

# Manipulation in Money Markets\*

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Interest rate derivatives are among the most actively traded financial instruments in the main currency areas. With values of positions reacting immediately to the underlying index of daily interbank rates, manipulation has become an increasing challenge for the operational implementation of monetary policy. To address this issue, we study a microstructure model in which a commercial bank may have strategic recourse to central bank standing facilities. We characterize an equilibrium in which market rates will be manipulated with strictly positive probability. Our findings have an immediate bearing on recent developments in the sterling and euro money markets.

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## 1. Introduction

In the recent past, many central banks have increasingly focused on steering some short-term money market interest rates in their implementation of the monetary policy stance. For example, this is the case of the Federal Reserve in the United States, the European Central Bank (ECB) in the euro area, and the Bank of England in the United Kingdom. More broadly, central banks seem to increasingly attach greater value to stable day-to-day and even intraday money market conditions. With this aim, so-called corridor systems have been adopted in several currency areas—for example, in Australia, Canada, the euro area, and New Zealand. More recently, the Bank of England has also adopted such a system (see Bank of England 2005).<sup>1</sup>

This paper wishes to contribute to the ongoing discussion on the appropriate design of corridor systems<sup>2</sup> by showing that manipulation is a potential issue in such money markets.<sup>3</sup> Specifically, a

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<sup>1</sup>In a corridor system (see, e.g., Woodford 2003), the central bank stands ready to provide overnight liquidity in unlimited amounts, generally against collateral, at a rate somewhat above market rates, and stands ready to absorb liquidity overnight in unlimited amounts at a rate somewhat below market rates. By setting a corridor around the central bank target or policy rate, the range of variation of overnight interest rates will be bounded, on a day-to-day basis, by the rates on the standing lending and deposit facilities, allowing short-term market interest rates to be steered with limited volatility around the desired level. The Federal Reserve has a semicorridor system following the introduction of its primary credit facility, 1 percentage point above the federal funds target, with zero being the standard lower bound.

<sup>2</sup>Furfine (2003) shows with a search model that the actual recourse of a lending facility may be less than suggested by the statistics of individual refinancing costs when the market attaches a stigma to its use, but also that the availability of a lending facility might reduce incentives for active participation in the interbank market. Pérez Quirós and Rodríguez Mendizábal (2006) conclude that the introduction of a deposit facility may lead to a stabilization of market rates. This is because the deposit facility reduces the costs of running into a “lock-in” situation, in which reserve requirements are satisfied before the last day of the reserve maintenance period.

<sup>3</sup>Manipulation in financial markets has attracted significant academic attention during the last two decades. Besides the contributions cited below, see for

commercial bank might hold a position that would gain from, say, a rise in policy rates, so that it would look like an attractive perspective if market rates would increase somewhat. To create temporarily higher rates, this bank may take up loans from the interbank market and deposit the funds with the central bank. Under certain conditions, this will cause a rise in the market rate, adding value to the manipulator's net position.<sup>4</sup> We will discuss under which conditions this and similar strategies are profitable, and which incentive effects are created by this possibility. We will also discuss some of the means at the disposal of the central bank to eliminate this kind of behavior.

In the euro area, variations of this type of manipulative strategy may have occurred on at least two occasions since the start of stage 3 of the Economic and Monetary Union in January 1999.

**Manipulation Episode at the End of the Maintenance Period May 24–June 23, 2000.**<sup>5</sup> In this maintenance period, the ECB raised key policy rates from 3.75 percent to 4.25 percent (cf. figure 1). Ahead of the decision, market participants were speculating on the timing of the policy change and also on the likely scale (25 basis points versus 50 basis points). Indeed, second-quarter turnover in interest rate swaps more than doubled in 2000. Already on the first day of the maintenance period, the money market index EONIA was at 4.06 percent, reflecting market expectations. On Monday, June 19, 2000, there was a EUR 4.999 billion (bn) total recourse to the deposit facility. This recourse occurred before the last main refinancing operation, and consequently did not affect market rates. However, at the close of trading on the next day, the allotment day of the last main refinancing operation in the maintenance period, there was another EUR 11.207 bn total recourse to the deposit facility. This recourse changed the liquidity conditions ahead of a crucial part of the maintenance period; thus the EONIA increased immediately. Made distinctly before the end of the reserve maintenance period, the recourse was indeed quite unusual. It is not

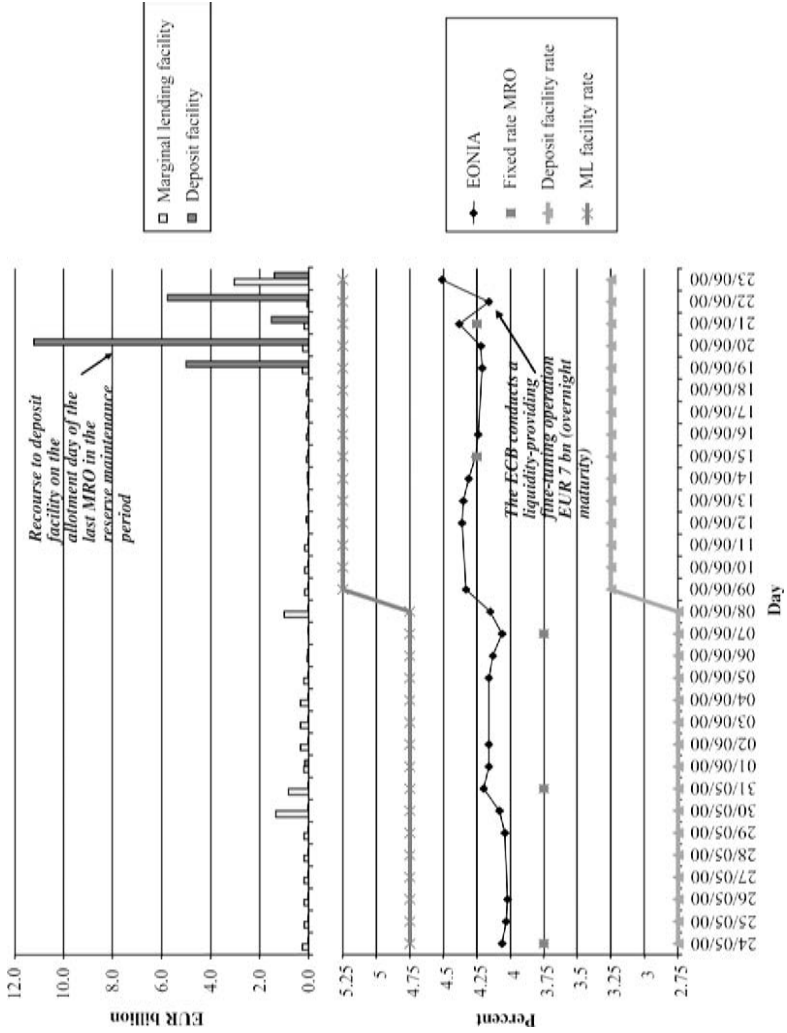
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instance Allen and Gale (1992), Allen and Gorton (1992), Bagnoli and Lipman (1996), Benabou and Laroque (1992), Gerard and Nanda (1993), and Vila (1989).

<sup>4</sup>As a side note, a similar strategy was attempted by screenplay adversary Auric Goldfinger of Ian Fleming's James Bond 007, when breaking into Fort Knox to "destroy" massive gold reserves.

<sup>5</sup>For explanations of technical terms in the context of the implementation framework of the Eurosystem, we refer the reader to section 2.

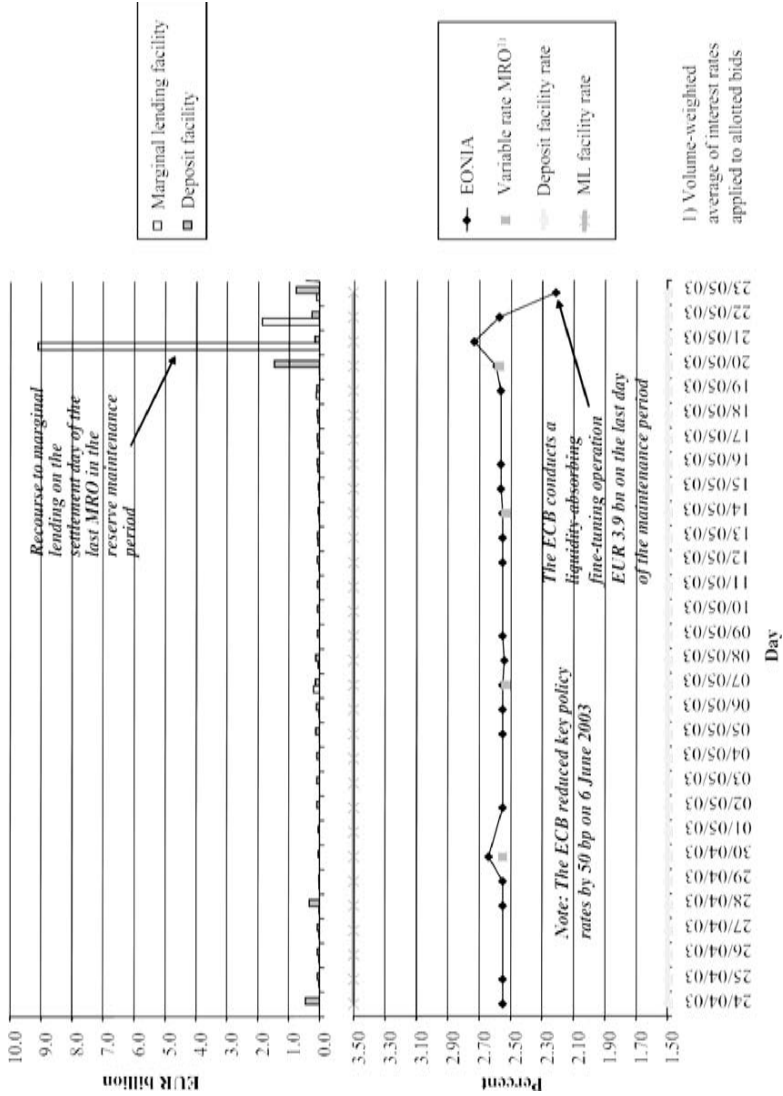
Figure 1. Daily Recourses to Standing Facilities in the Eurosystem, EONIA, and Key Policy Rates (May 24–June 23, 2000)



unlikely that an individual commercial bank had strong incentives to attempt manipulation. The Eurosystem launched a fine-tuning operation and provided EUR 7.000 bn at overnight maturity on Wednesday, June 21, achieving a temporary relief to market conditions. However, on the penultimate day, there was another large composite recourse to the deposit facility of a total size comparable to the fine-tuning operation. The Eurosystem did not perform an additional fine-tuning on the last day of the maintenance period. Overall, the maintenance period ended tight, with a net recourse to the marginal lending facility and EONIA 26 basis points above the new policy rate.

**Manipulation Episode at the End of the Maintenance Period April 24–May 23, 2003.** At the start of this maintenance period, the EONIA index was at 2.55 percent, and the minimum bid rate at the main refinancing operation was 2.50 percent (cf. figure 2). During the maintenance period, expectations were formed about a policy rate cut by the ECB in the subsequent period. The ECB indeed lowered key policy rates by 50 basis points on June 6, 2003. There was unusually strong activity in the swap market during May 2003. Then, on the allotment day of the last main refinancing operation in the maintenance period—Tuesday, May 20—there were several (nonstrategic) recourses to the deposit facility adding to a total of EUR 1.462 bn. Even though this is not a large total recourse, it seems that it had an impact on liquidity conditions because the EONIA was 23 basis points above the minimum bid rate on the next day. The movement of the market may have triggered the response that followed. On the next day, there was an active request by an individual market participant for lending from the Eurosystem of EUR 9.0 bn. This recourse apparently changed or even reversed liquidity conditions after the last main refinancing operation in the maintenance period. Thus the EONIA decreased immediately. On the following Thursday, there was another recourse to the lending facility of EUR 1.8 bn by the same market participant. Again, the timing of the recourses was unusual. Apparently, there had been an attempt to imitate the manipulation strategy employed in 2000, now in the context of expectations of decreasing policy rates. The ECB launched a liquidity-absorbing fine-tuning operation on Friday, the last day of the maintenance period, drawing EUR 3.850 bn from the market. Overall, the maintenance period ended slightly loose, with

**Figure 2. Daily Recourses to Standing Facilities in the Eurosystem, EONIA, and Key Policy Rates (April 24–May 23, 2003)**



a net recourse to the deposit facility and EONIA 29 basis points below the policy rate.

For a central bank, manipulation is undesirable mainly for two reasons. First, from an operational perspective, manipulation has the potential to add volatility to the overnight rate and to complicate the liquidity management of both commercial banks and the central bank. Second, manipulation may affect the market's confidence in a smooth implementation of monetary policy, which may have an impact on longer-term refinancing conditions and therefore on the effectiveness of monetary policy.

To address this issue, we consider a model in which a strategic trader with private information may trade in a swap market first and may then manipulate the market rate. The potential manipulator faces a trade-off between the costs of taking control of the market rates and the additional value for her derivatives position. It turns out that the trade-off will sometimes, but not always, induce the trader to leverage her derivatives position and to subsequently manipulate the money market. With several informed traders, a public-good problem reduces individual incentives for manipulation, yet does not eliminate the problem. We then discuss policy measures such as fine-tuning and the narrowing of the corridor set by the standing facilities. The discussion covers both elements of the new operational design by the Bank of England and recent experiences of the Eurosystem.

Technically, our analysis follows the microstructure literature on informed trade that is associated with the seminal work of Kyle (1985). In this literature, an individual trader with private information may cause a price effect because the market extracts the information contained in the aggregate order flow. In an important contribution to this literature, Kumar and Seppi (1992) have studied manipulation in futures markets with cash settlement. The present paper adapts the assumptions underlying the Kumar and Seppi model to reflect the institutional backdrop of a corridor system. The main adaptation concerns the way in which prices are moved in the market for the underlying. In Kumar and Seppi (1992), there is asymmetric information in the spot market. Because market participants may mistake uninformed trading for informed trading, the order flow created by the uninformed manipulator moves the price of the underlying asset. In contrast, there is no informed spot

trading in our model. Indeed, while we do not deny the existence of private information in the money market, the evidence discussed above suggests that private information was not necessarily the central element in its mechanics. In our model, it is assumed that the manipulator gains temporary control of the market rate by using the standing facilities.<sup>6</sup>

The difference in the way in which prices are moved in the market for the underlying asset causes a qualitative change in the predictions of the analysis. In our model, a recourse to one of the standing facilities incurs an immediate loss in net interest earnings equivalent to the difference between the corridor rate and the current market rate. To cause the market rate to change marginally, this spread of typically more than 100 basis points has to be paid on the absolute amount of the recourse, which may be unprofitable. Indeed, the strategy does not pay off unless the involved positions exceed a certain size. This is why, in our setup, the probability and the extent of equilibrium manipulation depends on the design of the corridor system, which enables us to discuss alternatives for policymakers within our formal framework.<sup>7</sup>

The rest of the paper is structured as follows. Section 2 provides some background on the euro money market. This section can be skipped by those that are already acquainted with the institutional details. Section 3 sets up the basic model. In section 4, we study the decision problem of the informed trader. Section 5 analyzes the strategic game between manipulator and market makers. In section 6, we consider welfare consequences, policy options, and the extension to several manipulators. Section 7 concludes. All proofs can be found in the appendix.<sup>8</sup>

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<sup>6</sup>In the past, market rates have also reacted to insufficient demand in central bank operations. It is not clear, however, that these so-called underbidding episodes have been deliberate attempts of manipulation (cf. Ewerhart 2002 and Nyborg, Bindseil, and Strebulaev 2002).

<sup>7</sup>Another difference is that in Kumar and Seppi (1992), the manipulator's order in the futures market does not convey any information. In our model, the private information of the manipulator causes the swap rate to exhibit a reaction to the order flow and implies an endogenously finite market order.

<sup>8</sup>Note that strategic recourses, while similar in nature, differ from short squeezes (cf. Nyborg and Strebulaev 2001, 2004). In contrast to a short squeeze, a



## 2. Institutional Background on the Euro Money Market

When a customer of a commercial bank A requests a transfer of money into another party's account at another commercial bank B, then by purely mechanical consideration, bank A's holdings of central bank money will diminish, and bank B's holdings will increase. This and similar types of transactions may, when accumulating over the business day, re-allocate significant amounts of liquidity between individual credit institutions, which is a motive for them to trade secured and unsecured short-term credit in the *euro money market*.

A central bank that has chosen to implement monetary policy by steering short-term interest rates may do so by seeking control of aggregate liquidity conditions in the money market and by using additional instruments to stabilize interest rates further. This is the approach favored by many modern central banks (cf. Bindseil 2004 or Borio 1997). In the case of the Eurosystem, the control of liquidity conditions is attained by the combination of open market operations, standing facilities, and reserve requirements.

Through its *open market operations*, the ECB provides the necessary refinancing to the banking system. The bulk of interbank liquidity in the euro area is offered in the weekly main refinancing operations (MROs), which are open to all eligible counterparties of the Eurosystem. Until March 2004, the maturity of these operations was two weeks; it has been one week since then. As a rule, funds extended through main refinancing operations are allotted on Tuesdays, with settlement on the following Wednesday. Other operations include the monthly longer-term refinancing operations (LTROs), with a maturity of three months, and so-called fine-tuning operations (FTOs). The latter can be used in a very flexible way, yet at the cost of addressing only a subpopulation of all eligible counterparties.

The Eurosystem's *standing facilities*—i.e., the marginal lending facility and the deposit facility—constitute the interest rate corridor in the euro area. There is no administrative procedure. That

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strategic recourse does not presuppose a temporary monopoly situation. Strategic recourses are also not directly related to bidding behavior in central bank operations.

is, a recourse to either the marginal lending facility or the deposit facility can be requested by any eligible counterparty to the Eurosystem, where intraday debit positions on the counterparty's settlement account with the national central bank are automatically considered as a request for recourse to the lending facility. The use of the facilities occurs after the close of the market (at 6:30 p.m.). By 9:15 a.m. on the subsequent trading day, the market is informed through Reuters page ECB40 about the aggregate recourses to each of the two standing facilities. Significant recourses are typically observed only on the last one or two days of the maintenance period, when demand and supply in the money market become increasingly inelastic.

*Reserve requirements* for credit institutions are expressed in terms of an average balance to be held over a so-called reserve maintenance period (usually about a month) on the counterparty's settlement account. Noncompliance with minimum reserve obligations implies sanctions. In contrast to the United States, required reserves are remunerated in the euro area at a rate close to funding costs.

The combination of the above instruments makes market conditions in the euro money market usually a very stable signal of the current monetary stance. Nevertheless, both the average level and the volatility of market rates may vary over time. Especially after the last main refinancing operation in the maintenance period, market rates may differ visibly from the middle of the corridor. Deviations of the EONIA from the middle of the interest rate corridor occur in response to liquidity flows, so-called autonomous factors, which are beyond the direct control of the central bank's liquidity management and which affect the aggregate liquidity position of the banking system. These factors include treasury accounts with some national banks, banknotes that are paid out or collected at counters of commercial banks, and changes to consolidated net foreign assets held by the Eurosystem. Movements of the market rates occur also at certain calendar dates such as the end of the quarter and the end of the year, when commercial banks manage their balance sheets more carefully, and in connection with events that are perceived by the market to have a potential effect on financial stability. Further deviations of the market index from the middle of the corridor have been observed occasionally.

The *derivatives market* provides the means to either hedge the risks of a change in short-term interest rates or to speculate on them. Among the most actively traded instruments in this market is the overnight interest rate swap (OIS) of various maturities, ranging from one week to two years. For instance, an institutional investor might speculate on the timing of an expected increase in policy rates using a swap contract with a maturity of one month. In contrast, a commercial bank that wishes to freeze refinancing conditions in the interbank market until the next main refinancing operation may prefer a swap with a maturity of only one week. In terms of payments streams, the OIS is an instrument that exchanges a fixed interest rate against an index of daily interbank rates (almost always EONIA). The OIS differs from the plain vanilla interest rate swap (cf., e.g., Bicksler and Chen 1986), which is used for longer maturities and with reference to the Euribor. Also, for plain vanilla interest rate swaps, the floating rate is determined at one settlement date and paid at the next. In contrast, the floating-rate leg of an OIS is determined and paid at maturity. Overnight interest rate swaps have been known in the United States for quite some time as call money swaps.

For many market participants, it is much easier to realize a short-term interest rate position with swaps than with transactions in the deposit market (see, e.g., Pelham 2003 or Elliott 1997). The swap is the more liquid instrument and involves less credit risk. As a consequence, the swap curve has emerged as one of the main benchmark yield curves for the euro area. In June 2005, Euribor FBE (the European Banking Federation) and Euribor ACI (the Financial Markets Association) launched the EONIA Swap Index, which is published over Telerate for maturities ranging from one week to twelve months. There has been a continued strong expansion of the EONIA swap market over the last few years.

The OIS market is a highly competitive, high-volume OTC market, with dominant players featuring in the main European financial centers. The market organization is highly concentrated, with a handful of dealers accounting for about half of the trading activity. Among the most active dealers are commercial banks that are headquartered in the euro area. Dealers contract both with other dealers and with customers. The range of institutions participating in the OIS market as customers is very broad, originating from both the financial sector (credit institutions, insurance companies,

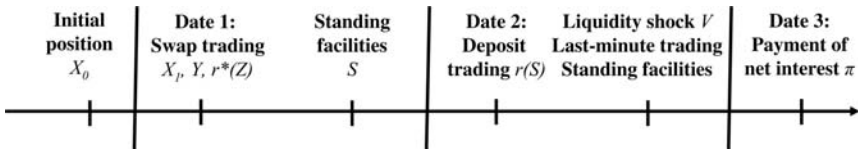
pension funds, hedge funds, money market funds, etc.) and the non-financial sector (European governments). Leveraged funds are especially active in this market. For further details on the euro money market and the overnight swap market, the reader is referred to descriptive studies by Remolona and Wooldridge (2003), Santillán, Bayle, and Thygesen (2000), Hartmann, Manna, and Manzaranes (2001), and to the ECB's annual euro money market study (2001–2006).

### 3. Formal Setup

Our market environment is an adapted version of Kumar and Seppi (1992), as discussed in the introduction. We envisage a developed money market with reserve requirements and averaging provision, embedded into a symmetric corridor system. Our analysis will focus on the last two days of the reserve maintenance period, when the regular (weekly) refinancing operations by the central bank have already established “neutral” conditions, and the market is essentially left on its own. Following the conventional terminology used in fixed-income and money markets, prices are replaced by interest rates. The setup is then as follows.

Three assets are traded in the money market: first, a riskless bond (“net interest”), which serves as a numeraire; second, a standardized overnight deposit contract (“liquidity”) with endogenous interest rate  $r$ ; and third, an overnight interest rate swap (“OIS”) on the deposit contract, traded at the swap rate  $r^*$ . Sign conventions for fixed-for-floating swaps tend to be ambiguous and depend on whether the hedging or the speculation motive is stressed. Throughout the present paper, we will adhere to the convention that the receive-floating party is long the swap, so that the position in her portfolio obtains a positive sign. This convention has the consequence that increasing market rates are desirable for the holder of a long position in the OIS, and conversely for a short position.

The sequence of events is summarized in figure 3. A swap market on date 1, organized in the late morning of the trading day, is followed by a spot market for the underlying deposit contract on date 2, organized at a similar time of the day. A liquidity shock hits the market shortly before the end of date 2. Following the shock, but still before the close of the market at the end of date 2, there

**Figure 3. Time Structure of the Model**

is last-minute trading in the deposit contract. All net interests on deposit and swap contracts are paid at date 3.

Altogether, five types of traders participate in these markets: first, a risk-neutral informed trader (a commercial bank attempting manipulation); then, risk-neutral discretionary traders in the deposit markets (nonspecialized commercial banks); third, nondiscretionary traders in the swap market that trade for exogenous reasons (non-financial firms); and finally, two groups of competitive risk-neutral market makers in the deposit market (money market specialists) and in the swap market (swap dealers). Commercial banks are subject to an individual minimum reserve requirement that must be fulfilled by the end of date 2. They also have access to the central bank's standing facilities at any date before date 3.<sup>9</sup>

The game between the traders has, then, the following structure. The informed trader obtains an initial endowment  $X_0$  in the swap before date 1. This initial position may be the result of OTC trading with nonbank customers, and is assumed to be private information.<sup>10</sup> At date 1, the informed trader submits a market order  $X_1$  to the swap dealers, where  $X_1 > 0$  when paying the fixed rate. The nonfinancial firms submit an independent order volume  $Y$  with mean  $E_Y[Y] = 0$ . The dealers observe the aggregate order

<sup>9</sup>Having commercial banks serving as market makers in the model would complicate the analysis without changing the conclusions. In a nutshell, the problem would be that an individual swap dealer, when receiving a too-large share of the total market order, may have an incentive to counteract the strategic recourse of the informed trader. This incentive is absent from the model if the swap dealers are not too few, or if there is not too much trading of swaps by nonfinancial firms at date 2.

<sup>10</sup>In practice, the initial position  $X_0$  may not necessarily be stochastic. A commercial bank could actively manage short-term interest rate positions vis-à-vis nonbank customers in a specific way, e.g., by asking customers to swap variable-interest income into fixed-interest income. That would make the initial position endogenous. However, a commercial bank may be able to do this once, but not several times, because it would openly steal profits from its customers.

flow  $Z = X_1 + Y$  in the swap market. Moreover, dealers will be understood to be informed about whether the informed trader submits a nonzero market order.<sup>11</sup> Formally, let  $b = 1$  if  $X_1 \neq 0$ , and  $b = 0$  otherwise, and assume that swap dealers observe  $b$ . The swap dealers are then willing to clear the swap market at the competitive rate  $r^*(Z, b)$ . Since only the case  $b = 1$  is interesting, we will henceforth drop the second argument and write simply  $r^*(Z)$  for  $r^*(Z, 1)$ .

With the close of the market at the end of date 1, the informed trader may have recourse  $S$  to the standing central bank facilities, where  $S > 0$  stands for a recourse to the lending facility, and  $S < 0$  for a recourse to the deposit facility. On date 2, the recourse  $S$  becomes public information. Commercial banks may then submit market orders for the deposit contract. As the order flow is not informative, the market specialists fix the interest rate on the deposit contract at date 2 to some value  $r(S)$  that depends only on  $S$ .

Shortly before the end of trading on date 2, a liquidity shock  $V$  may affect the aggregate liquidity position of the banking system, where  $V > 0$  stands for an absorption of liquidity. The dispersion of the shock over commercial banks does not matter under symmetric information and is therefore not explicitly modeled. The liquidity shock is assumed to be distributed independently from  $X_0$  and  $Y$ . Given that the central bank implements monetary policy in a neutral way, the median of the distribution of  $V$  will be zero. It is assumed that the distribution of  $V$  is given by a density  $\phi_V(\cdot)$  that is weakly increasing for  $S < 0$  and weakly decreasing for  $S > 0$ .

All commercial banks try to cover their positions at the end of date 2, so that the price in the last-minute trading equals either the central bank's lending rate  $r^L$  (when  $S - V < 0$ ) or the central bank's deposit rate  $r^D$  (when  $S - V > 0$ ), where  $0 < r^D < r^L$ . Under these conditions, the liquidity effect resulting from a recourse is determined exclusively by the change in the relative probabilities of a tight or a loose end of the maintenance period. Thus, the market for the overnight contract appears in the reduced form that has become standard in the literature since Poole (1968).

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<sup>11</sup>Non-anonymity significantly simplifies the analysis without affecting our main results. For a model without this assumption, the reader is referred to our working paper (Ewerhart et al. 2004).

POOLE'S LEMMA. *The market rate at date 2 for the deposit contract after a net recourse of  $S$  is given by a weakly decreasing function*

$$r(S) = \Phi_V(S)r^D + (1 - \Phi_V(S))r^L, \quad (1)$$

where  $\Phi_V(S) = \text{pr}\{V \leq S\}$  denotes the cumulative distribution function of the liquidity shock  $V$ . In particular, for  $S = 0$ , the market rate  $r(0)$  corresponds to the midpoint  $r^0 = (r^D + r^L)/2$  of the corridor.

Indeed, after the last main refinancing operation, money market rates in the euro area are generally expected to move in response to the release of public information about flows of liquidity that affect the aggregate liquidity position of the banking system, e.g., when a recourse to a standing facility of the central bank occurs.<sup>12</sup> As we will discuss now, it is this liquidity effect that opens the door for the profitable abuse of the credit and deposit facilities.

#### 4. Sporadic Manipulation

A market participant who intends to take temporary control of the market rate will be aware of the costs and benefits of such a strategy. There are costs because the use of standing facilities is bound to interest rate levels that almost always differ significantly from market conditions. There are benefits because short-term interest rate positions may gain in value. In this section, we analyze under which conditions a strategic recourse is profitable.

Formally, net interest income  $\pi$  for the informed trader is the sum of three components, as suggested by equation (2) below. First, there is the net return on the initial position  $X_0$  in the swap. The initial position is valued with the interest rate  $r(S)$  realized at date 2, while funding costs for this position are already sunk at date 1 and can be normalized to  $r^0$ . Next, there is the net return on the market order  $X_1$ . Here as well, the position is valued using the interest rate  $r(S)$  realized at date 2. Funding costs are given by the swap rate at date 1, which will be denoted by  $r^*$ . The third income component

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<sup>12</sup>The corresponding empirical evidence for the United States is mixed. See in particular Hamilton (1997), Thornton (2001), and Carpenter and Demiralp (2006).

is the net interest paid for the strategic recourse  $S$  to the standing facilities. This component is generally negative; e.g., a recourse to the credit facility costs  $r^L$  but yields only  $r(S) \leq r^L$ . Summing up, the informed trader obtains a net interest income

$$\begin{aligned} \pi(X_0, X_1, S) = & X_0(r(S) - r^0) + X_1(r(S) - r^*) \\ & + S(r(S) - r^{L/D}(S)), \end{aligned} \quad (2)$$

where we write

$$r^{L/D}(S) = \begin{cases} r^L & \text{if } S > 0 \\ r^0 & \text{if } S = 0 \\ r^D & \text{if } S < 0 \end{cases}$$

for the interest rate that the informed trader either pays for having recourse to the marginal lending facility or receives for depositing money with the central bank. The terms  $r^0$  and  $r^*$  correspond to the fixed leg of the swap positions  $X_0$  and  $X_1$ , respectively. The variable interest rate  $r(S)$  is received or paid on all three positions.

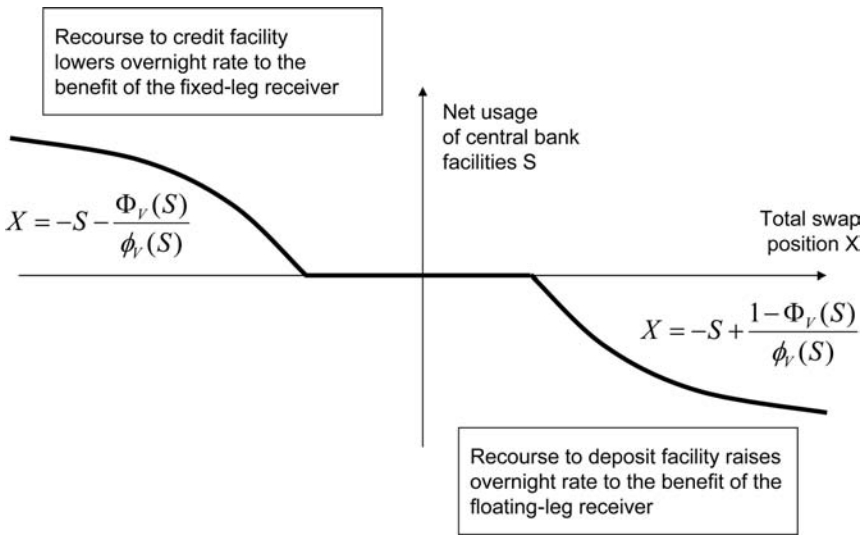
**Controlling the Market Rate.** The starting point for the analysis is to note that a strategic recourse to one of the standing facilities is not always optimal. Denote by  $X = X_0 + X_1$  the total swap position. For concreteness, assume a long position, i.e.,  $X > 0$ . The reader will note that it is never optimal in this situation to have recourse to the marginal lending facility. Indeed, to increase the value of the position, the market rate must go up and liquidity must become scarcer, so the informed trader will have recourse to the deposit facility ( $S < 0$ ). We differentiate the informed trader's objective function (2) with respect to  $S$ . Then the necessary first-order condition governing the informed trader's decision about  $S$  at date 1 becomes

$$-r'(S)(X_0 + X_1 + S) = r(S) - r^D. \quad (3)$$

As captured by the left-hand side of equation (3), the marginal benefit of manipulating the interest rate upward is the increase in the market value of the aggregate net position  $X + S$ . The marginal cost, on the other hand, is the interest rate differential between a deposit in the market and a deposit with the central bank. The resulting trade-off may be one-sided, however. Specifically, it turns out that



**Figure 4. Net Usage of Standing Facilities as a Function of the Informed Trader’s Swap Position**



if  $X$  is small enough in absolute terms, then the benefit will always be smaller than the cost, so that manipulation does not pay off. A similar consideration can be made for the case of a negative  $X$ . Thus, as depicted in figure 4, the optimal recourse to the standing facilities, drawn as a function of the informed trader’s position in short-term interest rate instruments, is zero for small absolute values of  $X$ .

**PROPOSITION 1.** *The optimal strategic use of standing facilities  $S^*(X)$  at the end of date 1 involves no recourse for  $|X| \leq \Delta/\rho$ , where  $\Delta = r^L - r^0$  is the half-width of the corridor, and  $\rho = |r'(0)|$  is the liquidity effect. Moreover, when  $|X| > \Delta/\rho$ , then  $S^*(X) \neq 0$ , and  $X$  and  $S^*(X)$  are of opposite sign.*

Thus, only a market participant with a sufficiently large exposure has an incentive to attempt manipulation. Such a trader may have built up a sufficiently large long position in the swap market ( $X > \Delta/\rho$ ) and will have recourse to the deposit facility ( $S < 0$ ) to cause prices to rise. In this case, it cannot be optimal to use

the lending facility, i.e., to manipulate the market rate downward, because the alternative choice  $S = 0$  avoids the nontrivial costs of the lending facility and does not lower the value of the swap position. In the other scenario, a trader with a sufficiently large short position ( $X < -\Delta/\rho$ ) will have recourse to the marginal lending facility ( $S > 0$ ) and will profit from the softening of the market.

Which size of position makes manipulation profitable? We have estimated elsewhere (see Ewerhart et al. 2003) that for the euro area under the system in use before March 2004,  $\rho \approx 0.09\%/bn$  EUR. The figure captures the response to a publicly observed, one-day liquidity shock of EUR 1 bn, which occurs immediately after the last main refinancing operation, and which is not corrected for by later fine-tuning. With this estimate, we can perform the following crude calibration. The corridor half-width being  $\Delta = 1\%$ , proposition 1 predicts that manipulation is the consequence of profit maximization for positions with a notional of at least

$$\frac{\Delta}{\rho} \approx \frac{1\%}{0.09\%/bn \text{ EUR}} = \text{EUR } 11.1 \text{ bn.}$$

The reader will note that this figure does not take into account expectations about potential central bank interventions after an attempted manipulation (these are difficult to quantify) and may therefore understate the actual threshold.

## 5. Trading in the Swap Market

Once a market participant considers manipulation as a profitable strategy, she will seek to improve the effectiveness of this strategy by leveraging her initial position in short-term interest-rate instruments. In the model, this possibility is reflected by the informed trader's endogenous choice of the swap market order  $X_1$ . As mentioned in section 2, an additional position taking could also be accomplished by satisfying reserve requirements unevenly over time. Which of these and possible other instruments is used by the individual treasurer is ultimately an empirical question. To keep the model tractable, we will focus in the sequel on the case of leverage using the swap market.

While position taking is essentially costless in the liquid OIS market, it cannot be accomplished at zero cost. In an equilibrium

with rational expectations, swap dealers will anticipate the possibility of manipulation and will extract the information contained in the order flow. For example, a large positive market order  $X_1 = X_1^*(X_0)$ , unless compensated by nondiscretionary trading, indicates to the dealers that the informed trader already has a relatively large initial long position  $X_0$ , making a recourse to the deposit facility more likely. Competition between swap dealers will force those dealers to set the swap rate close to market expectations about the deposit rate at date 2. More precisely, conditional on the total order volume  $Z = X_1^*(X_0) + Y$ , the swap rate will be set to

$$r^*(Z) = E_{X_0, Y}[r(S^*(X_0 + X_1)|Z)]. \quad (4)$$

Here, the swap rate  $r^*(Z)$  will typically be increasing in  $Z$ . Thus, as a consequence of the dealers' rational anticipation, creating significant leverage may be costly for the informed trader.

But when large trades can affect market expectations, it will be easier for the informed trader to leverage the existing position than to hedge it. Indeed, as our next result shows, neither hedging nor a change of the market side can be optimal in a symmetric market environment.

**PROPOSITION 2.** *Assume that  $X_0$ ,  $Y$ , and  $V$  are symmetrically distributed with mean zero, and that  $E_Y[r^*(X_1 + Y)]$  is increasing in  $X_1$ . Then the informed trader's optimal market order satisfies  $X_1^*(X_0) \geq 0$  for  $X_0 > 0$  and  $X_1^*(X_0) \leq 0$  for  $X_0 < 0$ .*

While the informed trader will never reduce her initial exposure in a symmetric market environment, we will see below that under general conditions, and in intuitive extension of proposition 1, she may choose to let the position unchanged. More precisely, if the initial position  $X_0$  is relatively small in absolute value, then the informed trader does not participate in the swap market and does not manipulate the deposit rate. The intuitive reason for this finding is that the informed trader needs a sizable total position  $X = X_0 + X_1$  to make a strategic recourse ex post optimal. But if  $X_0$  is small in absolute terms, a deal  $X_1$  of the required size would reveal too much information, which would make the whole plan unprofitable.

These considerations are reflected in our description of the equilibrium, which will be provided below. By an equilibrium, we mean functions for the market order  $X_1^*(.)$  and for the strategic recourse  $S^*(.)$ , and a pricing rule  $r^*(.)$  such that (i) for any  $X_0$  in the support of the initial distribution, the informed trader maximizes expected net income from interest  $E_Y[\pi]$  by choice of  $X_1 = X_1^*(X_0)$  and  $S = S^*(X_0 + X_1)$ , and (ii) for any  $Z = X_1 + Y$  in the support of the equilibrium distribution, dealers set the swap rate  $r^* = r^*(Z)$  competitively as captured by (4).<sup>13</sup>

The “sporadic” nature of manipulation, while valid much more generally, precludes the possibility of a tractable equilibrium with normally distributed random parameters. The point to note is that, as manipulation occurs only for sufficiently large initial positions, the conditional distribution of market orders is determined by the tails of the distribution of the initial position. However, tail distributions of normal distributions are not normal. Thus, despite the theoretical desirability of the normal distribution that has been pointed out by Nöldeke and Tröger (2001, 2006) and Bagnoli, Viswanathan, and Holden (2001), it is preferable in our situation to consider a setup with uniform distributions, just because the tail distribution of a uniform distribution is again uniform. We will assume therefore in the sequel that the random variables  $X_0$ ,  $Y$ , and  $V$  are uniformly distributed on intervals  $[-\delta_X, \delta_X]$ ,  $[-\delta_Y, \delta_Y]$ , and  $I_V = [-\delta_V, \delta_V]$ , respectively, where  $\delta_X, \delta_Y, \delta_V > 0$ .

In the uniform setup, boundary conditions must be considered explicitly. Two restrictions on the parameter values have to be imposed. First, we assume that the liquidity shock is sufficiently dispersed, as captured by

$$\delta_V > \frac{\delta_X + \delta_Y}{3}. \quad (5)$$

This restriction will ensure that the manipulated market rate does not reach the boundary of the corridor, which is a useful

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<sup>13</sup>In principle, the swap dealers should form expectations also off the equilibrium distribution of net aggregate orders. However, in our setup, these expectations can be assumed to extrapolate the linear price effect in the swap market, making a deviation by the informed trader unattractive.

simplification. Further, to focus on the interesting case of manipulation, we will assume that the initial position of the informed trader is sufficiently large with positive probability, i.e.,

$$\delta_X > \delta_V. \quad (6)$$

When these two conditions are satisfied, we find an explicit equilibrium with the following characteristics.

**PROPOSITION 3.** *Under conditions (6) and (5), there is an equilibrium in the manipulation game. For  $|X_0| < \delta_V$ , there is no manipulation, i.e.,  $X_1^*(X_0) = S^*(X_0) = 0$ . For  $|X_0| \geq \delta_V$ , however, the informed trader will leverage her position and subsequently manipulate the deposit rate.*

The proposition suggests that those commercial banks that are the most active traders of interest rate derivatives with nonbank customers—i.e., those with a large  $\delta_X$ —should be among the most likely to manipulate the deposit market. In practice, the decision to manipulate by a commercial bank will depend also on other factors, including (i) the overall trading and collateral capacities of the bank, (ii) the internal allocation of the bank's risk budget between markets, and (iii) its general readiness to take strategic measures in the search of profit opportunities, including the involved daringness vis-à-vis the monetary authority and potentially other regulatory institutions. For these reasons, we would expect that even in a large currency area, only a few commercial banks may be prepared for manipulative actions such as those described in this paper. Depending on the central bank's stance on this issue, it may also be difficult for an individual institution to repeat an unwanted manipulative strategy. Still, a central bank will have to formulate a credible response to such strategies.

## 6. Welfare Consequences, Policy Measures, and Further Discussion

### 6.1 Social Cost of Manipulation

As mentioned in the introduction, manipulation is not welcomed by a central bank because it might negatively affect the reputation of

the monetary authority and also because it may add volatility to money market conditions. We will use three different measures for the welfare loss: first, the *probability* of manipulation  $\text{pr}\{S^* \neq 0\}$ ; second, the expected *extent* of manipulation  $E[|S^*| | S^* \neq 0]$ , conditional on a strategic recourse; and finally, the *volatility* of the market rate at date 2, measured by the unconditional standard deviation

$$\sigma_M = \sqrt{E[(r(S) - r^0)^2]}.$$

The first two measures are related to central bank reputation, while the volatility measure captures the objective of smooth implementation.

## 6.2 Preventing Manipulation

What mechanisms could prevent the need of money markets to accommodate volatility caused by strategic recourses?

**The Width of the Corridor.** In the early discussion of the problem (cf. Vergara 2000), it had been suggested that a wider interest rate corridor should effectively defuse the risk of manipulation. Indeed, it is not implausible to conjecture that a larger average spread between the EONIA and the respective facility rate should increase the cost of affecting the market rate sufficiently to make manipulation unattractive. However, as our next result shows, this intuition is incorrect. Formally, let  $\Delta' > \Delta$  denote the enlarged half-width of the interest rate corridor. We increase the lending rate  $r^L = r^0 + \Delta$  to  $r^0 + \Delta'$  and lower the deposit rate from  $r^D = r^0 - \Delta$  to  $r^0 - \Delta'$ . These changes are then implemented consistently over the whole two-day maintenance period.

**PROPOSITION 4.** *Widening or narrowing the interest rate corridor has no effect either on the probability or on the extent of manipulation.*

To see why the proposition holds in the case considered in proposition 3, recall that by Poole's lemma, the size of the liquidity effect is proportional to the width of the corridor, i.e.,

$$\rho = |r'(0)| = |\Phi'_V(0)|(r^L - r^D) = 2|\Phi'_V(0)|\Delta. \quad (7)$$

Using (7) in the equilibrium conditions, one can verify that  $\Delta$  cancels out in all expressions, so that both the probability and the extent of manipulation remain unaffected by the size of the width.

Proposition 4 is much more general and does not depend on distributional assumptions. Intuitively, the interest rate corridor has two roles as an instrument in the implementation of monetary policy. On the one hand, the standing facilities impose an effective boundary to money market conditions. On the other, however, the width of the corridor is a linear scaling factor for the size of the liquidity effect. Once this double role of the corridor is taken into account, the above result should be ultimately straightforward: while a wider corridor makes strategic recourses more costly for the commercial bank, the gains are scaled up as well.

Of course, the volatility caused by manipulation could be lowered by having a tighter corridor. However, in practice, a corridor that is too tight would create incentives for exclusive trading with the central bank and would consequently dry out the interbank market. This would constitute a problem because considerations of credit risk imply a certain dispersion of the interest rates that are applied to bilateral transactions in the money market. The optimal size of the corridor should therefore reflect the central bank's trade-off between smoothing implementation and setting prudential incentives.

**Fine-Tuning.** The model can be extended in a straightforward way to incorporate the possibility of central bank intervention. Let  $\alpha_0 \in [0; 1]$  denote the probability that the central bank intends fine-tuning at the end of date 2.<sup>14</sup> The parameter values  $\alpha_0 = 0$  and  $\alpha_0 = 1$  correspond to no intervention and regular fine-tuning, respectively. In the case of the Eurosystem, the parameter  $\alpha_0$  has traditionally been close to zero. Indeed, before May 11, 2004, the ECB had generally been quite reluctant to use additional operations to correct for end-of-period imbalances, apparently because there had been no good reason for an intervention, and also because

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<sup>14</sup>In a more descriptive setup, the probability of fine-tuning would be correlated with the size of the liquidity imbalance on the last day of the reserve maintenance period. In this case, the manipulator may choose a lower  $S^*$  to avoid the fine-tuning. While the corresponding equilibrium would be intractable, we conjecture that the qualitative features of our predictions would be essentially unchanged.

some volatility seems to be desirable to provide incentives for bidding in the main refinancing operations (cf. ECB 2005). However, following the initial experiences with the new operational framework, the ECB gradually increased its willingness to intervene on the last day, with fine-tuning after February 2005 occurring almost regularly at the end of the maintenance period. Thus, nowadays, with quasi-regular fine-tuning at the end of each maintenance period,  $\alpha_0$  should be much closer to one.

Fine-tuning operations may not always lead to the desired result. This indeed happened in the euro area when market participants found the condition in a liquidity-draining fine-tuning operation not sufficiently attractive to participate (cf. ECB 2006). Formally, we assume a conditional probability  $\pi > 0$  that a given fine-tuning operation does not lead to the desired outcome. The probability of successful fine-tuning is then given by  $\alpha = \alpha_0(1 - \pi)$ . We assume that an unsuccessful operation fails completely, while a successful operation ensures that the market rate at the end of date 2 is  $r^0$ . The market rate after manipulation will then be  $r(S)$  with probability  $1 - \alpha$ , and  $r^0$  with probability  $\alpha$ . In expected terms, a recourse of  $S$  in the context of a central bank reaction captured by  $\alpha$  implies a market rate at date 2 of

$$r(S, \alpha) = \alpha r^0 + (1 - \alpha)r(S). \quad (8)$$

In particular, with fine-tuning, the deviation of the market rate from  $r^0$  at date 2 is bounded by  $(1 - \alpha)\Delta$ . Thus, in a sense, fine-tuning attenuates the liquidity effect more effectively than a more dispersed liquidity shock. Adapting proposition 3, we arrive at the following result.

*PROPOSITION 5. Assume that the probability  $\pi$  of an operational failure is smaller than one. Then a higher probability of fine-tuning  $\alpha_0$  lowers the probability and the extent of manipulation, as well as the volatility of money market conditions.*

Thus, deviations of the money market rate caused by strategic recourses can be effectively reduced by an appropriate and immediate reaction of the central bank, and should therefore be expected to be a transient phenomenon in practice. An immediate



reaction is needed, however, because if the recourse is not compensated promptly in the morning of the subsequent day, then the market rate could have moved, and a gain for the manipulator would result.

**The BoE Design.** Our analysis may throw some light on the innovative design of the standing facilities in the new operational framework of the Bank of England (see Macgorain 2006). The final design involves having a corridor half-width of 1 percent, as in the case of the Eurosystem, but having the corridor narrowed down to 0.25 percent on the final day of the reserve maintenance period. This design element effectively drives a wedge between the cost of the strategic recourse on the right-hand side of equation (3), which remains high, and the benefit from the interest rate movement on the left-hand side of (3), which is significantly reduced. Formally, for  $t = 1, 2$ , denote by  $r_t^L$  and  $r_t^D$  the facility rates at date  $t$ , and by  $\Delta_t = (r_t^D - r_t^L)/2$  the corridor half-width at date  $t$ .

**PROPOSITION 6.** *Narrowing of the corridor only on date 2 by some factor  $\beta = \Delta_1/\Delta_2 > 1$  is equivalent to successful fine-tuning with probability  $\alpha = 1 - 1/\beta$ .*

The narrowing of the corridor on the last day of the reserve maintenance period may therefore become a complement or substitute for fine-tuning, for instance, when the probability  $\pi$  of an operational failure is not negligible. The Bank of England has combined narrowing of the corridor with a regular fine-tuning policy and flexible reserve requirements. Preliminary evidence from the sterling money market suggests that this combination of policy measures is indeed quite powerful.<sup>15</sup>

### 6.3 Several Manipulators

Intuitively, the possibility of profitable manipulation should provoke imitation or climbing on the bandwagon by other major players in

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<sup>15</sup>Information policy does not appear to us as a useful instrument to combat manipulation. While, in principle, the central bank could withhold information about recourses, the manipulator has an incentive to actively disseminate this information. Moreover, the resulting ambiguity might lead to even more gaming.

the interbank market. To study this possibility in formal terms, we generalize our model to the case of  $N \geq 2$  informed traders  $i = 1, \dots, N$ . Consider an informed trader  $i$  with an initial position  $X_0^i$  and a submitted market order  $X_1^i$ . There is an interaction with the other informed traders at the end of date 1 because the value of  $i$ 's position does not only depend on her own recourse  $S^i$ , but also on the aggregate net recourse

$$S^{-i} = \sum_{j \neq i} S_j$$

of the other informed traders. Formally, this interdependence is reflected in the net income from interest for trader  $i$ , which is given by

$$\begin{aligned} \pi^i(X_0^i, X_1^i, S^i, S^{-i}) &= X_0^i(r(S^i + S^{-i}) - r^0) + X_1^i(r(S^i + S^{-i}) - r^*) \\ &\quad + S^i(r(S^i + S^{-i}) - r^{L/D}(S^i)), \end{aligned}$$

in straightforward generalization of (2). To keep the model tractable, we focus on the second stage of the manipulation game. Formally, we will disallow swap trading at date 1 and assume that initial swap positions  $X_0^i$  are perfectly correlated.

**PROPOSITION 7.** *There exists an equilibrium in the second stage of the manipulation game with  $N \geq 2$  informed traders. Similar to the case  $N = 1$ , there is no recourse provided that  $|X^i| \leq \delta_V$ . If, however,  $\delta_V < |X^i| < (2 + 1/N)\delta_V$ , the informed trader  $i$ 's equilibrium recourse  $S^{i,*}$  is given by*

$$S^{i,*} = -\text{sign}(X^i) \frac{|X^i| - \delta_V}{N + 1}.$$

Thus, with  $N \geq 2$  informed traders, there is a public-good problem between the informed traders because all informed traders will benefit from an individual trader's strategic recourse. However, it must be conjectured that competition among potential manipulators alone will not preclude the possibility of manipulation.

## 7. Conclusion

In this paper, we have pointed out that in money markets that are embedded in a corridor system, composed of central bank lending

and deposit facilities, there is the potential for manipulative action that abuses these facilities. Anecdotal evidence for the euro area suggests that this strategy may be perceived by the market as more than just a theoretical possibility. We have used a microstructure model to show that manipulation can be profitable for a commercial bank with suitable *ex ante* characteristics. Manipulation remains a feature of the equilibrium even if dealers in the derivatives market form rational expectations about potential manipulation. A widening of the interest rate corridor over the whole reserve maintenance period is not helpful. Instead, regular fine-tuning fights manipulation effectively, as does narrowing the corridor on the last day of the maintenance period. Indeed, these measures ensure that the costs of manipulation remain high, while the benefits decrease. Our analysis supports the common perception that the monetary authority has powerful instruments to combat manipulation, but also that further vigilance in these operational matters appears recommendable.

### Appendix. Proofs

**Proof of Poole's Lemma.** The distribution of  $V$  having no mass points, the probability that the maintenance period ends with ample liquidity amounts to

$$\text{pr}\{S - V > 0\} = \text{pr}\{V < S\} = \text{pr}\{V \leq S\} = \Phi_V(S).$$

Similarly, the probability that  $S - V < 0$  is given by  $1 - \Phi_V(S)$ . This proves (1). The monotonicity of  $r(S)$  follows from

$$r(S) = r^L - \Phi_V(S)(r^L - r^D).$$

As the median of the distribution of  $V$  is zero, we have  $\Phi_V(0) = 1/2$ , which proves the lemma.

**Proof of Proposition 1.** Assume first that  $X \geq 0$ . Then any  $S > 0$  is strictly inferior to no recourse, i.e., to  $S = 0$ . Thus,  $S^*(X) \leq 0$ . Using (3) and Poole's lemma, we find the necessary first-order condition

$$X = -S + \frac{1 - \Phi_V(S)}{\phi_V(S)}, \quad (9)$$

where  $S < 0$  and such that  $\phi_V(S) > 0$ . It is easy to check that the right-hand side of equation (9) is strictly decreasing in  $S < 0$  and approaches  $\Delta/\rho$  for  $S \rightarrow 0$ . Hence, equation (9) has a unique solution  $S^*(X) < 0$  for any  $X > \Delta/\rho$ . Clearly, this is the global optimum when the support interval  $I_V$  of the distribution of  $V$  is not bounded from below. Assume now a finite lower boundary  $\underline{V} < 0$  of  $I_V$ . Then clearly, any  $S < \underline{V}$  is inferior to  $S = \underline{V}$ , so that also in this case, the global optimum is determined by (9). For  $0 \leq X \leq \Delta/\rho$ , an interior solution is not feasible. Therefore,  $S^*(X) = 0$  when  $I_V$  is unbounded from below. When  $I_V$  is bounded from below, then  $\underline{V} \leq -\Delta/\rho$  because  $\Phi_V(S)$  is convex for  $S < 0$ . But then,

$$\pi(X_0, X_1, \underline{V}) - \pi(X_0, X_1, 0) = X(r^L - r^0) + \underline{V}(r^L - r^D) < 0.$$

Thus, also when  $I_V$  is bounded from below,  $S^*(X) = 0$  for  $0 \leq X \leq \Delta/\rho$ . The case  $X < 0$  can be treated in an analogous way. Hence the assertion.

**Proof of Proposition 2.** Without loss of generality, assume  $X_0 > 0$  (the other case follows by symmetry). Consider first a change in the market side, i.e., a market order  $X_1 < 0$  such that  $X_0 + X_1 < 0$ . We claim that submitting this market order is suboptimal, even if followed by  $S = S^*(X_0 + X_1)$ . As an alternative plan of action, consider  $\hat{X}_1 = -2X_0 - X_1$ , followed by  $\hat{S} = -S$ . Indeed, in this case  $X_0 + \hat{X}_1 = -(X_0 + X_1)$ , so that in a symmetric market environment,

$$\begin{aligned} &\pi(X_0, \hat{X}_1, \hat{S}) - \pi(X_0, X_1, S) \\ &= \underbrace{X_1(E_Y[r^*(X_1 + Z)] - r^0)}_{> 0} - \underbrace{\hat{X}_1(E_Y[r^*(\hat{X}_1 + Z)] - r^0)}_{> 0}. \end{aligned} \tag{10}$$

Clearly,  $|X_1| > |\hat{X}_1|$  and consequently also

$$|E_Y[r^*(X_1 + Z)] - r^0| > |E_Y[r^*(\hat{X}_1 + Z)] - r^0|.$$

Thus, (10) is positive, proving our claim. Consider now the case of hedging, i.e.,  $-X_0 \leq X_1 < 0$ . Then  $X_0 + X_1 \geq 0$  and therefore  $S^*(X_0 + X_1) \leq 0$  by proposition 1. We claim that a deviation to  $\hat{X}_1 = 0$  *without changing*  $S = S^*(X_0 + X_1)$  is already a

better trading strategy. To see why, note that  $r(S) \geq r^0$  and that  $r^0 > E_Y[r^*(X_1 + Z)]$ . But then,

$$\pi(X_0, \hat{X}_1, \hat{S}) - \pi(X_0, X_1, S) = -X_1(r(S) - E_Y[r^*(X_1 + Z)]) > 0.$$

Thus, also hedging cannot be optimal.

**Proof of Proposition 3.** This result follows immediately from lemma A.1 below for  $\alpha = 0$  (i.e., no fine-tuning).

**Proof of Proposition 4.** Start from an equilibrium in the manipulation game. Assume first that the half-width of the corridor is scaled up from  $\Delta > 0$  to some  $\Delta' > 0$ , where  $\Delta' < r^0$ . Let  $\gamma = \Delta'/\Delta > 1$ . Then, by Poole's lemma, the liquidity effect  $r(S) - r^0$  is scaled up by the factor  $\gamma$ . Consider now, as an equilibrium candidate in the model with corridor  $\Delta'$ , a competitive swap spread  $r^* - r^0$  that is scaled up by the factor  $\gamma$ . It is then straightforward to check that the objective function (2) of the manipulator is multiplied by  $\gamma$ . The optimal strategy of the informed trader concerning the choice of  $X_1$  and  $S$  as a function of  $X_0$  remains unchanged. Thus, neither the distribution of aggregate market orders  $Z$  arriving at the dealer's desk nor the dealer's posterior belief on  $S$  given his observation of  $Z$  is affected. From equation (4), we get that the scaled-down pricing function in the swap market is indeed competitive. A similar argument can be made for a narrowing of the corridor. Hence the assertion.

**Proof of Proposition 5.** This result follows immediately from lemma A.1 below.

LEMMA A.1. For  $\alpha < 1$ , let  $\hat{\delta}_V = \delta_V/(1 - \alpha)$ . Assume

$$\delta_X > \hat{\delta}_V \text{ and } \delta_X + \delta_Y < 2\delta_V + \hat{\delta}_V. \tag{11}$$

Then the following is an equilibrium in the manipulation game with fine-tuning. For  $|X_0| < \hat{\delta}_V$ , there is no manipulation, i.e.,

$$X_1^*(X_0, \alpha) = S^*(X_0 + X_1^*(X_0, \alpha), \alpha) = 0.$$

For  $|X_0| \geq \hat{\delta}_V$ , however, the informed trader will submit a market order

$$X_1^*(X_0, \alpha) = \theta \text{sign}(X_0)(|X_0| - \hat{\delta}_V) \tag{12}$$

and will have recourse to the standing facilities

$$S^*(X_0 + X_1^*(X_0, \alpha), \alpha) = -\frac{1 + \theta}{2} \text{sign}(X_0)(|X_0| - \hat{\delta}_V) \tag{13}$$

at the end of date 1. Here,  $\theta = \delta_Y / (\delta_X - \hat{\delta}_V) > 0$  is a measure for the informational advantage of the informed trader. The swap dealers set the competitive rate to

$$r^*(Z, \alpha) = r^0 + \frac{\rho}{4} \frac{1 + \theta}{\theta} Z. \tag{14}$$

The probability of manipulation, the extent of manipulation, and the volatility of the market rate at date 2 are respectively given by

$$\text{pr}\{S^* \neq 0\} = \frac{\delta_X - \hat{\delta}_V}{\delta_X}, \tag{15}$$

$$E[|S^*| | S^* \neq 0] = \frac{\delta_X - \hat{\delta}_V + \delta_Y}{4}, \text{ and} \tag{16}$$

$$\sigma_M = \frac{\delta_X - \hat{\delta}_V + \delta_Y}{2\delta_V} \sqrt{\frac{\delta_X - \hat{\delta}_V}{3\delta_X}} (r^L - r^0). \tag{17}$$

*Proof.* We have to show that conditions (i) and (ii) in the definition of the equilibrium are satisfied. First, we consider the decision problem of the informed trader. Let  $X_0 \in [-\delta_X; \delta_X]$ . Assume that the swap dealers apply the linear pricing rule

$$r^*(X_1 + Y) = r^0 + \lambda(X_1 + Y), \tag{18}$$

where  $\lambda > 0$  is a constant. We will show at a later stage of the proof that

$$\lambda > \frac{\rho}{2} \frac{\delta_V}{2\delta_V + \hat{\delta}_V - \delta_X}. \tag{19}$$

But then, by lemma A.2 below, the informed trader does not participate in the swap market for  $|X_0| < \hat{\delta}_V$ , and submits the bid

$$X_1^*(X_0, \alpha) = \frac{\rho}{4\lambda - \rho} (X_0 - \hat{\delta}_V \text{sign}(X_0)) \tag{20}$$

for  $|X_0| \geq \hat{\delta}_V$ . From (20), the distribution of  $X_1$ , conditional on  $b = 1$ , is uniform on the interval  $[-\delta_1; \delta_1]$ , where

$$\delta_1 = \frac{\rho}{4\lambda - \rho}(\delta_X - \hat{\delta}_V) \tag{21}$$

is the maximum market order of the informed trader. By theorem 3.1 in Bagnoli, Viswanathan, and Holden (2001), a linear equilibrium requires  $\delta_Y = \delta_1$ . This proves (12). Solving (21) for  $\lambda$  and subsequently using (18) proves (14). Further, inequality (19) is equivalent to

$$\frac{4\lambda}{\rho} = 1 + \frac{\delta_X - \hat{\delta}_V}{\delta_Y} > 1 + \frac{\delta_X - \hat{\delta}_V}{2\delta_V + \hat{\delta}_V - \delta_X}.$$

Subtracting one on both sides and invoking (11) shows that (19) is indeed satisfied. Finally, lemma A.3 below and (12) deliver (13). Checking the expressions (15), (16), and (17) is a straightforward exercise. This completes the proof of lemma A.1.

**LEMMA A.2.** *Assume (11) and (19). Then the informed trader does not participate in the swap market for  $|X_0| < \hat{\delta}_V$ , and submits the bid (20) whenever  $|X_0| \geq \hat{\delta}_V$ .*

*Proof.* When the central bank fine-tunes successfully with probability  $\alpha$ , then the net interest for the informed trader amounts to

$$\begin{aligned} \pi(X_0, X_1, S, \alpha) &= X_0(r(S, \alpha) - r^0) \\ &\quad + X_1(r(S, \alpha) - r^*) + S(r(S, \alpha) - r^{L/D}(S)). \end{aligned} \tag{22}$$

Assuming (18), the expected profit for the informed trader is given by

$$E_Y[\pi] = -\lambda X_1^2 + (X_0 + X_1 + S)(r(S, \alpha) - r^0) + S(r^0 - r^{L/D}(S)).$$

Using lemma A.3, the informed trader’s objective function reads

$$\begin{aligned}
 h(X_1) &= E_Y[\pi(X_0, X_1, S^*(X_0 + X_1), \alpha)] \\
 &= \begin{cases} -\lambda X_1^2 & \text{if } |X| < \hat{\delta}_V \\ -\lambda X_1^2 + \frac{\rho}{4}(|X| - \hat{\delta}_V)^2 & \text{if } \hat{\delta}_V \leq |X| < \hat{\delta}_V + 2\delta_V \\ -\lambda X_1^2 + \rho\delta_V(|X| - \delta_V - \hat{\delta}_V) & \text{if } |X| \geq \hat{\delta}_V + 2\delta_V. \end{cases} \tag{23}
 \end{aligned}$$

From (11) and (19), we obtain  $4\lambda > \rho$ . Under this condition, the objective function  $h(X_1)$  is continuously differentiable and strictly concave on  $\mathbb{R}$ . The necessary and sufficient condition for the optimum is therefore  $h'(X_1) = 0$ . Note that the third case,  $|X_0 + X_1^*| \geq \hat{\delta}_V + 2\delta_V$ , is not possible. This is because in this case the first-order condition would imply  $|X_1^*| = \rho\delta_V/(2\lambda)$ , but then, using (19), we obtain  $|X| \leq |X_0| + |X_1^*| < \hat{\delta}_V + 2\delta_V$ , a contradiction. Assume now  $|X_0| \geq \hat{\delta}_V$ . By straightforward extension of proposition 2, we have  $\text{sign}(X_1^*) = \text{sign}(X_0)$  for  $X_0 \neq 0$ . But then clearly  $|X| < \hat{\delta}_V$  is impossible, which yields (20). Consider now  $|X_0| < \hat{\delta}_V$ . Formula (20) would imply a reversed sign for  $X_1$ , so this is clearly not feasible. Hence  $X_1^* = 0$  in this case. This proves the assertion.

LEMMA A.3. *In the uniform model with fine-tuning, let  $\hat{\delta}_V = \delta_V/(1 - \alpha)$ , as before. Then  $S^*(X, \alpha) = 0$  for  $|X| < \hat{\delta}_V$ , while  $S^*(X, \alpha) = -\text{sign}(X)(|X| - \hat{\delta}_V)/2$  for  $\hat{\delta}_V \leq |X| < \hat{\delta}_V + 2\delta_V$ , and  $S^*(X, \alpha) = -\text{sign}(X)\delta_V$  for  $|X| \geq \hat{\delta}_V + 2\delta_V$ .*

*Proof.* Consider first the case  $X > 0$ . Without fine-tuning, Poole’s lemma implies  $r(S) = r^0 - S\Delta/\delta_V$  for  $|S| \leq \delta_V$ . Using (8) yields  $r(S, \alpha) = r^0 - S\Delta/\hat{\delta}_V$  for  $|S| \leq \delta_V$ . Clearly,  $S^*(X, \alpha) \leq 0$ . The necessary first-order condition for an interior solution reads  $X = \hat{\delta}_V - 2S$ . Thus, an interior solution of the informed trader’s problem at the end of date 1 exists and is given by  $S^*(X, \alpha) = -(X - \hat{\delta}_V)/2$  for  $\hat{\delta}_V < X < \hat{\delta}_V + 2\delta_V$ . Otherwise, there is a boundary solution. From

$$\pi(X_0, X_1, -\delta_V, \alpha) - \pi(X_0, X_1, 0, \alpha) = (1 - \alpha)(X - 2\hat{\delta}_V)\Delta$$



it is obvious that  $S^*(X, \alpha) = 0$  for  $|X| \leq \hat{\delta}_V$ , and  $S^*(X, \alpha) = -\delta_V$  for  $X \geq \hat{\delta}_V + 2\delta_V$ . An analogous consideration can be made for the case  $X < 0$ . Hence, the assertion.

**Proof of Proposition 6.** Write  $r(S, \alpha, \Delta)$  for the market rate in a corridor system with half-width  $\Delta$  at date 2 around  $r^0$ , after a recourse of  $S$ , and with a probability  $\alpha$  of successful fine-tuning. Using (8) and Poole's lemma yields

$$\begin{aligned} r(S, \alpha, \Delta) &= \alpha r^0 + (1 - \alpha) \{ \Phi_V(S) r_2^D + (1 - \Phi_V(S)) r_2^L \} \\ &= \Phi_V(S) (\alpha r^0 + (1 - \alpha) r_2^D) \\ &\quad + (1 - \Phi_V(S)) (\alpha r^0 + (1 - \alpha) r_2^L) \\ &= r(S, 0, (1 - \alpha)\Delta). \end{aligned}$$

Thus, fine-tuning with probability  $\alpha = 1 - 1/\beta$  is equivalent to narrowing the interest rate corridor on date 2 from  $\Delta_1$  to  $\Delta_2 = (1 - \alpha)\Delta_1 = \Delta_1/\beta$ .

**Proof of Proposition 7.** Under the assumptions made, trader  $i$ 's objective function reads

$$\bar{\pi}^i(X^i, S^i, S^{-i}) = X^i(r(S^i + S^{-i}) - r^0) + S^i(r(S^i + S^{-i}) - r^{L/D}(S^i)),$$

where  $X^i = X_0^i$ . Consider first the case  $X^i \geq 0$ . Then, for any  $S^{-i}$ , choosing  $S^i > 0$  is always (weakly) inferior for trader  $i$  than  $S^i = 0$  because

$$\begin{aligned} \bar{\pi}^i(X^i, S^i, S^{-i}) - \bar{\pi}^i(X^i, 0, S^{-i}) &= X^i \underbrace{(r(S^i + S^{-i}) - r(S^{-i}))}_{\leq 0} + S^i \underbrace{(r(S^i + S^{-i}) - r^L)}_{\leq 0} \leq 0. \end{aligned}$$

Thus,  $S^i \leq 0$ . Moreover, in the uniform case, choosing  $S^i$  such that  $S^i + S^{-i} < -\delta_V$  is inferior to choosing  $S^i$  equal to  $-\delta_V - S^{-i}$  because

$$\begin{aligned} \bar{\pi}^i(X^i, S^i, S^{-i}) - \bar{\pi}^i(X^i, -\delta_V - S^{-i}, S^{-i}) &= (S^i + S^{-i} + \delta_V)(r^L - r^D) < 0. \end{aligned}$$

Thus, if  $S^{-i} < -\delta_V$  then an optimal recourse is given by  $S^{i,*}(X^i, S^{-i}) = 0$ . Moreover, if  $S^{-i} \geq -\delta_V$  then  $S^{i,*}(X^i, S^{-i}) \in$

$J^- = [-\delta_V - S^{-i}; 0]$ . For values  $S^i \in J^-$ , trader  $i$ 's objective function is differentiable and strictly concave with respect to  $S^i$ . The interior solution  $S^{i,*}(X^i, S^{-i}) = (\delta_V - X^i - S^{-i})/2$  stays within  $J^-$  provided that  $\delta_V - S^{-i} \leq X^i \leq 3\delta_V + S^{-i}$ . In a symmetric equilibrium,  $S^{-i} = (N-1)S^i$ . Thus  $S^{i,*}(X^i) = (\delta_V - X^i)/(N+1)$  for  $\delta_V \leq X^i \leq 2 + \delta_V/N$ , and we have established an equilibrium. The case of  $X^i \leq 0$  can be treated in an analogous way.

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