
<td>(.35, 1.15, 0)</td>
</tr>
<tr>
<td>Relative Loss</td>
<td>1.116</td>
<td>1.116</td>
<td>1.004</td>
<td>1.008</td>
</tr>
<tr>
<td>(q = q_H)</td>
<td>(.3, 3, 0)</td>
<td>(.25, 3, 0)</td>
<td>(.4, 2.6, 0)</td>
<td>(.35, 0)</td>
</tr>
<tr>
<td>Relative Loss</td>
<td>1.027</td>
<td>1.054</td>
<td>1.021</td>
<td>1.046</td>
</tr>
</tbody>
</table>

\(^a\)The denominator of “relative loss” is the value of social loss that would be attained under commitment policy with the appropriate loan rate smoothing objective.

\(^{22}\)The examined parameter ranges are as follows: \([0, .95]\) for \(\rho\), \([0, 3]\) for \(\phi_{\pi}\), and \([0, 1]\) for \(\phi_x\). The increment size of the grid is .05.
under loan rate stabilization is greater than or equal to that under $\psi_r = 0$. This implies that the introduction of a loan rate smoothing term into the welfare function tends to strengthen the desirability of policy rate smoothing, although this smoothing effect seems to be limited, especially when loan rate stickiness is not severe. This argument is also confirmed by the value of social loss under an optimal policy rule relative to the loss under timeless commitment. It turns out that the cost of putting a “too-small” weight on the previous policy rate is at most 2.6 percent.

6.3.2 Commitment

Next, let us turn to the case of commitment. Under commitment, the central bank is assumed to minimize a given loss function. In order to investigate the desirability of policy inertia, we specify here the central bank’s one-period loss function as follows:

$$L_t^{smooth} = x_t^2 + \psi_\pi \pi_t^2 + \tilde{\psi}_r (\Delta r_t)^2.$$ 

In this case, the value of $\tilde{\psi}_r$ that minimizes social loss can be interpreted as a proxy for the optimal degree of policy inertia.

The optimal values of $\tilde{\psi}_r$ under alternative values of $q$ are shown in table 2. It shows that as long as the value of $q$ is not that small, the optimal value of $\tilde{\psi}_r$ tends to be larger under loan rate stabilization than under $\psi_r = 0$. This reconfirms the previous result that the introduction of a loan rate stabilization term justifies conventional policy rate smoothing. However, the quantitative importance of this smoothing effect is again not very large. The cost of ignoring policy rate stability is at most 3.6 percent.

Table 2. Optimal Weights on $(\Delta r_t)^2$

<table>
<thead>
<tr>
<th>$(\psi_\pi, \psi_r)$</th>
<th>(13.58, $\psi_r$)</th>
<th>(13.58, 0)</th>
<th>(4, $\psi_r$)</th>
<th>(4, 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q = q_L$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Relative Loss</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$q = q_M$</td>
<td>.05</td>
<td>0</td>
<td>.05</td>
<td>0</td>
</tr>
<tr>
<td>Relative Loss</td>
<td>1.001</td>
<td>1.014</td>
<td>1.001</td>
<td>1.026</td>
</tr>
<tr>
<td>$q = q_H$</td>
<td>.05</td>
<td>0</td>
<td>.05</td>
<td>0</td>
</tr>
<tr>
<td>Relative Loss</td>
<td>1.007</td>
<td>1.026</td>
<td>1.014</td>
<td>1.051</td>
</tr>
</tbody>
</table>
Figure 3. Policy Rate Responses under Commitment

Figure 3 illustrates the policy rate responses under timeless commitment. It turns out that the optimal policy rate responses under alternative values of $q$ differ only slightly. Nevertheless, it can be said from the figure that the initial reduction of the policy rate is largest (smallest) when $q = q_H$ ($q_L$). This is because an increase in $q$ mitigates the cost-channel effect, the direct impact of a policy change on inflation. A reduction in the policy rate is needed in the face of a positive productivity or preference shock, but such an expansionary policy necessarily entails a negative effect on inflation due to the presence of the cost channel. As $q$ increases, however, such an undesirable aspect becomes less important, and thereby the central bank can set the policy rate at a level closer to the natural rate of interest. The figures show that the cost-channel effect is quantitatively more influential than the policy rate smoothing effect, which stems from the change in the value of $\psi_r$.

In order to clarify the strength of the policy rate smoothing effect, figure 4 illustrates policy rate responses under commitment with and without loan rate smoothing.\footnote{The obtained results are essentially the same in the case of a preference shock as well.} It can be confirmed that the initial reduction in the policy rate is smaller in the presence of loan rate
stabilization. As expected, however, the difference between the two cases is not significantly different.

In summary, in the face of a productivity shock and/or a preference shock, the presence of a loan rate stabilization term itself supports, to some extent, the idea of conventional policy rate smoothing. However, it appears that the optimal policy is more strongly influenced by the cost-channel effect than the policy rate smoothing effect that stems from the presence of a loan rate stabilization term. In the next section, we reexamine the role of loan rate stabilization by introducing a loan rate premium shock, which directly changes the markup in loan rates.

6.4 Undesirability of Policy Rate Smoothing: The Case of a Loan Rate Premium Shock

In the above analysis, we investigated the optimal policy response in the face of a productivity shock or a preference shock. In such
an environment, retail loan rates are determined solely by the policy rate, although those shocks have an indirect influence through the policy rate. In practice, however, it is usual for loan rates to fluctuate for reasons that are not directly linked to the policy rate behavior. One possible case is a shift in the loan rate premium triggered by changes in financial market conditions. While here we do not emphasize a particular cause of loan premium fluctuations, optimal policy in the face of such kinds of shocks is worth considering.

A loan rate premium shock can be introduced by modifying the first-order condition of the commercial banks’ problem as follows:

\[
E_t \sum_{s=0}^{\infty} (q \beta)^s \frac{C_t \sigma^{1-\sigma} A_t \varphi_{t+s}}{P_t} (R_t^i - \varphi_{t+s} R_t^i) = 0,
\]

where \( \varphi_{t+s} > 0 \) and \( E[\varphi_{t+s}] = 1 \). It follows that

\[
\Delta r_t^i = \beta E_t \Delta r_{t+1}^i + \lambda_B (r_t - r_t^i) + \lambda_B \hat{\varphi}_t,
\]

where \( \hat{\varphi}_t \equiv \log(\varphi_t) \). \( \hat{\varphi}_t \) is actually a shock to the change in the loan rate but can also be interpreted as a shock to the level of the loan rate once redefined as \( \tilde{\varphi}_t \equiv \frac{\lambda_B}{1-\lambda_B} \hat{\varphi}_t \).

Figure 5 illustrates optimal policy rate responses under commitment to a 1 percent (at the annual rate) positive loan rate premium shock. It is assumed that this shock is a temporary one. It clearly shows that the presence of a loan rate stabilization term now plays a critical role in the conduct of monetary policy. Under optimal policies with a loan rate stabilization objective, the policy rate needs to be drastically reduced in the face of a rise in the loan rate premium. This is because a reduction in \( r_t \) can partially offset a rise in \( \hat{\varphi}_t \), as is evident from the above equation. On the other hand, such a drastic policy rate reduction cannot be observed when the central

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24 Another way of introducing an exogenous loan rate shock is to assume a time-varying subsidy rate, \( \tau_t^h \). Provided that \( E[\tau_{t+s}^h] = \tau^h \), then the modified loan rate adjustment equation leads to

\[
\Delta r_t^i = \beta E_t \Delta r_{t+1}^i + \lambda_B (r_t - r_t^i) - \lambda_B \tau_t^h,
\]

where \( \tau_t^h = \log((1 + \tau_t^h)/(1 + \tau^h)) \). In this case, a below-the-average subsidy rate will act as a positive loan rate shock. See also Woodford (2003, ch. 6) for a discussion of time-varying markups in goods prices.
bank conducts “optimal” policy rate smoothing, which is incorrect from the point of view of welfare maximization. In fact, since the optimal weight on the policy rate is very small (.05), this result will also hold even in the case where the central bank pays no attention to policy rate smoothing.

Figure 5 also illustrates the behavior of the average loan rate. As is clear from the figure, the response of the average loan rate is mitigated as $q$ increases, since the fraction of newly adjusted loan rates declines. Along with this, the required amount of policy rate reduction turns out to be smaller under $q = q_H$ than under $q = q_M$. Although a rise in $q$ has the effect of requiring more drastic policy shifts by increasing the size of the relative weight on the loan rate, such an effect is relatively small.

To sum up, the role of a loan rate stabilization term fundamentally alters according to the underlying nature of shocks. In the face of a shock that would directly shift inflation and output, the presence of the loan rate stabilization objective itself requires inertial policy. This is because loan rates are determined based only on the policy rate, in which case the only way to avoid fluctuations in loan rates is
to avoid fluctuations in the policy rate. However, a shock that would directly give rise to loan rate fluctuations should be dampened by a drastic policy shift. Policy rate smoothing is not needed (and in fact is even harmful) when there is requirement for loan rate stabilization. From this point of view, it can be said that conventional policy rate smoothing is no longer a panacea. The case of a loan rate premium shock is an example for which policy rate smoothing should be abandoned.

7. Concluding Remarks

The main findings of this paper can be summarized as follows. First, when the pass-through from the policy rate to retail loan rates is incomplete, fluctuations in the average loan rate will reduce social welfare. This is because shifts in the average loan rate immediately give rise to a loan rate dispersion across firms, which ultimately yields an inefficient dispersion in hours worked. Accordingly, the central bank faces a policy trade-off in stabilizing inflation, an output gap, and the rate of change in the average loan rate.

Second, the introduction of a loan rate stabilization term in the central bank’s loss function causes the optimal policy rate to become more inertial in the face of a productivity shock and a preference shock. In this sense, loan rate smoothing is closely parallel to conventional policy rate smoothing. However, such a smoothing effect turned out to be less influential than the cost-channel effect.

Third, the presence of a loan rate stabilization term requires a drastic policy reaction in the face of an exogenous shock that directly shifts retail lending rates, such as a shift in the loan rate premium. This result is counter to the conventional wisdom that the policy rate must be adjusted gradually in short steps. However, given the fact that the standard dynamic stochastic general equilibrium model usually ignored the cost of loan rate dispersion, this disagreement is not so surprising. The case of a loan premium shock is an example for which the central bank has to clearly distinguish between policy rate smoothing and loan rate smoothing.

We conclude by noting several points that should be addressed in future research. First, a more realistic framework for long-term interest contracts should be introduced. In the present paper, loan rate determination is specified as Calvo-type pricing. However, a
more plausible situation would be that the length of maturity is determined at the time of contract and is allowed to differ across borrowers. Second, although our model treats the frequency of loan rate adjustments as exogenous, there is a possibility that the frequency of loan rate adjustments depends on the policy rate behavior. Finally, stickiness in deposit rates as well as in loan rates should also be considered, since many previous studies have reported that deposit rates are also sticky. Although this paper treats deposit rates as equivalent to the policy rate, the relaxation of this assumption may affect the desirability of policy rate smoothing.

Appendix 1. Derivation of the Demand for Funds

From (4), (5), and (7), labor wage $W_t(i)$ can be expressed as

$$W_t(i) = \xi_t^{\sigma-1} P_t C_t^{\sigma} L_t^{\omega}(i)$$

$$= \xi_t^{\sigma-1} P_t C_t^{\sigma} \left( \frac{Z_t(i)}{A_t} \right)^{\omega}$$

$$= \xi_t^{\sigma-1} P_t C_t^{\sigma} \left( \frac{P_t^{z}(i)}{P_t^{z}} \right)^{-\omega \theta_z} \frac{Y_t(V_t^y)^{\omega}}{A_t^{\omega}}$$

$$\equiv \Xi_t P_t^{z}(i)^{-\omega \theta_z}. \quad (20)$$

Inserting this equality into (6) gives

$$P_t^{z}(i) = \frac{\Xi_t R_t^i P_t^{z}(i)^{-\omega \theta_z}}{R A_t}$$

$$= \frac{\Xi_t R_t^i}{R A_t} \left( \frac{1}{1+\omega \theta_z} \right). \quad (21)$$

Therefore, the amount of funds demanded by intermediate-goods firm $i$, $W_t(i)L_t(i)$, leads to

$$W_t(i)L_t(i) = \xi_t^{\sigma-1} P_t C_t^{\sigma} L_t(i)^{1+\omega}$$

$$= \xi_t^{\sigma-1} P_t C_t^{\sigma} \left( \frac{P_t^{z}(i)}{P_t^{z}} \right)^{-\theta_z(1+\omega)} \frac{Y_t^{1+\omega}(V_t^y)^{1+\omega}}{A_t^{1+\omega}}$$

$$\equiv (R_t^i)^{-\frac{(1+\omega)\theta_z}{1+\omega \theta_z}} A_t,$$
where $\Lambda_t \equiv \xi_t^{\sigma-1} P_t C_t^{\sigma} A_t^{(1+\omega)(\theta_z-1)} (P_t^z)^{(1+\omega) \theta_z} (Y_t V_t^y)^{1+\omega} \Xi_t^{-(1+\omega) \theta_z} R_t^{(1+\omega) \theta_z}$. 

Appendix 2. Proof of Proposition 1

Given the definition of long-term interest rates, the newly adjusted loan rate, $\tilde{r}_t$, should be expressed as

$$\tilde{r}_t = (1 - q\beta) E_t [r_t + q\beta r_{t+1} + (q\beta)^2 r_{t+2} + \ldots]$$

$$= \left( \sum_{s=0}^{\infty} \delta_s \right)^{-1} E_t \left[ r_t + \delta_1 \left( \frac{r_t + \beta r_{t+1}}{1 + \beta} \right) + \delta_2 \left( \frac{r_t + \beta r_{t+1} + \beta^2 r_{t+2}}{1 + \beta + \beta^2} \right) + \ldots \right].$$

Accordingly, comparing the coefficients on $r_t$ and on $E_t r_{t+1}$, respectively, yields

$$1 - q\beta = \left( \sum_{s=0}^{\infty} \delta_s \right)^{-1} \left( 1 + \frac{\delta_1}{1 + \beta} + \frac{\delta_2}{1 + \beta + \beta^2} + \ldots \right)$$

and

$$(1 - q\beta) q\beta = \left( \sum_{s=0}^{\infty} \delta_s \right)^{-1} \left( \frac{\delta_1 \beta}{1 + \beta} + \frac{\delta_2 \beta}{1 + \beta + \beta^2} + \ldots \right).$$

Summarizing these two equations leads to

$$\left( \sum_{s=0}^{\infty} \delta_s \right)^{-1} = (1 - q)(1 - q\beta).$$

Therefore, comparing the coefficients on $E_t r_{t+k}$ and on $E_t r_{t+k+1}$, respectively, yields

$$(1 - q\beta)(q\beta)^k = (1 - q)(1 - q\beta) \left( \frac{\delta_k \beta^k}{\sum_{s=0}^{k} \beta^s} + \frac{\delta_{k+1} \beta^k}{\sum_{s=0}^{k+1} \beta^s} + \ldots \right)$$
and

\[(1 - q\beta)(q\beta)^{k+1} = (1 - q)(1 - q\beta) \left( \frac{\delta_{k+1}\beta^{k+1}}{\sum_{s=0}^{k+1} \beta^s} + \frac{\delta_{k+2}\beta^{k+1}}{\sum_{s=0}^{k+2} \beta^s} + \ldots \right) \]

for all \( k \geq 1 \). Summarizing these two equations, we have

\[\delta_k = q^k \sum_{s=0}^{k} \beta^s,\]

which is the desired result.

Next, let us derive a condition that attains \( \delta_{k+1} < \delta_k \) for all \( k \geq 0 \), where \( \delta_0 \equiv 1 \). From the expression of \( \delta_k \), we have

\[\delta_k - \delta_{k+1} = q^k \sum_{s=0}^{k} \beta^s - q^k \sum_{s=0}^{k} \beta^s = q^k \left[ 1 - \frac{1}{1 - \beta} \right].\]

Then, the following condition has to be satisfied for this to be positive for all \( k \geq 0 \):

\[q < \frac{1 - \beta^{k+1}}{1 - \beta^{k+2}} \equiv \varpi(k), \text{ for all } k \geq 0.\]

At this point, note that \( \frac{\partial \varpi(k)}{\partial k} = -\beta^{k+1}(1-\beta) \ln \beta / (1-\beta^{k+2})^2 > 0 \), and \( \varpi(0) = (1 + \beta)^{-1} \). Therefore, the condition \( \delta_{k+1} < \delta_k \) is satisfied for all \( k \geq 0 \) if and only if \( q(1 + \beta) < 1 \).

**Appendix 3. Derivation of \( p^*_t - p_t = r^t_t + (\sigma + \omega)x_t \)**

From the household’s optimality condition (4), it is obvious that

\[w_t(i) - p_t = \sigma y_t + \omega l_t(i) - (1 - \sigma)\hat{\xi}_t.\]

Using this equality and the pricing rule of intermediate-goods firms, (6), we have

\[p^*_t - p_t - r^t_t + a_t = \sigma y_t + \omega l_t - (1 - \sigma)\hat{\xi}_t, \quad (22)\]
A linear approximation of (7) leads to \( z_t = y_t \), and the production function (5) implies \( l_t = z_t - a_t \). Notice that, as is shown in Galí and Monacelli (2005), the term \( v_t^y = d \log \int_0^1 \left( \frac{P_j(t)}{P_t} \right)^{\theta} dj \) is of second order. It follows that
\[
l_t = y_t - a_t.
\]
Inserting this condition into equation (22) yields
\[
p_t^z - p_t = r_t^l + (\sigma + \omega) \left[ y_t - \left( \frac{1 + \omega}{\sigma + \omega} \right) a_t - \left( \frac{1 - \sigma}{\sigma + \omega} \right) \xi_t \right]. \tag{23}
\]
Let us define \( z_t^f \) as the flexible-price equilibrium of an arbitrary variable \( z_t \). It follows from (23) that
\[
y_t^f + \left( \frac{1}{\sigma + \omega} \right) r_t^{lf} = \left( \frac{1 + \omega}{\sigma + \omega} \right) a_t + \left( \frac{1 - \sigma}{\sigma + \omega} \right) \xi_t \equiv \tilde{y}_t^f.
\]
Let us call \( \tilde{y}_t^f \) the quasi-flexible-equilibrium output. This relation states that the sum of the flexible-equilibrium output and the flexible-equilibrium loan rate can be expressed in terms of a productivity shock and a preference shock. By defining \( x_t \equiv y_t - \tilde{y}_t^f \), equation (23) can be rewritten as
\[
p_t^z - p_t = r_t^l + (\sigma + \omega) x_t.
\]

**Appendix 4. Derivation of Equation (17)**

A second-order approximation of \( u(C_t) \) and \( v(L_t(i)) \), respectively, leads to
\[
u(C_t; \xi_t) = u' \bar{C} \left[ c_t + \frac{1}{2} (1 - \sigma) c_t^2 + \frac{u'^c \xi}{u'^c} c_t \xi_t \right] + t.i.p. \tag{24}\]
\[
v(L_t(i)) = v' \bar{L} \left[ l_t(i) + \frac{1}{2} (1 + \omega) l_t^2(i) \right] + t.i.p.
\]
From the relation \( l_t(i) = z_t(i) - a_t \), the latter can be written as
\[
v(L_t(i)) = v' \bar{L} \left[ z_t(i) + \frac{1}{2} (1 + \omega) z_t^2(i) - (1 + \omega) a_t z_t(i) \right] + t.i.p.
\]
It immediately follows that
\[
\int_0^1 v(L_t(i))\,di = v'\bar{L}\left\{[1 - (1 + \omega)a_t]\int_0^1 z_t(i)\,di \\
+ \frac{1}{2}(1 + \omega)\int_0^1 z_t^2(i)\,di\right\} + t.i.p. \tag{25}
\]

It turns out that the disutility of labor depends on \(\int_0^1 z_t(i)\,di\) and \(\int_0^1 z_t^2(i)\,di\). We focus on these expressions in turn.

From a second-order approximation of the definition of intermediate-goods price index, we have
\[
\int_0^1 p^*_z(i)\,di = p^*_z - \left(\frac{1 - \theta_z}{2}\right)\text{var}_i p^*_z(i).
\]

Inserting this into a linearized version of equation (7) yields
\[
\int_0^1 z_t(i)\,di = \frac{\theta_z(1 - \theta_z)}{2}\text{var}_i p^*_z(i) + y_t + v_t^y. \tag{26}
\]

Thus, the total intermediate goods can be expressed as a function of the variance of individual prices, \(\text{var}_i p^*_z(i)\).

Next, we show that the variance of intermediate-goods price can be written in terms of the variance of loan rates. Based on equation (6), we can establish that
\[
p^*_r(i) = r^*_t + w_t(i) - a_t \\
= r^*_t - a_t - (1 - \sigma)\xi_t + p_t + \sigma c_t + \omega l_t(i) \\
= r^*_t - a_t - (1 - \sigma)\xi_t + p_t + \sigma c_t + \omega\left[-\theta_z(p^*_z(i) - p^*_z) + y_t - a_t\right],
\]

where the second and the third equalities follow from (4) and (7), respectively. Since only \(p^*_z(i)\) and \(r^*_t\) are dependent on index \(i\), the variance of \(p^*_r(i)\) leads to
\[
\text{var}_i p^*_r(i) = \left(\frac{1}{1 + \omega\theta_z}\right)^2 \text{var}_i r^*_t. \tag{27}
\]
Meanwhile, the term $\int_0^1 z_t^2(i)di$ can be rewritten as follows:

$$
\int_0^1 z_t^2(i)di = var_zz_t(i) + \left[\int_0^1 z_t(i)di\right]^2
$$

$$
= var_zz_t(i) + y_t^2
$$

$$
= \theta_z^2 var_zp_t^2(i) + y_t^2
$$

$$
= \left(\frac{\theta_z}{1 + \omega \theta_z}\right)^2 var_zr_t^i + y_t^2, \quad (28)
$$

where the last line comes from (27).

Therefore, from equations (25)–(28), the disutility of labor leads to

$$
\int_0^1 v(L_t(i))di = \frac{v'\bar{L}}{2} \left\{(1 + \omega)(y_t^2 - 2a_t y_t) + 2y_t + 2v_t^y
$$

$$
+ \left(\frac{\theta_z}{1 + \omega \theta_z}\right) var_zr_t^i\right\} + t.i.p.
$$

Since $u'\tilde{C} = v'\bar{L}$ holds in the efficient steady state, the utility of the representative household can be expressed as

$$
U_t = u(C_t; \xi_t) - \int_0^1 v(L_t(i))di
$$

$$
= \frac{v'\bar{L}}{2} \left\{(1 - \sigma)y_t^2 + \frac{2u'_c}{u'_c} \xi_t y_t - (1 + \omega)(y_t^2 - 2a_t y_t)
$$

$$
- 2v_t^y - \left(\frac{\theta_z}{1 + \omega \theta_z}\right) var_zr_t^i\right\} + t.i.p.
$$

$$
= \frac{v'\bar{L}}{2} \left\{(\sigma + \omega) \left[y_t^2 - 2 \left(\frac{1 + \omega}{\sigma + \omega}\right) a_t y_t - 2 \left(\frac{u'_c/\xi_t}{\sigma + \omega}\right) \xi_t y_t\right]
$$

$$
+ 2v_t^y + \left(\frac{\theta_z}{1 + \omega \theta_z}\right) var_zr_t^i\right\} + t.i.p.
$$

Note that $v_t^y$ can be approximated as $(\theta/2) var_jp_t(j)$. In addition, the specification of the total utility function yields $u'_c/\xi_t = 1 - \sigma$ and $v'\bar{L} = \bar{L}^{1+\omega}$, which establishes equation (17).
References


