Is Moderate-to-High Inflation Inherently Unstable?*

Michael T. Kiley
Federal Reserve Board

The data across time and countries suggest the level and variance of inflation are highly correlated. This paper examines the effect of trend inflation on the ability of the monetary authority to ensure a determinate equilibrium and inflation stability in a sticky-price model. Trend inflation increases the importance of future marginal costs for current price setters in a staggered-price-setting model. The greater importance of expectations makes it more difficult for the monetary authority to ensure stability in two senses. First, equilibrium determinacy is more difficult to achieve through reasonable specifications of nominal-interest-rate (Taylor) rules at moderate-to-high levels of inflation (e.g., at levels around 4 percent per year). In addition, the volatility of inflation induced by cost-push shocks is, all else equal, higher at higher rates of inflation under a reasonable specification of the nominal-interest-rate rule. If monetary policymakers have followed these types of policy rules in the past, these results may explain, in part, why moderate-to-high inflation is associated with inflation volatility.

JEL Codes: E3, E5.

*The author would like to thank two anonymous referees and participants in seminars at the Federal Reserve Board and North American Summer Meetings of the Econometric Society in 2004 for their comments. The author would also like to thank Hasan Bakshi and Pablo Burriel-Lombart for discussions that led to this research, and Athanasios Orphanides, John Roberts, Mike Woodford, and especially John Driscoll for comments on previous drafts. The views expressed herein are those of the author and do not represent those of the Federal Reserve Board or its staff. Author contact: Mail Stop 76, Federal Reserve Board, Washington, DC 20551. E-mail: mkiley@frb.gov. Tel: 202-452-2448.

1. Introduction

The relationship between trend inflation and macroeconomic volatility is examined below in a standard New Keynesian sticky-price model.[1] Positive rates of trend inflation raise the importance of expected future marginal costs for current price setters in a staggered-price-setting model. The greater importance of expectations makes it more difficult for the monetary authority to ensure stability through simple policy rules. For example, much recent research has emphasized that monetary policymakers should raise real interest rates in response to an increase in inflation. In the context of a simple rule relating nominal interest rates to inflation (e.g., Taylor 1993), this requires that the coefficient on inflation exceed unity, a condition referred to as the Taylor principle. It is demonstrated below that the Taylor principle can be violated at fairly moderate rates of trend inflation. Equilibrium indeterminacy, and hence the possibility of sunspot fluctuations and increased macroeconomic instability, occurs for an increasingly large proportion of the range of policy settings when trend inflation rises to moderate levels (e.g., from 0 to 4 percent per year).[2]

This result may have implications for monetary policy practices and the interpretation of past events. With regard to practices, the results suggest that a focus on trend inflation in discussing policy rules is central, in that inflation stability requires that policymakers commit to low inflation and respond vigorously to inflation fluctuations around that trend level. With respect to past experience, Clarida, Galí, and Gertler (2000) have suggested that insufficient responsiveness of nominal interest rates to expected inflation in the 1970s—i.e., violations of the Taylor principle that allowed

[1] Most recent work on sticky-price models abstracts from positive trend inflation rates (e.g., Woodford 2003). Two exceptions are Bakhshi et al. (2002) and Ascari (2004). These authors illustrate how the equilibrium in a Calvo staggered-price-setting model (Calvo 1983) may not exist for high values of trend inflation, because the infinite sum on which the currently chosen nominal price is based may not converge for high rates of inflation.

[2] It should be emphasized that throughout this analysis the equilibria are always locally determinate and stable, or locally indeterminate. The term "instability" herein will often refer to indeterminacy and hence the possibility that sunspot shocks increase macroeconomic volatility. This is the same notion as in Clarida, Galí, and Gertler (2000).

real interest rates to fall with an increase in expected inflation—contributed to macroeconomic volatility. During that period, inflation in many countries was at least moderate. The results herein suggest that the level of inflation may have contributed to volatility, because moderate-to-high inflation is more difficult to stabilize (in two senses formalized below) when prices are rigid. This conclusion is also consistent with a substantial body of evidence showing a very strong correlation between the level and variance of inflation across countries and time (e.g., Kiley 2000)—if policymakers in that sample could be interpreted as following Taylor-type rules, as suggested in earlier work for Germany, Japan, and the United States (e.g., Clarida, Galí, and Gertler 1998, 2000).

The analysis herein will focus exclusively on the possible effects of trend inflation on economic volatility, and does not consider any other costs or benefits associated with moderate inflation. However, the finding of potentially pernicious effects of moderate-to-high inflation on economic volatility adds further weight to research suggesting that low inflation is desirable due to steady-state distortions to relative prices, interactions between trend inflation and nominal tax systems, and the classical costs associated with the area under the money-demand curve stemming from positive nominal interest rates.[3]

The next section briefly discusses the association between the level and variance of inflation. Section 3 presents a simple model illustrating the effect of trend inflation on price-setting behavior in a New Keynesian staggered-price-setting model. The model's implications for macroeconomic volatility are examined in section 4. The final section discusses necessary further work.

2. Inflation and Its Variance

There is a long history documenting a positive relationship between the level and variance of inflation. Okun (1971) and Taylor (1981) are classic examples. Both authors demonstrate through international

[3]The effects of trend inflation on various aspects of economic performance in New Keynesian models like that considered herein has attracted increased interest recently (e.g., Ascari 2004, Ascari and Ropele 2004, Blake and Fernandez-Corugedo 2006, and Cogley and Sbordone 2006).

Table 1. Basic Statistics for Consumer Price Inflation in the G-7

	1974–1985		1986–2000	
	Average	**Std. Deviation**	**Average**	**Std. Deviation**
Canada	8.5	2.3	2.7	1.4
France	10.7	2.5	2.3	1.1
Germany	4.3	1.7	1.9	1.3
Italy	16.0	3.6	4.9	1.7
Japan	6.7	5.5	0.7	1.2
United Kingdom	12.1	5.7	3.9	2.1
United States	6.9	2.5	2.8	1.1

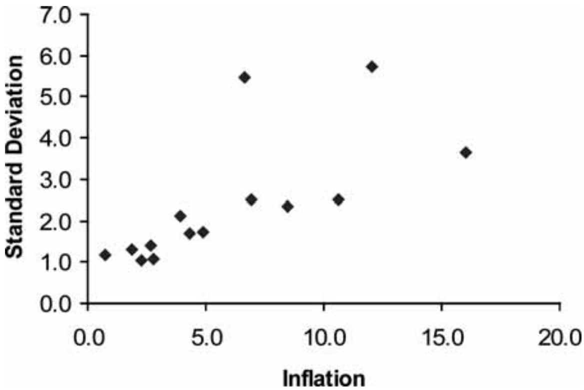
Note: Inflation is measured as the percent change in the annual average of the personal consumption deflator.

cross-sectional comparisons that high inflation is volatile inflation. Kiley (2000) recently reports a similar correlation across forty-three countries.

For our purposes, it is important to emphasize that this finding is not driven by inclusion of very high inflation economies; rather, it is true of the G-7 economies over the past thirty years, as well as during the moderate inflation conditions of the last fifteen years.[4] For example, table 1 presents average inflation rates, as measured by the annual percent change in the personal consumption deflator from the national accounts, and the standard deviation of inflation for the seven economies that make up the G-7 over two periods, 1974–85 and 1986–2000. The break between time periods was chosen to roughly correspond to the period after disinflation from the higher levels of the 1970s was completed in most countries and the period of increased macroeconomic stability identified in McConnell and Perez-Quiros (2000) and OECD (2002).

[4]We focus on low-to-high inflation, where high is defined as 10 percent per year. The positive association between very high inflation and its variance will not be considered, as very high rates of inflation make a New Keynesian sticky-price model implausible.

Figure 1. Scatter Plot of the Volatility of Inflation against Its Level in the G-7



In every country, the period of low inflation was also a period of more-stable inflation. This relationship is also apparent across countries: figure 1 presents the scatter plot of the fourteen country/time-period pairs for average inflation and its standard deviation. The simple correlation between the level and standard deviation of inflation is 0.7 in the fourteen country/time-period observations. This correlation is not driven by the high-inflation period—it remains 0.7 in the seven country-level observations over 1986–2000.

Finally, it is important to emphasize that the positive association between the level and variance of inflation is distinct from the positive association between inflation and the dispersion of relative prices at a point in time—which is also quite strong, as documented in, for example, Fischer (1981) and Stockton (1988). These earlier studies have demonstrated that relative price dispersion and the level of inflation are positively correlated largely because relative price shocks—particularly to food and energy—increase price dispersion and inflation in the short run. This literature has also found some positive effect of trend inflation on relative price dispersion; in other words, the causality runs in both directions, albeit more strongly from relative price shocks to dispersion and inflation than vice versa.

In contrast, the positive association between the level of inflation and its variance is not dominated by relative price shocks. Two factors suggest this interpretation: (i) the cross-country experience partially controls for energy price shocks (e.g., common global movements in oil prices and Okun's [1971] findings, which predate the 1970s' supply shocks), and (ii) the correlation between the level and variance is strong between 1986 and 2000, a period of greater stability in oil prices. Two hypotheses were offered in the earlier literature for the relationship between the level of inflation and its variance. Milton Friedman's Nobel lecture (1977) suggests that high inflation causes inefficient, and hence more variable, macroeconomic policies—perhaps reflecting a diminution of political consensus; much of Friedman's conjecture concerns inflation at the high end of the range we consider, and does not address the correlation between the rate of inflation and its variance at the moderate levels emphasized herein. Taylor (1981) suggests that accommodative monetary policies may lead to high inflation and greater variability in response to supply shocks. Our analysis will echo Taylor in emphasizing policy. But it will differ in demonstrating that moderate-to-high levels of inflation will potentially increase macroeconomic instability even when policymakers are not accommodative, because moderate-to-high inflation may both amplify the effects of intrinsic shocks and open up the possibility of self-fulfilling inflation fluctuations.

3. A Model

The model is a standard New Keynesian description of the macroeconomy, similar to the baseline case in Clarida, Galí, and Gertler (1999). The aggregate-demand side of the model—the IS curve and the monetary-policy reaction function—is not affected by trend inflation and hence its derivation from microeconomic behavior will not be discussed.

Focusing first on aggregate supply, the economy consists of a large number of (symmetric) monopolistically competitive firms producing intermediate goods that are aggregated with a constant-elasticity-of-substitution aggregation function to produce the final consumption good. The demand function facing firm j in period t is given by

$$D_{j,t}\left(\frac{X_{j,t}}{P_t}\right) = \left(\frac{X_{j,t}}{P_t}\right)^{-\theta} Y_t, \quad \theta > 1,$$

where $X_{j,t}$ is the price charged by the firm, P_t is the aggregate price level (defined below), and Y_t is aggregate demand. For simplicity, aggregate demand and consumption are used interchangeably, and the effects of investment by firms or consumer durable purchases are ignored (following, e.g., Woodford 2003).

Firms set nominal prices for two periods. We will fix this period of price rigidity across all the levels of inflation considered below. It is reasonable to suppose that the degree of price rigidity would vary with trend inflation, as demonstrated empirically by Kiley (2000). But the observed variation across countries and time has been for large differences in trend inflation, and we will consider more-modest variations in trend inflation. Future work may wish to reexamine the role of *endogenous* selection of price rigidity at different levels of trend inflation, along the lines pursued in Kiley (2000), for example. However, previous research suggests that the results herein will not be affected to a significant extent. Fischer (1981) notes that menu cost models will imply that the degree of price stickiness will decrease with the average inflation rate, but that this decrease will generally be partial in the sense that relative price dispersion increases with trend inflation; in the model below, the increase in relative price dispersion with trend inflation is the important mechanism driving our results. Recent research also suggests that, within calibrated dynamic general equilibrium models, the degree of price rigidity is not likely to be very sensitive to the average level of inflation for the differences across the G-7 since the 1970s (e.g., Klenow and Kryvtsov 2003).

In our model, there are two classes of firms, each with mass equal to one-half the total. The firms differ in that they alternate the period in which they adjust their price; i.e., price setting is staggered as in Taylor (1980). This sticky-price assumption preserves tractability and avoids some of the problems associated with the popular Calvo model.[5] The firm's profit

[5]A staggered-price-setting model with prices rigid for four periods, rather than two periods, was also examined. The results were very similar. However, the four-period model resulted in even more-complicated relationships between trend inflation and the coefficients in the aggregate supply relations of the model; as a result, the discussion focuses solely on the two-period model, where some intuition and very limited analytical results are more easily obtained. On the Calvo model, Kiley (2002a, 2002b) discusses a number of issues illustrating how this model fails to provide a good approximation to staggered-price-setting models with finite maximum lags.

maximization problem involves choosing the nominal price X_t that maximizes

$$\Lambda_t \left[\left(\frac{X_t}{P_t} \right)^{1-\theta} Y_t - \Gamma \left(\left(\frac{X_t}{P_t} \right)^{-\theta} Y_t \right) \right] + E \left\{ \Lambda_{t+1} \left[\left(\frac{X_t}{P_{t+1}} \right)^{1-\theta} Y_{t+1} - \Gamma \left(\left(\frac{X_t}{P_{t+1}} \right)^{-\theta} Y_{t+1} \right) \right] \middle| t \right\},$$

where $E\{.|t\}$ is the expectations operator conditional on period t information, Λ_t is the marginal utility of consumption for the firm's owners in period t (and hence the appropriate discount rate), and $\Gamma(.)$ is the firm's cost function.

Manipulating the first-order condition yields an expression for the optimal price

$$\frac{X_t}{P_t} = \frac{\theta}{\theta - 1} \frac{\Lambda_t MC_t \left(\frac{X_t}{P_t} \right)^{-\theta} Y_t + E \left\{ \Lambda_{t+1} MC_{t+1} \left(\frac{X_t}{P_{t+1}} \right)^{-\theta} Y_{t+1} | t \right\}}{\Lambda_t \left(\frac{X_t}{P_t} \right)^{-\theta} Y_t + E \left\{ \Lambda_{t+1} \Pi_{t+1}^{-1} \left(\frac{X_t}{P_{t+1}} \right)^{-\theta} Y_{t+1} | t \right\}}, \tag{1}$$

where Π_{t+1} is inflation (P_{t+1}/P_t). The real price equals a constant markup over the weighted average of marginal cost (MC) during the period for which the price is fixed, where the weights incorporate the effects of discounting and trend inflation.

The aggregate price level P_t is given by the standard equation

$$P_t = \left[\frac{1}{2} X_t^{1-\theta} + \frac{1}{2} X_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{2}$$

In order to complete the specification of the firm's problem, expressions for marginal cost and the discount rate are necessary. Assuming that households' preferences are separable in consumption and leisure, insurance markets are complete, and preferences are of the constant-relative-risk-aversion (CRRA) form with risk aversion equal to σ implies that the discount rate for period $t + j$ (Λ_{t+j}) is $\beta^{j-t} Y_{t+j}^{-\sigma}$. If the disutility from labor supply takes the typical power function form, household j's decision regarding its hours supply $(H_{j,t})$ is governed by the intratemporal optimality condition

$$\frac{W_{j,t}}{P_t} = H_{j,t}^{\phi} Y_t^{\sigma},$$

where $W_{j,t}/P_t$ is the real wage and $1/\phi$ is the labor-supply elasticity. Note that our assumption regarding wages has assumed that household j is attached to firm j (i.e., labor markets are sector specific); this is done to introduce a degree of "real rigidity" into the model, is equivalent to a yeoman-farmer specification (in its reduced-form implications, e.g., Woodford 2003), and has no effect on our qualitative results. Finally, the production function of firm j is given by

$$Y_{j,t} = H_{j,t}^{a},$$

implying that total costs for firm j in period t are

$$\Gamma(Y_{j,t}) = \frac{W_{j,t}}{P_t} H_{j,t} = \frac{W_{j,t}}{P_t} Y_{j,t}^{\frac{1}{a}}.$$

Substituting the demand curve in this expression, differentiating, and then substituting the real wage equation above yields marginal cost for a firm charging nominal price X_t in period k:

$$MC_{t+k} = \frac{1}{a}\left(\frac{X_t}{P_{t+k}}\right)^{-\omega\theta} Y_{t+k}^{\omega+\sigma}, \quad \omega = \frac{\phi}{a} + \frac{1}{a} - 1, \quad k = 0, 1. \quad (3)$$

The parameter ω represents the elasticity of marginal cost with respect to the firm's own output.

Inserting equation (3) into equation (1) along with the expression for the discount factor yields the solution for the optimal price chosen by a firm in period t:

$$\frac{X_t}{P_t} = \left[\frac{\theta}{\theta-1}\frac{1}{a}\frac{Y_t^{1+\omega} + E\{\beta\Pi_{t+1}^{\theta+\omega\theta}Y_{t+1}^{1+\omega}|t\}}{Y_t^{1-\sigma} + E\{\beta\Pi_{t+1}^{\theta-1}Y_{t+1}^{1-\sigma}|t\}}\right]^{\frac{1}{1+\omega\theta}}. \quad (4)$$

Log-linearizing equations (4) and (2) around the steady-state values of relative prices, output, and inflation ($P_{t+1}/P_t = \Pi$) yields (with lowercase letters denoting log-deviations from steady-state levels)

$$x_t - p_t = d_1 y_t + E\{d_2 y_{t+1} + d_3 \pi_{t+1}|t\},$$
$$d_1 = \frac{1}{1+\omega\theta}\left[\frac{1+\omega}{1+\beta\Pi^{\theta+\omega\theta}} - \frac{1-\sigma}{1+\beta\Pi^{\theta-1}}\right],$$
$$d_2 = \frac{1}{1+\omega\theta}\left[\frac{(1+\omega)\beta\Pi^{\theta+\omega\theta}}{1+\beta\Pi^{\theta+\omega\theta}} - \frac{(1-\sigma)\beta\Pi^{\theta-1}}{1+\beta\Pi^{\theta-1}}\right],$$
$$d_3 = \frac{1}{1+\omega\theta}\left[\frac{(\theta+\omega\theta)\beta\Pi^{\theta+\omega\theta}}{1+\beta\Pi^{\theta+\omega\theta}} - \frac{(\theta-1)\beta\Pi^{\theta-1}}{1+\beta\Pi^{\theta-1}}\right].$$
$$\pi_t = \Pi^{1-\theta}[x_t - p_t] + x_{t-1} - p_{t-1}. \tag{5}$$

These expressions look cumbersome but should be familiar for the case where trend inflation equals 0 ($\Pi = 1$) and there is no discounting ($\beta = 1$); in that case, these equations simplify to

$$x_t - p_t = \frac{1}{2}\frac{\omega+\sigma}{1+\omega\theta}[y_t + E\{y_{t+1}|t\}] + \frac{1}{2}E\{\pi_{t+1}|t\}$$
$$\pi_t = x_t - p_t + x_{t-1} - p_{t-1},$$

which are equivalent to the staggered-price-setting specification in Taylor (1980), Chari, Kehoe, and McGrattan (2000), and Kiley (2002a, 2002b). However, in the present case, trend inflation raises the importance of future output and inflation in decisions regarding the current price (i.e., d_2 and d_3 are increasing in trend inflation, as noted by Ascari 2000). This occurs because firms realize that demand for their product will be higher in the future, after inflation has eroded the real value of their nominal prices; hence, firms place a larger weight on future developments in setting current prices when inflation is higher. Note that these responses are related to the increase in relative price dispersion that accompanies higher inflation in the staggered-price-setting model, consistent with earlier empirical work and the emphasis in recent research (e.g., Woodford 2003) on the importance of this channel in staggered-price-setting models for aggregate welfare.

For the analysis of equilibrium determinacy, the set of stochastic disturbances affecting the economy can be ignored. But the analysis of volatility will require some set of exogenous disturbances. Both for simplicity and in line with earlier work (Clarida, Galí, and Gertler 1999), a cost-push shock is appended to the log-linearized equation for relative prices, yielding

$$x_t - p_t = d_1 y_t + E\{d_2 y_{t+1} + d_3 \pi_{t+1}|t\} + u_t, \tag{6}$$

where u is an i.i.d. disturbance term.

The remainder of the model follows the New Keynesian literature. The IS equation links the deviation of current output from its steady-state or potential value (y) to the real-interest-rate deviation (the nominal rate i minus future inflation) and future output:

$$y_t = E\{y_{t+1}|t\} - \frac{1}{\sigma}[i_t - E\{\pi_{t+1}|t\}]. \tag{7}$$

Microfoundations for this equation can be found in the consumption Euler equation (equation (7) is a log-linearized version of such an equation). As noted above, output replaces consumption for simplicity (reflecting its dominance in aggregate demand). A more thorough discussion can be found in Woodford (2003).

The aggregate-demand side of the model is closed with a specification of monetary policy, which follows a forward-looking (with respect to inflation) Taylor rule:

$$i(t) = \gamma_\pi E\{\pi_{t+1}|t\} + \gamma_y y_t. \tag{8}$$

This specification is relatively standard. A substantial body of earlier work (e.g., Clarida, Galí, and Gertler 2000; Bullard and Mitra 2002; and Woodford 2003) has demonstrated that equilibrium determinacy, and hence macroeconomic stability, can be achieved in this framework if the real interest rate increases with inflation ($\gamma_\pi > 1$). This property has been labeled the Taylor principle, following the influential work of Taylor (1993, 1999).[6] Two considerations drive our emphasis on forward-looking behavior. First, Clarida, Galí, and

[6]The condition for a determinate equilibrium depends on the response coefficients involving both inflation and output. Bullard and Mitra (2002) describe the Taylor principle in terms of the set of values for both response coefficients. The discussion herein focuses on the less formal notion emphasized by John Taylor in his contributions—i.e., that real interest rates should rise with inflation. We will return to the formal conditions ensuring determinacy in the next section.

Gertler (1998, 2000) and Orphanides (2002) both argue that monetary policy behavior in Germany, Japan, and the United States has been well described by this type of behavior. In addition, central banks that have adopted inflation targeting have placed increased focus on inflation expectations and have characterized their behavior in this regard as consistent with an equation like (8) (for example, the Reserve Bank of New Zealand models its own policy in this way [see page 39 of Black et al. 1997]; also see the summary of twenty countries' experience with inflation targeting in Schmidt-Hebbel and Tapias 2002). Of course, decisions in actual practice invariably include factors not included in the model. Section 4 will discuss the sensitivity of the results to alternative specifications of the monetary policy rule.

The model can be compactly expressed as a second-order expectational difference equation:

$$AE\{z_{t+1}|t\} + Bz_t + Cz_{t-1} = Du_t, \tag{9}$$

where z_t is a 4×1 vector containing the relative price set at t, inflation, output, and the nominal interest rate. A, B, C, and D are matrices containing structural coefficients. Equation (9) has a unique rational-expectations solution in which fluctuations are driven solely by the cost-push shock (u) when the number of roots of the matrix polynomial on the left-hand side that lie inside the unit circle equals the number of predetermined endogenous variables (one in this case, reflecting the lagged relative price in the inflation equation). When more than one of these roots lie inside the unit circle, rational-expectations solutions in which sunspots—nonfundamental shocks—can drive fluctuations are also possible; this multiplicity is termed indeterminacy herein.[7] Such indeterminacy is undesirable, as nonfundamental shocks could increase the variability of the economy—a notion pursued, for example, in Clarida, Galí, and Gertler (2000).

[7] Farmer (1993) provides a good introduction to indeterminacy and the possibility of sunspot equilibria in rational-expectations models of the sort discussed in this research.

4. Results on Equilibrium Determinacy and Volatility

4.1 Indeterminacy

We focus first on determinacy of equilibrium and return later to volatility under policy settings consistent with a unique equilibrium. Our results are derived from an extensive set of numerical exercises, in which the model is solved using the AIM algorithm originally developed by Anderson and Moore (for a recent presentation, see Anderson 2000).[8] We first assign a baseline set of parameter values to the model. Since the sticky-price model assumes that nominal prices are fixed for two periods, a period is assumed to correspond to one-half year. Table 2 presents values for most of the parameters. The discount factor (β) is set at 0.96 per year, implying a real interest rate of approximately 4 percent. The coefficient of relative risk aversion (σ) is set at $^1/_4$; while this value is quite low, Woodford (2003) justifies a low value by noting that the inverse of this parameter governs the interest sensitivity of aggregate demand and that this sensitivity is substantially higher than the intertemporal elasticity of substitution of consumption once investment in business capital and consumer durables—both absent from the model—are considered. The elasticity of output with respect to labor input (a) is set at $^2/_3$, approximately the value in U.S. data. The baseline setting for the markup of prices over marginal cost ($1/(\theta - 1)$) is 10 percent (Woodford 2003 typically uses a similar value, and other values of this parameter are discussed below). The labor-supply elasticity ($1/\phi$) equals 1 in the baseline calibration. This value for the labor-supply elasticity lies above traditional estimates (MaCurdy 1981, Altonji 1986, and Abowd and Card 1989) but below the common assumption in dynamic general equilibrium models that labor supply

[8] We have introduced a very simple model, and one might imagine that analytical results could be obtained. For example, it is straightforward to collapse the system of equations reported as equation (9) to a system that can be expressed as $AE\{z_{t+1}|t\} = z_t + Bu_t$, where z is a 2×1 vector containing relative prices $(x - p)$ and output (y). In this case, determinacy depends solely upon the eigenvalues of A, and relatively simple conditions on the trace and determinant of A can be checked to ensure determinacy. Unfortunately, the elements of A are extremely complicated functions of the structural parameters in the case where trend inflation is nonzero, making analytic results too difficult to obtain (at least for this author).

Table 2. Baseline Parameter Values

Parameter	Description	Value
β	Discount Factor	0.96
σ	Coefficient of Relative Risk Aversion	$^1/_4$
a	Elasticity of Y with Respect to H	$^2/_3$
$1/(\theta - 1)$	Markup (at Zero Inflation)	.10
$1/\phi$	Labor-Supply Elasticity	1

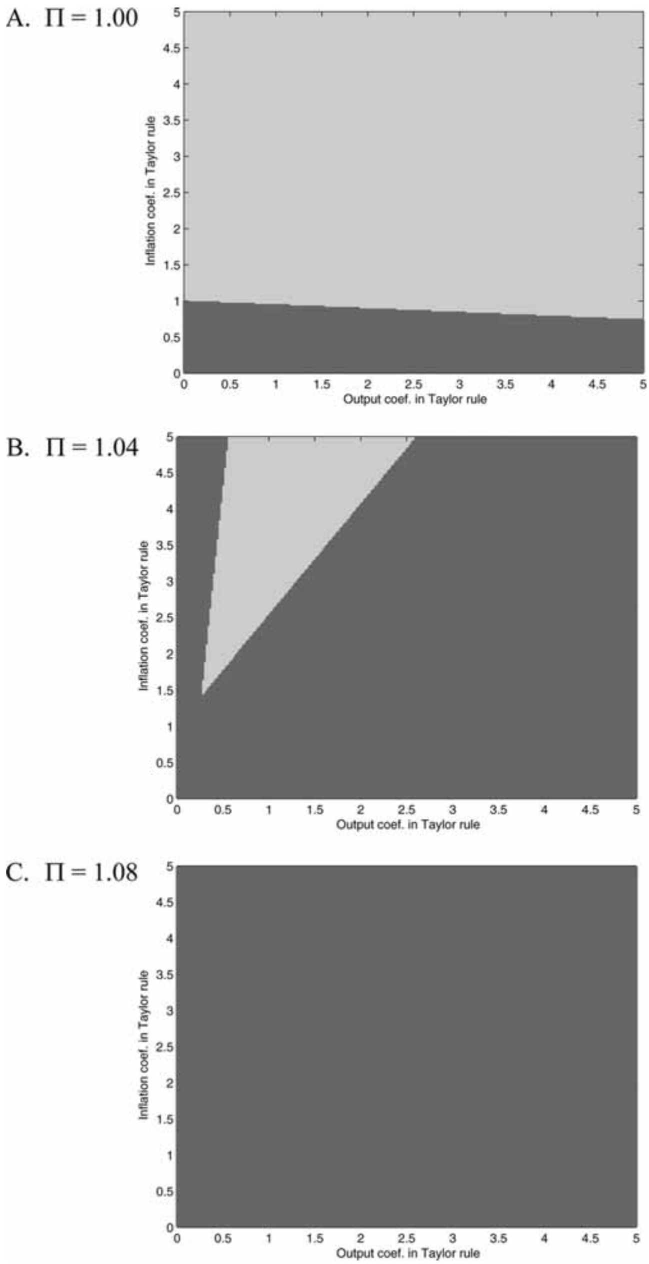
Note: The discount factor is expressed at an annual rate; as there are two periods in a year, β equals $\sqrt{}0.96$.

is perfectly elastic (an assumption associated with indivisible labor supply). This baseline value is near the recent estimates of Mulligan (1998), and alternatives are considered below.

The remaining parameter values are the trend inflation rate (Π) and the coefficients in the Taylor rule. Our experiments consider the possibility of indeterminacy for different values of these parameters. For each set of parameters, the indeterminacy of equilibrium is examined numerically, and the results for a range of inflation rates and policy settings are summarized in figure 2. The three panels of figure 2 correspond to trend inflation rates of 0 percent, 4 percent, and 8 percent, respectively. (The value for inflation refers to the steady-state percent change in the price level *at an annual rate.*) Each panel presents whether indeterminacy arises for coefficients in the Taylor rule ranging from 0 to 5 for both output and the one-period-ahead inflation forecast; indeterminacy is indicated by the dark-shaded region.

The results for a trend inflation rate of 0, shown in panel A, are closely related to the findings in the previous literature. In particular, indeterminacy, and hence the possibility of sunspot fluctuations, arises with trend inflation equal to 0 when nominal interest rates are insufficiently responsive to expected inflation. It is possible to derive an analytical characterization of the region in which a determinate equilibrium exists for the case of trend inflation equal to 0 by following the steps in Bullard and Mitra (2002) or Woodford (2003). A determinate equilibrium occurs when the coefficients in the nominal-interest-rate rule (assuming both coefficients are positive) satisfy the inequality $\gamma_\pi + \frac{1}{2}\frac{1-\beta}{1+\beta}\frac{1+\omega\theta}{\omega+\sigma}\gamma_y > 1$.

Figure 2. Equilibrium Determinacy for Different Trend Inflation Rates (Π)



This condition implies the loose version of the Taylor principle emphasized earlier—that nominal interest rates must respond to expected inflation with a coefficient exceeding unity—in the absence of discounting or when the coefficient on output in the rule equals 0. The restrictions on coefficients are very similar to those that arise in the Calvo pricing model.[9]

Panels B and C illustrate how this condition changes with trend inflation. For trend inflation of 4 percent, indeterminacy is possible for a much wider range of the parameter space: to ensure determinacy, the output response (γ_y) must be slightly positive (but not too large) and the inflation response (γ_π) must be greater than 1 (and by a significant amount for larger values of the output response). When trend inflation is 8 percent, no set of responses considered in the rule yields determinacy.

A comparison of the various panels of figure 2 also reveals that determinacy depends on both coefficients in the nominal-interest-rate rule in a complicated manner. For zero trend inflation (panel A), determinacy is ensured if the nominal interest rate is sufficiently responsive to expected inflation, and the degree of responsiveness required is slightly decreasing in the responsiveness of policy to the output gap. (This is consistent with the analytic expression presented earlier and is only a slightly different region than under the Calvo pricing model.) For positive trend inflation, the determinacy region shrinks from both sides (panel B), so that nominal interest rates can be neither too unresponsive nor too responsive to expected inflation for a given coefficient on the output gap.

4.2 Sensitivity to Important Structural Parameters

Unfortunately, the structural coefficients in the system represented by equation (9) are much more complex in the case of positive trend inflation, and analytic results could not be found. As mentioned earlier, higher trend inflation increases the role of expectations in the dynamic system. This appears to be the source that raises the

[9] As shown elsewhere (e.g., Woodford 2003), the coefficients in the interest rate rule with expected inflation as in equation (8) must satisfy another restriction to ensure determinacy (and this additional condition involves the response coefficients not being "too large"). This type of additional restriction does not arise in the staggered-price-setting model examined herein.

possibility of sunspot fluctuations. Other parameters influence how trend inflation impacts the coefficients on expectations and hence are important as well. (As noted above, other parameters are basically not important for determinacy when trend inflation is 0, as determinacy is ensured so long as $\gamma_\pi + \frac{1}{2}\frac{1-\beta}{1+\beta}\frac{1+\omega\theta}{\omega+\sigma}\gamma_y > 1$, which is always satisfied for γ_π greater than 1 and γ_y greater than 0.)

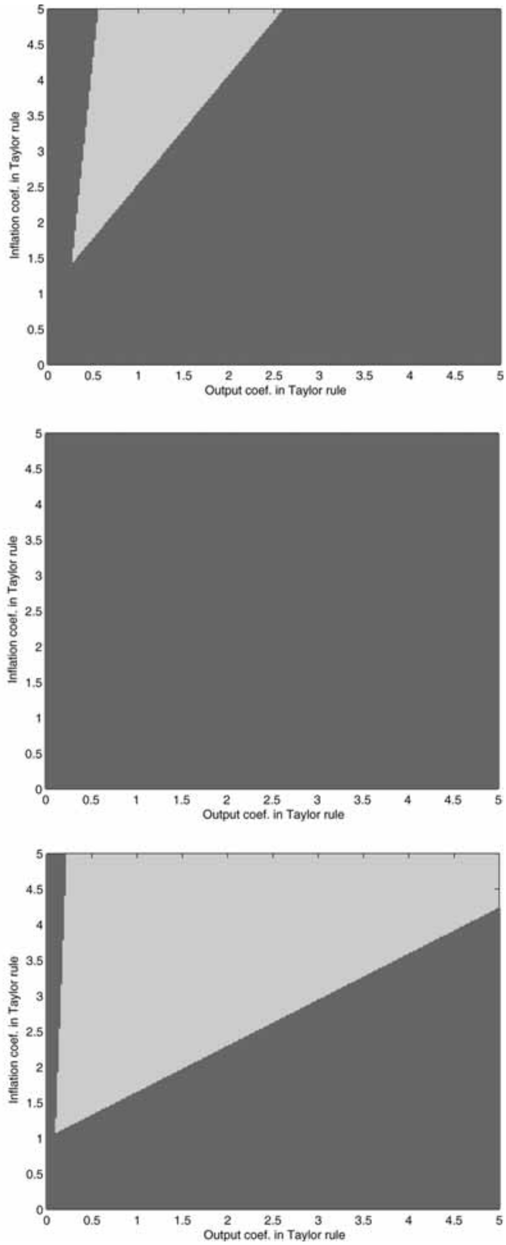
Two important parameters are the labor-supply elasticity and the elasticity of demand for a firm's product. Firms know that demand will be higher when inflation erodes their real price—i.e., in the future. The strength of this effect is driven by the demand elasticity; and low labor-supply elasticities imply that marginal cost is more sensitive to high demand, which implies that future profits are eroded to a greater extent by price rigidity with high inflation and lower labor-supply elasticities. Therefore, the demand elasticity and labor-supply elasticity play important roles in determining the importance of inflation for forward-looking behavior.

Figure 3 presents the indeterminacy regions for different values of the labor-supply elasticity, holding all other parameters at their baseline values and fixing trend inflation at 4 percent ($\Pi = 1.04$). Panel A reproduces the baseline results. Panel B shows that a low labor-supply elasticity (1/4, similar to estimates cited above) makes indeterminacy likely, while a high labor-supply elasticity (infinity, panel C) makes indeterminacy less of a problem for policymakers. Again, higher weight on expectations (in this case, through a lower labor-supply elasticity) makes indeterminacy a larger potential problem.

Figure 4 presents the results on indeterminacy for the baseline demand elasticity and a lower elasticity (consistent with a markup of 20 percent), again with trend inflation of 4 percent. A lower elasticity (higher markup) lowers the importance of future demand in price setting, and this shrinks the region of policy settings over which indeterminacy is a concern.

In summary, sunspot fluctuations are a possible concern when trend inflation is moderate, at least under the type of forward-looking Taylor rule that has been discussed as a reasonable characterization of behavior for some central banks. Even values of trend inflation of 4 percent per year substantially shrink the range of policy settings that deliver equilibrium determinacy. At trend inflation of 8 percent, no policy settings within the forward-looking Taylor-rule

Figure 3. Equilibrium Determinacy for Different Labor-Supply Elasticities ($1/\phi$)

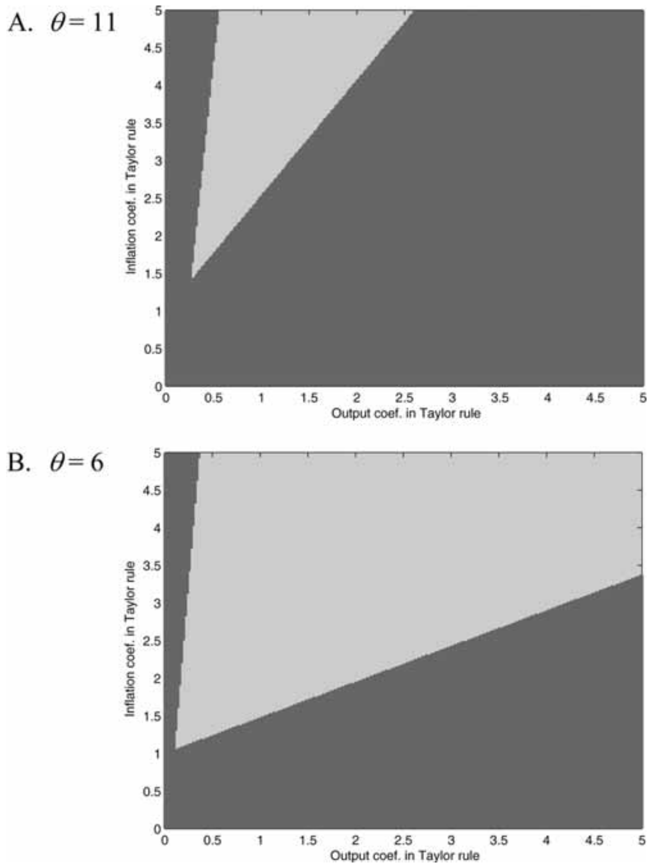


A. $1/\phi = 1.00$

B. $1/\phi = 1/4$

C. $1/\phi = \infty$

Figure 4. Equilibrium Determinacy for Different Demand Elasticities (θ)

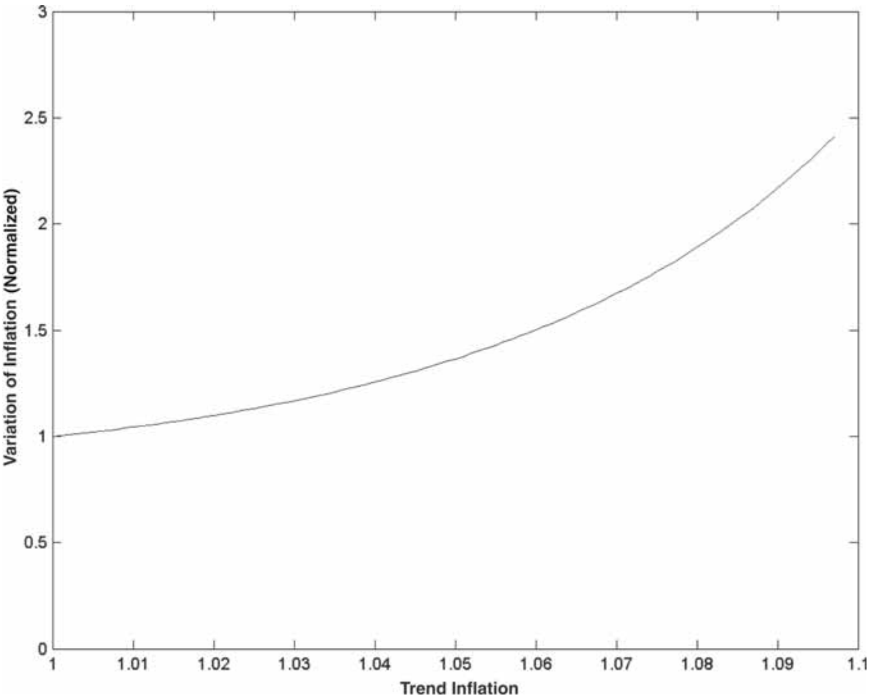


framework can ensure determinacy for the range of parameter values considered. Alternative policy rules are discussed in subsection 4.4 below.

4.3 Volatility in the Determinate Region

The previous section demonstrated that the increasing importance of forward-looking behavior with trend inflation in sticky-price models raises the potential problem of indeterminacy, which could increase

Figure 5. The Variance of Inflation from Cost-Push Shocks at Different Values of Trend Inflation



macroeconomic volatility through the possibility of sunspot fluctuations. Even within the parameter space consistent with a determinate equilibrium, greater forward-looking behavior could contribute to increased volatility in response to fundamental shocks.

The implications of trend inflation for the volatility of inflation in our model are examined numerically using a markup of 20 percent, an infinite labor-supply elasticity, and traditional values for the coefficients in the Taylor rule ($\gamma_\pi = 1.5, \gamma_y = 0.5$). Other parameters equal their values in table 2. (The high markup and high labor-supply elasticity were chosen to allow a substantial range for inflation within which the equilibrium is determinate; the qualitative results do not depend on the specific values chosen for these parameters.) Figure 5 presents the variances of inflation for different trend inflation rates, normalized to equal 1 at a 0 percent inflation rate.

The results show that inflation volatility rises with trend inflation when the monetary policy settings are held fixed. (Note that the variances are presented up to trend inflation somewhat below 10 percent, as equilibrium determinacy fails prior to that level of inflation.) This suggests that moderate trend inflation may contribute to macroeconomic instability both through its effect on the transmission of fundamental shocks and through the possibility of sunspot-induced volatility. (As a side note, output volatility increases as well in this case.)

Some intuition for the source of increased volatility can be seen from the form of the relative price equation and inflation equations (5). It is straightforward to show that the lags in equation system (9) can be eliminated by substituting the inflation equations into the relative price and nominal-interest-rate equations and recasting the system solely in terms of output, relative prices, the nominal interest, and the cost-push shock (e.g., a system like that in footnote 8, with no lags). With i.i.d. disturbances and no lags in the system, the coefficients on contemporaneous values for the variables are the sole determinants of equilibrium behavior (assuming a determinate equilibrium). The relative price equation after substitution of the inflation equation and ignoring date $t+1$ variables is given by

$$x_t - p_t = \left(\frac{1}{1-d_3}\right)(d_1 y_t + u_t). \tag{10}$$

As the coefficient d_3 is increasing in inflation, the variability in relative prices induced by cost-push shocks is increasing in trend inflation. This directly implies more-volatile inflation at higher trend inflation rates from equation (5). (Of course, the behavior of output is important as well, but the simple intuition holds.)

4.4 Robustness to Alternative Policy Rules

The analysis has held fixed the policy rule. An alternative to a forward-looking Taylor rule is the Taylor rule with contemporaneous inflation:

$$i(t) = \gamma_\pi \pi_t + \gamma_y y_t.$$

Figure 6. Equilibrium Determinacy under a Contemporaneous Taylor Rule

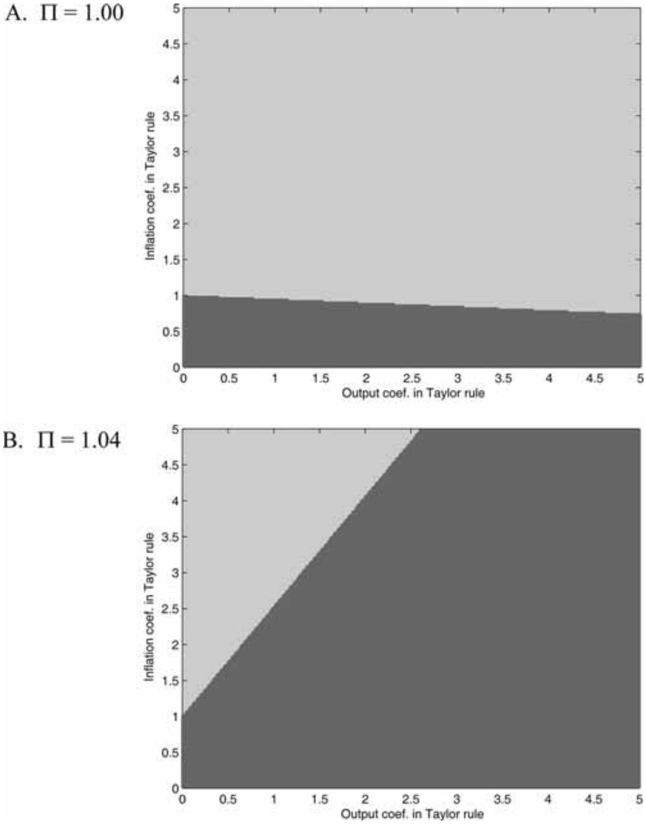


Figure 6 presents the indeterminacy regions for trend inflation rates of 0 and 4 percent at the baseline parameter values in table 2 using this alternative Taylor rule. While the indeterminacy region is slightly different, the picture is much the same as that with the forward-looking rule: trend inflation substantially increases the parameter space over which indeterminacy and sunspot fluctuations are a possible concern.

This examination of a slightly different form of the Taylor rule is only a tiny fraction of the very general set of rules that could be considered. A growing body of research has emphasized how different assumptions regarding policy rules can affect equilibrium determinacy.[10] An analysis of optimal rules under discretion and commitment by Ascari and Ropele (2004) and Blake and Fernandez-Corugedo (2006) has been performed for a Calvo sticky-price model since the initial drafts of this paper circulated. These authors illustrate that optimal rules are subject to the same effects as documented herein for the Taylor rule. While subsequent research may discover policy rules that perform well in the presence of trend inflation, the results herein remain important given the central role that Taylor-type rules have played in central bank practice according to a number of researchers (e.g., Clarida, Galí, and Gertler 1998, 2000 and Orphanides 2002).

4.5 Empirical Relevance

The analysis has illustrated that the impact of trend inflation on determinacy and volatility may be apparent at quite moderate levels of inflation, suggesting that equilibrium indeterminacy and the instability possible from sunspot fluctuations may be a serious concern for moderate trend inflation. This effect may be relevant for some historical episodes, in light of the different average rates of inflation witnessed in the G-7 since the 1970s.

It is clear from table 1 (in section 2) that average inflation rates in the earlier period were well within the range that can lead to equilibrium indeterminacy or affect macroeconomic volatility from fundamental shocks. This may suggest that the conclusion of Clarida, Galí, and Gertler (2000)—that the failure of policymakers to increase real interest rates in response to increases in expected inflation was a source of aggregate instability in the 1970s—should be reinterpreted. In fact, moderate-to-high inflation can contribute to instability under the Taylor-type rules currently in vogue in two ways—by potentially allowing for sunspot fluctuations and by boosting the volatility of inflation induced by fundamental shocks. Hence, it may have been the combination of the high level of inflation

[10]For example, Benhabib, Schmitt-Grohé, and Uribe (2002) and Carlstrom and Fuerst (2000) present alternative models in which forward-looking policymaking can lead to indeterminate equilibria; the latter authors emphasize the desirability of backward-looking policy, while the former authors highlight an important role for nominal-interest-rate inertia—i.e., a role for lagged interest rates in the policy rule.

and the policy actions attempting to stabilize fluctuations around that high level that contributed to macroeconomic volatility in the 1970s.[11]

We have focused on instability under monetary rules that have been suggested as summaries of the behavior of most inflation-targeting central banks (Black et al. 1997 and Schmidt-Hebbel and Tapias 2002). A substantial number of such inflation targeters, particularly in developing countries, pursue targets that are moderate to high. Lower target inflation rates may contribute to macroeconomic stability.

5. Conclusion

Evidence across time and countries suggests that moderate-to-high inflation tends to be less stable, but it has not been clear from previous work whether this is an intrinsic feature of such regimes. The analysis herein suggests that moderate-to-high inflation may contribute to instability of inflation in two ways. First, moderate-to-high inflation makes equilibrium indeterminacy more likely in a standard New Keynesian model with staggered price setting, raising the possibility of volatility induced by sunspot fluctuations. In addition, the variance of inflation induced by cost-push shocks is higher at higher trend inflation rates (even when the equilibrium is determinate) in the same model. These findings may provide some of the explanation for the evidence across time and countries on trend inflation and inflation volatility.

[11] This result may also be relevant in light of the work of Orphanides (2002), which finds that the Federal Reserve may have been following a forward-looking interest rate rule in the 1970s that satisfied the Taylor principle, but relied too heavily on output-gap estimates that suggested the economy was operating substantially below potential and hence pursued a policy that was excessively loose. To the extent such actions can be characterized as a medium-term shift in the inflation target (a reasonable description, as a persistently large and negative output-gap estimate inserted into a Taylor rule can be expressed algebraically as a rule with the correct output-gap estimate and a new, higher inflation target by simple substitution into equation (8)), Orphanides' conclusions, in conjunction with the result herein that moderate-to-high inflation targets can generate instability, are consistent with the notion that monetary policy contributed to macroeconomic volatility in the 1970s, as argued for different reasons by Clarida, Galí, and Gertler (2000).

Relatedly, these findings may suggest a reinterpretation of the 1970s. Clarida, Galí, and Gertler (2000) and Orphanides (2002) have agreed that monetary policy in the United States was well characterized by a Taylor-type rule at that time, but disagree with respect to whether the Taylor principle was followed and therefore whether volatility was increased by sunspot fluctuations. It has been shown herein that high rates of inflation contribute to the possibility of sunspot fluctuations and, within the model examined, amplify the effect of cost-push shocks on macroeconomic volatility. Given this, the vigorous responses of nominal interest rates to expected inflation called for by Clarida, Galí, and Gertler (2000) may not have been sufficient to deliver significantly increased stability in the 1970s until a commitment to low inflation had been put in place. This result is also relevant for central banks in countries with moderate-to-high inflation that have recently adopted (or are considering for the future) inflation targeting: stability under such regimes is only possible with low trend inflation rates in the New Keynesian model.

The analysis herein was kept as simple as possible to convey the main ideas. Extensions to more fully specified general equilibrium models may be useful, especially consideration of different assumptions regarding labor and product markets on the sensitivity of firms' price-setting behavior to future conditions. It is also important to note that the analysis herein assumed that firms' prices were rigid for two periods. Some recent work has assumed that firms index their nominal prices to inflation (e.g., Christiano, Eichenbaum, and Evans 2005). The main results of this paper do not generalize to that case, as the effect of inflation on relative prices is key to firms' increased sensitivity to future conditions. We do not view such indexation as a plausible characterization of price setting in the developed countries considered for several reasons (the first two were offered by Ascari and Ropele 2004). Survey evidence on price rigidity points to fixed nominal prices without indexation. Gray (1976) demonstrated that full indexation is unlikely to be optimal. And empirical findings such as those in Christiano, Eichenbaum, and Evans (2005)—which suggest a role for indexation across long samples of U.S. history—should be interpreted carefully, as their econometric work clearly mixes data from at least two different periods of trend inflation, as suggested above, while their theoretical model is a linearized approximation of a model around a single rate of trend inflation. The role of indexation found in some past empirical work may have reflected failure to account for a time-varying rate of trend inflation, as suggested in Cogley and Sbordone (2006). Kiley (2007) finds much greater support for a model without indexation in recent decades.

Finally, there is no evidence that the range of inflation experienced in the United States over the postwar period has been associated with differences in price rigidity sufficient to affect the analysis herein, indicating that the mechanisms identified in this study may be operative in similar economies. But the data suggest that nominal price rigidities are less important at much higher rates of inflation (e.g., Kiley 2000).

References

Abowd, J. M., and D. Card. 1989. "On the Covariance Structure of Earnings and Hours Changes." *Econometrica* 57 (2): 411–45.

Altonji, J. G. 1986. "Intertemporal Substitution in Labor Supply: Evidence from Micro Data." *Journal of Political Economy* 94 (3, part 2): S176–S215.

Anderson, G. 2000. "The Anderson-Moore Algorithm." Occasional Staff Study No. 4, Board of Governors of the Federal Reserve System. Available at www.federalreserve.gov/pubs/oss/oss4/aimindex.html.

Ascari, G. 2000. "Optimising Agents, Staggered Wages and Persistence in the Real Effects of Money Shocks." *Economic Journal* 110 (465): 664–86.

———. 2004. "Staggered Prices and Trend Inflation: Some Nuisances." *Review of Economic Dynamics* 7 (3): 642–67.

Ascari, G., and T. Ropele. 2004. "The Optimal Stability-Oriented Monetary Policy: Optimal Monetary Policy Under Low Trend Inflation." Mimeo.

Bakhshi, H., P. Burriel-Llombart, H. Khan, and B. Rudolf. 2002. "Endogenous Price Stickiness, Trend Inflation, and the New Keynesian Phillips Curve." Mimeo.

Benhabib, J., S. Schmitt-Grohé, and M. Uribe. 2002. "Designing Monetary Policy: Backward-Looking Interest-Rate Rules and Interest Rate Smoothing." Mimeo, Rutgers University (October).

Black, R., V. Cassino, A. Drew, E. Hansen, B. Hunt, D. Rose, and A. Scott. 1997. "The Forecasting and Policy System: The Core Model." Research Paper No. 43, Reserve Bank of New Zealand (August).

Blake, A., and E. Fernandez-Corugedo. 2006. "Optimal Monetary Policy with Non-Zero Steady-State Inflation." Mimeo, Bank of England.

Bullard, J., and K. Mitra. 2002. "Learning About Monetary Policy Rules." *Journal of Monetary Economics* 49 (6): 1105–29.

Calvo, G. A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12 (3): 383–98.

Carlstrom, C. T., and T. J. Fuerst. 2000. "Forward-Looking Versus Backward-Looking Taylor Rules." Working Paper No. 0009, Federal Reserve Bank of Cleveland.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan. 2000. "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?" *Econometrica* 68 (5): 1151–79.

Christiano, L. J., M. Eichenbaum, and C. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113 (1): 1–45.

Clarida, R., J. Galí, and M. Gertler. 1998. "Monetary Policy Rules in Practice: Some International Evidence." *European Economic Review* 42 (6): 1033–1106.

———. 1999. "The Science of Monetary Policy." *Journal of Economic Literature* 37 (4): 1661–1707.

———. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics* 115 (1): 147–80.

Cogley, T., and A. M. Sbordone. 2006. "Trend Inflation and Inflation Persistence in the New Keynesian Phillips Curve." Mimeo, University of California, Davis (July).

Farmer, R. 1993. *The Macroeconomics of Self-Fulfilling Prophecies.* Cambridge, MA: MIT Press.

Fischer, S. 1981. "Relative Shocks, Relative Price Variability, and Inflation." *Brookings Papers on Economic Activity* 2:381–441.

Friedman, M. 1977. "Nobel Lecture: Inflation and Unemployment." *Journal of Political Economy* 85 (3): 451–72.

Gray, J. A. 1976. "Wage Indexation: A Macroeconomic Approach." *Journal of Monetary Economics* 2 (2): 221–35.

Kiley, M. T. 2000. "Price Stickiness and Business Cycle Persistence." *Journal of Money, Credit, and Banking* 32 (1): 28–53.

———. 2002a. "Partial Adjustment and Staggered Price Setting." *Journal of Money, Credit, and Banking* 34 (2): 283–98.

———. 2002b. "The Lead of Output over Inflation in Sticky Price Models." *Economics Bulletin* 5 (5): 1–7.

———. 2007. "A Quantitative Comparison of Sticky-Price and Sticky-Information Models of Price Setting." *Journal of Money, Credit, and Banking* 39 (1): 101–25.

Klenow, P. J., and O. Kryvtsov. 2003. "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?" Mimeo, Federal Reserve Bank of Minneapolis.

MaCurdy, T. E. 1981. "An Empirical Model of Labor Supply in a Life-Cycle Setting." *Journal of Political Economy* 89 (6): 1059–85.

McConnell, M. M., and G. Perez-Quiros. 2000. "Output Fluctuations in the United States: What Has Changed Since the Early 1980s?" *American Economic Review* 90 (5): 1464–76.

Mulligan, C. B. 1998. "Substitution over Time: Another Look at Life Cycle Labor Supply." In *NBER Macroeconomics Annual*, Vol. 13, ed. B. S. Bernanke and J. J. Rotemberg. Cambridge, MA: MIT Press.

Okun, A. 1971. "The Mirage of Steady Inflation." *Brookings Papers on Economic Activity* 2:485–98.

Organisation for Economic Cooperation and Development (OECD). 2002. *Economic Outlook,* Vol. 71.

Orphanides, A. 2002. "Monetary Policy Rules and the Great Inflation." Finance and Economics Series Discussion Paper No. 2002-8, Board of Governors of the Federal Reserve System.

Schmidt-Hebbel, K., and M. Tapias. 2002. "Monetary Policy Implementation and Results in Twenty Inflation Targeting Countries." Working Paper No. 166, Central Bank of Chile.

Stockton, D. J. 1988. "Relative Price Dispersion, Aggregate Price Movement, and the Natural Rate of Unemployment." *Economic Inquiry* 26 (1): 1–22.

Taylor, J. B. 1980. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88 (1): 1–23.

———. 1981. "On the Relationship Between the Variability of Inflation and the Average Inflation Rate." *Carnegie-Rochester Conference Series on Public Policy* 15:57–85.

———. 1993. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* 39 (December): 195–214.

———, ed. 1999. *Monetary Policy Rules.* NBER Conference Report Series. Chicago and London: University of Chicago Press.

Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton, NJ: Princeton University Press.