Estimating a Phillips Curve for South Africa: A Bounded Random-Walk Approach*

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In this paper we estimate a Phillips curve for South Africa using a bounded random-walk model. Central bank credibility, the slope of the Phillips curve, the natural rate of unemployment, and the central bank's inflation target band are time varying. We find that the slope of the Phillips curve has flattened since the mid-2000s—particularly after the Great Recession—which is in line with the findings in most advanced countries. Our results do not lend support to the hypothesis that the ability of the South African Reserve Bank to hit its inflation target has decreased. With respect to the faith in the IT regime as measured by the degree to the extent of which inflation expectations are anchored to the target, our results indicate that inflation persistence has increased from 1994 to 2001, remained constant from 2001 to 2008, and eventually increased around 2008. This pattern is different from that of advanced countries where expectations have become better anchored relatively early in the IT regime. Moreover, we find that the increased stability of inflation expectations after 2008—which coincides with the Great Financial Crisis—is not only a result of good policy but also of "good luck."

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1. Introduction

The relationship between inflation and activity which was dubbed the "Phillips" curve by Samuelson and Solow (1960), following the celebrated contribution by Phillips (1958), remains one of the cornerstones of modern macroeconomics. For example, it features prominently in rational expectations guise in New Keynesian models such as Galí, Smets, and Wouters (2012) and Galí (2015a).

On the fiftieth anniversary of Phillips' paper, Gordon (2011) provides a roadmap of its evolution since 1975. According to Gordon (2011), in the post-1975 era, the Phillips curve has evolved into two strands: a "left fork" and a "right fork." The left fork constitutes a framework that allows policy to respond to shocks in the context of price stickiness. This involves an inflation model where expectations are backward looking. The "left fork road" emphasizes the role of demand shock, inflation inertia, and supply shocks in explaining the dynamics in inflation. Demand shocks are proxied by the unemployment gap, whereas supply shocks include variables such as changes in the relative prices of food, energy, and imports. And inflation inertia is represented by the formation of expectations and persistence due to fixed duration in wage and price contracts. The New Keynesian model follows the right fork whereby, in line with rational expectations versions of the Phillips curve, forward-looking expectations can jump in response to policy changes.[1]

A recent contribution by Blanchard, Cerutti, and Summers (2015)—clearly along the left fork of the road—investigates the relationship between output, unemployment, and inflation over the course of about fifty years for twenty-three advanced economies. They estimate for each country a benchmark relation between inflation, long-term inflation expectations, lagged inflation, and the unemployment gap. Their specification allows for the natural rate of unemployment as well as the coefficients to change over time. Focusing on the Great Financial Crisis, they find that the shift to inflation targeting and stable inflation for the two decades preceding the crisis have led forecasts of future inflation to put less weight on past inflation and more weight on the perceived target of the central

[1] See Kydland and Prescott (1977) and Sargent (1980).

bank. In addition, they find that the slope of the Phillips curve has flattened over time—with much of the decrease taking place from the mid-1970s to the mid-1990s and the slope being insignificant for most countries today. This then leads to a puzzle and a potential challenge for inflation targeting. At the same time as inflation expectations have become more anchored, the ability of central banks to affect inflation through the unemployment gap has decreased. Put another way, the faith in the ability of central banks to achieve their target has increased, while the effectiveness of monetary policy has decreased.

In this paper we investigate this puzzle for South Africa. Accordingly we build a "left fork of the road" Phillips-curve model in unemployment–inflation space along the lines of Chan, Koop, and Potter (2016) (CKP). We allow the slope of the Phillips curve, the central bank's perceived inflation target, the natural rate of unemployment, and inflation persistence to be time varying and estimate the model with Bayesian methods using a Markov chain Monte Carlo algorithm. As in CKP, we constrain some of our state variables to be consistent with economic theory or with the structure of the South African economy. For example, the South African Reserve Bank's (SARB's) official inflation targeting band is 3–6 percent. Therefore, we constrain the central bank's perceived target to fall within this band. Moreover, since the inflation persistence parameter in our model is interpreted as a measure of the degree of central bank credibility, we constrain this parameter to be between 0 and 1, where 0 captures full credibility and 1 is complete lack of credibility. Thus, unlike CKP, we explicitly tie the inflation persistence parameter to private agents' inflation expectations formation process and link trend inflation with the central bank's perceived inflation target. The above interpretation allows us to extend Kabundi and Schaling (2013) (KS) and Kabundi, Schaling, and Some (2015) (KSS) with both a time-varying implicit inflation target (band) and evolving central bank credibility.

KS examine the relationship between inflation and inflation expectations in South Africa. They find that economic agents' inflation expectations largely depend on lagged inflation. This suggests that the South African Reserve Bank has not been successful in anchoring expectations of the private sector since the adoption of the inflation-targeting (IT) regime in 2000. They also find evidence that the SARB's implicit inflation target lies above the upper bound of the official IT band. KSS disaggregate KS and allow for three groups of agents in the model: financial analysts, trade unions, and business. Results of that paper indicate that expectations of price setters (business and unions) are closely related to each other and are higher than the upper bound of the official target band, while expectations of analysts are within the target band. Further, they find that the SARB has successfully anchored expectations of analysts but that price setters have not sufficiently used the focal point implicit in the IT regime.

However, neither KS nor KSS estimate a Phillips curve for South Africa in a time-varying parameter framework. This paper attempts to close this gap and, because of the richer framework, is able to answer several questions in the spirit of Blanchard, Cerutti, and Summers (2015) and Gillitzer and Simon (2015) such as: How has the slope of the Phillips curve evolved over time and what can be said about the faith in the IT regime as proxied by the evolving level of central bank credibility and the perceived inflation target? Has the ability of the central bank to hit its inflation target decreased?

Our results show that in South Africa inflation persistence and the slope of the Phillips curve are time varying. Specifically, the inflation persistence parameter has decreased since the Great Recession in 2008. This suggests that the SARB's credibility has increased since 2008. This is consistent with the view that inflation persistence has decreased since the 1980s and that the adoption of inflation targeting in most advanced economies has led to less persistent inflation. As for the slope parameter, results show that there is a slight decrease since 2008. This means that inflation has become less responsive to demand-side factors in South Africa. This is in line with the results of Matheson and Stavrev (2013) and Blanchard, Cerutti, and Summers (2015), who find that the slope of the Phillips curve in the United States and several OECD economies has significantly decreased since the 1980s.

The remainder of the paper is organized as follows: section 2 introduces the unobserved-components model. In section 3 we discuss the data, while our empirical results are given in section 4. Section 5 concludes and suggests avenues for further research.

2. The Unobserved-Components Model

The model we use is a standard unemployment-based Phillips curve (see, for example, Ball and Mazumder 2011, Matheson and Stavrev 2013, Ball 2015, and Gillitzer and Simon 2015). More specific, we assume that the economy is characterized by the following Phillips curve:[2]

$$\pi_t = \pi_t^e - \alpha_t(u_t - u_t^n) + \varepsilon_t, \tag{1}$$

where π_t is the inflation rate between time $t-1$ and t, π_t^e is expected inflation, u_t is the time-t unemployment rate, α_t is the slope of the Phillips curve which is allowed to be time varying, and ε_t is a residual capturing other factors such as supply (cost-push) shocks. To account for this fact, we specify a stochastic process for the conditional volatility of the disturbance term ε_t in the Phillips curve as

$$\varepsilon_t \rightsquigarrow N(0, h_t^2)$$
$$\log(h_t) = \log(h_{t-1}) + \varepsilon_t^h,$$

where ε_t^h is an iid process with mean 0 and variance σ_h^2 and uncorrelated with ε_t. We also assume that each error term is uncorrelated with the others. Here u_t^n is the natural rate of unemployment that prevails when inflation is equal to expected inflation ($\pi_t = \pi_t^e$) and when shocks are absent ($\varepsilon_t = 0$).[3]

Note that in equation (1) the variable u_t^n is unobserved. Moreover, we need to make an assumption about how inflation expectations are formed in (1) to fully describe the dynamics of inflation. Kabundi, Schaling, and Some (2015) model inflation expectations in South Africa as a weighted average of lagged inflation and the South African Reserve Bank's (SARB) implicit inflation target.[4] We follow

[2] Ball and Mazumder (2011, p. 346) define core inflation as the part of inflation explained not by supply shocks, but rather by expected inflation and economic activity. Thus in equation (1) core inflation can be written as $\pi_t^c = \pi_t^e - \alpha_t\,(u_t - u_t^n)$.

[3] For u_t^n to be well identified, we will impose $\alpha_t > 0\ \forall$ t.

[4] More specifically, in Kabundi, Schaling, and Some (2015), inflation expectations are modeled as $\pi_t^e = \rho\pi_{t-1} + (1-\rho)\pi^*$, where the weight parameter ρ and the inflation target π^* are constant over time.

the same logic to specify the inflation expectations process. However, we take into account the fact that the inflation expectations formation process may vary over time. In fact, Kabundi, Schaling, and Some (2015) interpret the weight on lagged inflation as the parameter that captures the level of credibility of monetary policy. The idea is that the closer this weight is to zero, the better inflation expectations are anchored to the target. The credibility of monetary policy may change over time and thus we allow the weight on lagged inflation to be time varying. Moreover, the SARB does not have an explicit point target but instead has a target band of 3–6 percent. Thus, we assume that the SARB's implicit inflation target point is unobserved and is time varying. Accordingly, the inflation expectation process is given by

$$\pi_t^e = \rho_t \pi_{t-1} + (1 - \rho_t)\pi_t^*, \tag{2}$$

where π_t^* is a proxy for the SARB's inflation target point (in the spirit of Kabundi, Schaling, and Some 2015), which is assumed to be time varying. ρ_t is a time-varying parameter that captures the weight agents put on past inflation. As mentioned before, when $\rho_t = 0$, inflation expectations are completely anchored to the central bank's inflation target ($\pi_t^e = \pi_t^*$). In this case the central bank policy is fully credible, whereas it totally lacks credibility when $\rho_t = 1$.[5]

The reason the SARB should focus on anchoring can be inferred from equations (1) and (2) of the paper. It has to do with the so-called second-round effects of the supply shock ε_t. Via equation (1) an adverse shock to inflation raises the actual inflation rate in the same period. Then (assuming $0 < \rho < 1$) inflation expectations for time $t+1$ increase. These in turn raise inflation in period $t+1$. This is the second-round effect. Of course, the feedback loop does not stop here and goes on indefinitely. What can be seen from equation

[5] See Kabundi, Schaling, and Some (2015) for further discussion on the link between ρ and central bank credibility. Using our notation, Matheson and Stavrev (2013) estimate $\pi_t^e = \rho_t \pi_{t-1}^4 + (1 - \rho_t)\bar{\pi}_t$, where $\bar{\pi}_t$ is long-run inflation expectations (sourced from the Federal Reserve Board) and π_{t-1}^4 is year-over-year headline CPI inflation (lagged one quarter). Thus, they anchor inflation expectations to *long-run* inflation expectations, not to the central bank's inflation target. However, to the extent that the former are influenced by the Fed's (implicit) inflation target, our specification is not that dissimilar.

(2) is that if inflation expectations are fully anchored (i.e., if $\rho = 0$), there are no second-round effects.

We assume that π_t^* (unobserved) follows the random-walk process[6]

$$\pi_t^* = \pi_{t-1}^* + \varepsilon_t^*. \tag{3}$$

Combining equations (1), (2), and (3), the reduced form of the inflation process can be written as

$$\pi_t - \pi_t^* = \rho_t(\pi_{t-1} - \pi_{t-1}^*) - \alpha_t(u_t - u_t^n) + \nu_t, \tag{4}$$

where $\nu_t = \varepsilon_t - \rho_t \varepsilon_t^*$, with variance $\sigma_{v,t}^2 = h_t + \rho_t^2 \sigma_{\varepsilon^*}^2$.[7]

Equation (4) is a Phillips curve in terms of the cyclical component of inflation (if we assume that the central bank target π_t^* is a good proxy for trend (or core) inflation). Basically, the dynamics of the cyclical component of inflation, $\pi_t - \pi_t^*$, are explained by built-in persistence due here to agents' expectations formation process; excess demand, $u_t - u_t^n$; and other supply shocks ν_t. This formulation of the Phillips curve based on cyclical inflation has been adopted in previous work, including Stock and Watson (2007), Stella and Stock (2013), and Chan, Koop, and Potter (2016). The difference with our specification is that we explicitly tie the persistence parameter ρ_t to economic agents' inflation expectations instead of just an ad hoc statistical parameter to be estimated.[8]

Note that when $(1 - \rho_t L)(\pi_t - \pi_t^*) = 0$ and $\nu_t = 0$, we have $u_t = u_t^n$ (where L is the lag operator). Thus, the expression for u_t^n as the natural rate of unemployment is slightly different from the traditional definition of the non-accelerating inflation rate of unemployment (NAIRU), where one only needs $\pi_t = \pi_{t-1}$. Our specification

[6]Note that the random-walk process was adopted to account for slow change in the inflation target.

[7]Note that the error term v_t in the reduced form of the inflation equation (4) is correlated with the error term of the inflation target equation (3) ε_t^*, as $cov(v_t, \varepsilon_t^*) = cov(\varepsilon_t - \rho_t \varepsilon_t^*, \varepsilon_t^*) = -\rho_t \sigma_{\varepsilon^*}^2$. Note that independence between the two terms can be reasonably assumed if the $\rho_t \sigma_{\varepsilon^*}^2$ term is negligible (both terms of the product are between 0 and 1), which appears to be the case in South Africa since $\sigma_{\varepsilon^*}^2$ is small (we estimate it at 0.003).

[8]For a recent specification related to our work, see Blanchard, Cerutti, and Summers (2015).

does not explicitly include a supply shock variable as an explanatory factor in the Phillips curve, and thus differs somewhat from the triangle specification of Gordon (1998).[9] However, as said before, we allow the residual ν_t to follow a stochastic volatility process to correct for potential heteroskedasticity introduced by the omission of supply-side variables.

We assume that the parameters ρ_t and α_t follow random-walk processes

$$\rho_t = \rho_{t-1} + \varepsilon_t^{\rho} \tag{5}$$

$$\alpha_t = \alpha_{t-1} + \varepsilon_t^{\alpha}, \tag{6}$$

where ε_t^{ρ} and ε_t^{α} are error terms whose processes will be made clear shortly.

Since the natural rate of unemployment, u_t^n, is unobserved, the cyclical unemployment $u_t - u_t^n$ is unobserved as well. Following Chan, Koop, and Potter (2016), we specify an AR(2) process for cyclical unemployment as follows:

$$u_t - u_t^n = \varphi_1(u_{t-1} - u_{t-1}^n) + \varphi_2(u_{t-2} - u_{t-2}^n) + \varepsilon_t^u, \tag{7}$$

where φ_1 and φ_2 are constant parameters and ε_t^u is an identically independently distributed (iid) error term with mean 0 and variance σ_u^2.[10] Since we want to use inflation and unemployment data to estimate the model, we also specify a process for the natural rate of unemployment as a random-walk process:[11]

$$u_t^n = u_{t-1}^n + \varepsilon_t^n, \tag{8}$$

where ε_t^n is the error term to be defined below. Note that this specification of the natural rate discards the possibility of the hysteresis hypothesis of unemployment where the natural rate of unemployment depends on past unemployment rates.[12]

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[9] For an analysis about optimal monetary policy in a triangle model, see Bullard and Schaling (2001).

[10] In a preliminary analysis, we find that, using a Hodrick-Prescott (HP) filter, the cyclical unemployment follows an AR(2) process and there is no evidence of time variation of the coefficients φ_1 and φ_2. Matheson and Stavrev (2013) assume that the persistence of the unemployment gap is constant.

[11] Equation (8) allows for a slow variation in the NAIRU from one quarter to another.

[12] For a recent analysis of unemployment hysteresis, see Galí (2015b).

Although supply shocks such as cost-push pressure by unions or "oil sheiks" are not explicitly modeled, we stress that our Phillips-curve model is close to the "left fork of the road" (Gordon 2011) or econometrically sound mainstream triangle approach (as opposed to the "right fork" or rational expectations approach). This can be seen by combining (4) and (7). This yields

$$\pi_t = \rho_t L \pi_t - \alpha_t \left(\varphi_1 L + \phi_2 L^2\right)(u_t - u_t^n) + (1 - \rho_t L)\pi_t^* - \alpha_t \varepsilon_t^u + \nu_t. \tag{9}$$

This expression is similar to equation (13) of Gordon (2011).

The system of equations (4), (3), (5), (6), (7), and (8) can be viewed as a bivariate unobserved-components model of inflation and unemployment where the parameters ρ_t and α_t are time varying.

Note that without further restrictions on the state variables u_t^n, π_t^*, ρ_t, and α_t and with the standard distributional assumptions on the error terms, this system can be estimated using a Kalman-filter algorithm. However, this would mean that these variables are in principle unbounded when the variances of the error terms are significantly large. We depart from this assumption for the following reasons. First, the SARB has a clearly specified inflation target range between 3 and 6 percent. Even if a target point is not clearly specified, it would not be reasonable to assume that the SARB's unobserved target point π_t^* can take any value. Therefore we have to impose a restriction of the type $3\% \leq \pi_t^* \leq 6\%$ at any time t. Second, here the inflation persistence parameter ρ_t in (4) captures the perceived credibility of monetary policy or the extent to which inflation expectations are anchored. Perfect credibility is associated with $\rho_t = 0$, whereas a complete lack of credibility implies $\rho_t = 1$ with imperfect credibility in between. So here, a priori, we have the constraint $0 \leq \rho_t \leq 1$.

A preliminary analysis using an HP filter shows that the cyclical component of unemployment follows a stationary AR(2) process. This implies that the parameters φ_1 and φ_2 in equation (7) must satisfy $\varphi_1 + \varphi_2 < 1$, $\varphi_1 - \varphi_2 < 1$ and $-1 < \varphi_2 < 1$. Moreover, the natural rate of unemployment is a steady-state concept and should be bounded. Finally, the Phillips-curve slope parameter α_t should be positive on theoretical grounds.

In general the random-walk trend variables in this model are bounded from below and above as

$$a_x \leq x_t \leq b_x, \tag{10}$$

where $x_t = u_t^n, \pi_t^*, \rho_t, \alpha_t$, and a_x and b_x are constant real numbers. Using the random-walk property of these variables, the inequalities in equation (10) imply that

$$a_x - x_{t-1} \leq \varepsilon_t^x \leq b_x - x_{t-1}, \tag{11}$$

where $\varepsilon_t^x = \varepsilon_t^n, \varepsilon_t^*, \varepsilon_t^\rho$, and ε_t^α.

Thus, for the boundedness restrictions of the random-walk processes to be satisfied in (3), (5), (6), and (8), we can simply bound the error terms below and above by time-varying bounds. Chan, Koop, and Potter (2016) use a similar framework to study inflation and unemployment trends for the U.S. economy. Matheson and Stavrev (2013) and Blanchard, Cerutti, and Summers (2015) estimate a similar Phillips curve with time-varying coefficients for twenty OECD countries where the slope coefficient and the inflation persistence parameter are constrained. As in Chan, Koop, and Potter (2016), we assume that the error terms $\varepsilon_t^x = \varepsilon_t^n, \varepsilon_t^*, \varepsilon_t^\rho$ follow a truncated normal distribution, that is, we have

$$\varepsilon_t^x \rightsquigarrow TN(a_x - x_{t-1}, b_x - x_{t-1}, 0, \sigma_x^2),$$

where $TN(\theta, \beta, \mu, \sigma^2)$ is the normal distribution of mean μ and variance σ^2 and truncated within $[\theta \quad \beta]$.

We estimate several parameters of the model by a Bayesian method. Let the time-invariant parameters be summarized in the following vector:

$$\psi = (\varphi_1, \varphi_2, \sigma_u^2, \sigma_h^2, \sigma_{u^n}^2, \sigma_{\pi^*}^2, \sigma_\rho^2, \sigma_\alpha^2, a_{u^n}, a_{\pi^*}, a_\rho, a_\alpha, b_{u^n}, b_{\pi^*}, b_\rho, b_\alpha)'.$$

Note that the bounded parameters can be fixed or estimated. During the estimation, we will fix the bounded parameters $a_\rho, b_\rho, a_\alpha, b_\alpha$ and estimate the inflation target and the natural rate of unemployment bounded parameters a_{π^*}, a_{u^n}, and b_{π^*}, b_{u^n}, respectively. So, the parameter vector to be estimated becomes $\psi = (\varphi_1, \varphi_2, \sigma_u^2, \sigma_h^2, \sigma_{u^n}^2, \sigma_{\pi^*}^2, \sigma_\rho^2, \sigma_\alpha^2, a_{u^n}, a_{\pi^*}, b_{\pi^*}, b_{u^n})'$.

With these "boundedness" restrictions, the model is highly nonlinear and standard Kalman-filter algorithms do not apply. We follow the algorithm proposed by Chan, Koop, and Potter (2016) to estimate the model. The algorithm is a Bayesian estimation method using a Markov chain Monte Carlo (MCMC) drawing procedure which takes into account the restrictions imposed on the state variables.

Before we give a brief summary of the algorithm, some notation needs to be introduced. For each variable z, we denote by $z = (z_1, z_2, \ldots, z_T)^{'}$ the sample vector of z where T is the sample size and $y = (\pi^{'}, u^{'})^{'}$. After specifying the priors for the parameters vector ψ and the initial state variables, the drawing procedure is a six-step MCMC algorithm and can be summarized as follows:

1. $p(\pi^* | y, u^n, \rho, \alpha, h, \psi)$
2. $p(u^n | y, \pi^*, \rho, \alpha, h, \psi)$
3. $p(\rho | y, \pi^*, u^n, \alpha, h, \psi)$
4. $p(\alpha | y, \pi^*, u^n, \rho, h, \psi)$
5. $p(h | y, \pi^*, u^n, \rho, \alpha, \psi)$
6. $p(\psi | y, \pi^*, u^n, \rho, \alpha, h)$

For the specification of the priors, we assume that all the standard deviations follow inverse gamma (IG) distributions, the bounded parameters $a_{\pi^*}, b_{\pi^*}, a_{u^n}, b_{u^n}$ follow uniform distributions, and the AR(2) parameters φ_1, φ_2 follow normal distributions. The initial state variables u_0^n, π_0^* , ρ_0, α_0 follow truncated normal (TN) distributions. In general, we use relatively non-informative priors in the analysis. To save space, we do not present here all the details of the algorithm and refer the reader to Chan, Koop, and Potter (2016) for a detailed description of the drawing procedure of each of the six steps above.

3. Data Analysis

The analysis uses quarterly unemployment rate data obtained from the South African Reserve Bank (SARB) covering the period

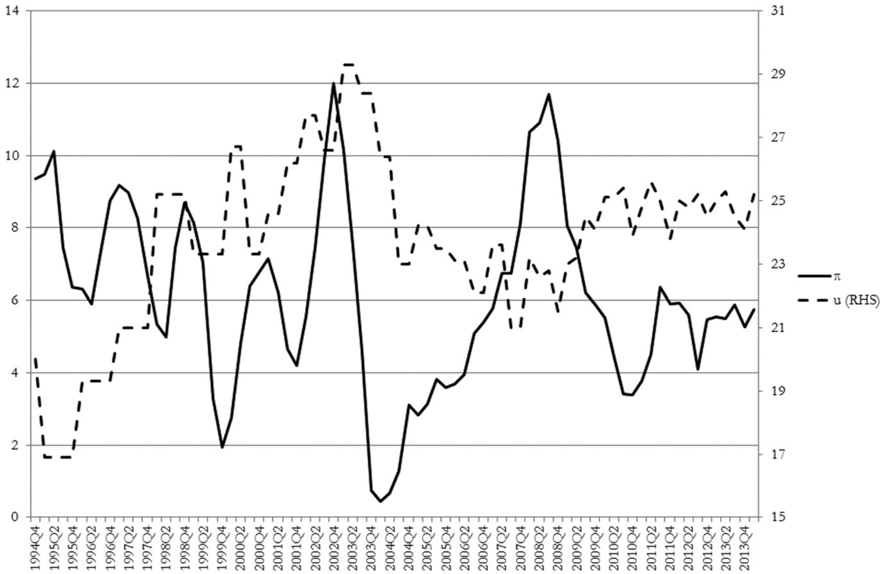
1994:Q1 to 2014:Q1. It is important to note that labor market data in South Africa are somewhat unreliable. Therefore, the sample size is determined by the availability of the unemployment data. The SARB constructs the unemployment time series by linking different labor surveys conducted by Statistics South Africa (Stats SA) from 1994 to most recent. In addition, the weights used in the different surveys are different. The first sample, from 1994 to 1999, is based on the annual October household survey. From 2000 to 2007 both the frequency and the approach used changed. The new sample is biannual, and it is based on the Labour Force Survey (LFS). Finally, from 2008 to present Stats SA publishes quarterly series of unemployment based on the Quarterly Labour Force Survey (QLFS). The QLFS is a household-based survey on the labor market activities of population aged fifteen to sixty-four years. It uses the strict definition of unemployment, that is, it does include discouraged jobseekers.

We emphasize that this is the only available unemployment series for South Africa that goes as far back as 1994. The current analysis necessitates an adequate number of observations in order to obtain statistically meaningful results. Moreover, starting in 1994 enables the analysis of the evolution of the Phillips curve before and during the inflation-targeting (IT) regime, which started in 2000. Viegi (2015) avoids altogether using the unemployment series in the estimation of his Phillips curve and instead uses the more reliable employment series. However, the unemployment rate is an essential variable especially for policymaking. It is crucial for the monetary policy authority to have an idea of the nature of relationship between inflation and unemployment. In addition, the estimation of the non-accelerating inflation rate of unemployment (NAIRU) is important for the conduct of monetary policy.

For inflation we use the year-on-year headline inflation data obtained from the South African Reserve Bank. However, we are aware that from 2000 to 2008 the SARB was targeting CPIX-inflation instead of the headline CPI.[13] In 2008 the SARB reverted back to targeting headline CPI inflation. We use headline CPI

[13]CPIX refers to headline CPI excluding mortgage costs.

Figure 1. Unemployment and Inflation



inflation to avoid statistical issues such as structural breaks that are inherent when two different series are used.[14]

Figure 1 depicts the unemployment and inflation rates from 1994:Q4 to 2014:Q1. It is clear from the figure that unemployment displays a discrete pattern from 1994 to 2007. The series becomes continuous from 2008 until the end of the sample. In 1994:Q4 South Africa registered its lowest unemployment rate at 16.9 percent. During this period inflation was around 10 percent. Then unemployment increased markedly, attaining the maximum of 29.3 percent in 2003:Q2. At the same time, inflation decreased first and reached its minimum value of 1.9 percent in 1999:Q4, then reversed sharply and reached a maximum of 12.00 percent in the second quarter of 2002. The rise in inflation was mainly due to a strong depreciation of the South African rand. Interestingly, the two series trend upwardly in early 2000 until late 2002; then inflation declined first, followed by

[14] The results are qualitatively the same when we use both the headline CPI inflation and the CPIX inflation.

unemployment. Hence, the graphical representation points to evidence of a Phillips curve in the beginning of the sample, but the curve flattened in the early part of 2000 until the later part of 2003 when the negative relationship recurred again all the way to the end of the sample. The surge in inflation in 2004 was caused by many factors—among others, an increase in demand, the rise in petrol price, and food prices. Unemployment decreased and reached a minimum of 21 percent in the fourth quarter of 2007; in the meantime, inflation peaked at 11.69 percent in 2008:Q3. The trend reversed when the impact of the financial crisis in the United States spilled over to South Africa, driving the economy into recession, which subsequently pushed unemployment to about 25 percent, and at the same time inflation plummeted to 3.39 percent in 2010:Q4 before stabilizing around the 6 percent level. The analysis based on the graphical representation of these two variables points to the changing nature of the Phillips curve in South Africa.

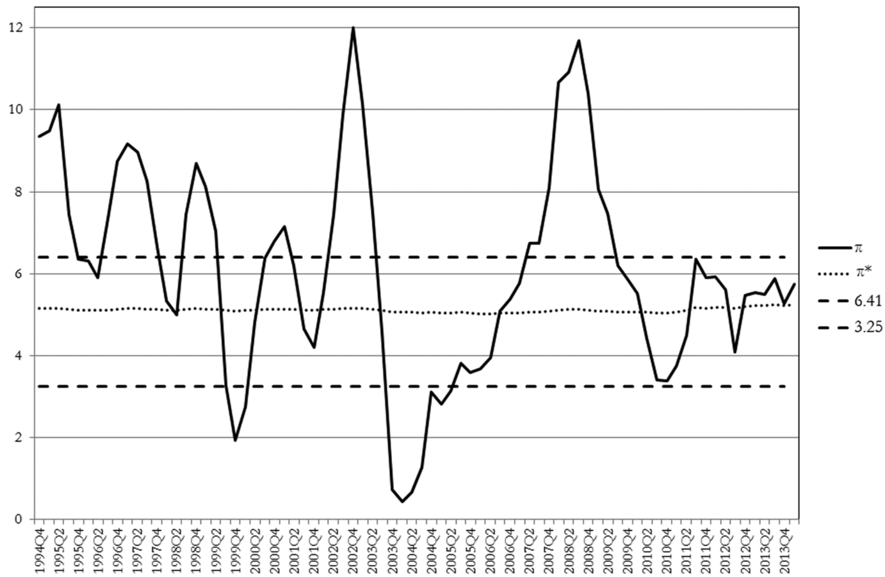
4. Empirical Results

This section presents the results obtained by estimating equation (4). We first discuss the extracted trends which represent, on the one hand, the implicit inflation target and the estimated target band and, on the other hand, the trend unemployment rate or NAIRU. Secondly, we discuss the relationship between the inflation cycle and the unemployment cycle. The empirical section concludes with the analysis of the estimated parameters—among others, inflation persistence, the slope of the Phillips curve, and lastly the conditional volatility of inflation.

4.1 Trend Inflation and Trend Unemployment

The estimated trends of inflation and unemployment from the state-space model follow a bounded random-walk process. The initial values are set in a way that is consistent with the two series. We set trend inflation between 3 percent and 7 percent, and trend unemployment between 15 percent and 30 percent. Figure 2 represents inflation together with the implicit inflation target and the estimated target bands. Interestingly, the estimated band is consistent with the official target between 3.25 percent and 6.41 percent. The

Figure 2. Inflation, Implicit Inflation Target, and Implicit Target Bands

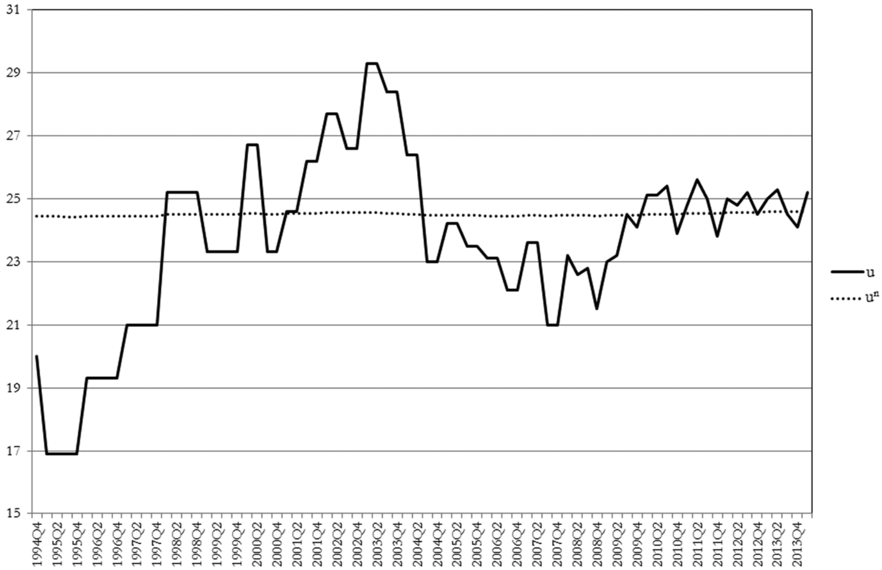


estimated inflation target (π^*) is relatively constant with a mean of 5.12 percent, and maximum and minimum values of 5.25 percent and 5.03 percent, respectively. These results are consistent with an implicit target of financial analysts of 5.51 percent, estimated by Kabundi, Schaling, and Some (2015). Note that Kabundi, Schaling, and Some (2015) use a different sample, from 2000 to 2013. Unlike these authors, the approach used in this paper allows the estimation of the target band. It is not surprising that the mean of the target is in line with the midpoint of the estimated band—in this case, 5 percent.

Similarly, the estimated unemployment trend (u^n) or NAIRU in figure 3 is relatively flat throughout the entire sample period, around 24.5 percent. Previous work suggests that the NAIRU for South Africa is about 25 percent.[15] Figure 7 in the appendix shows that the NAIRU estimated using the Phillips curve (PC) is slightly different from the estimation obtained from an atheoretical approach—in this

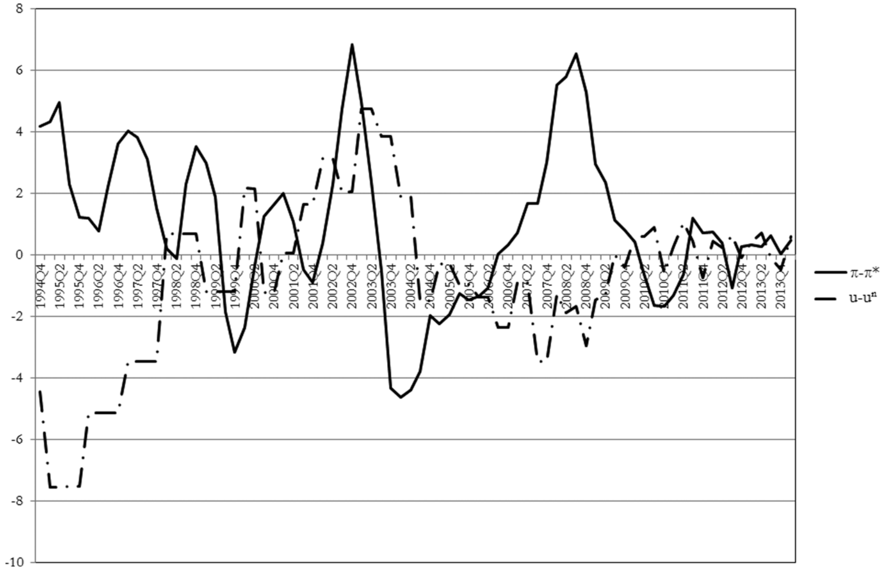
[15]See, for example, Viegi (2015).

Figure 3. Unemployment and NAIRU



case, an AR(2) process. The PC estimates are consistently higher than the AR ones and the difference widens from 2008:Q4 onward. Figure 3 shows that the unemployment rate was lower than the NAIRU from 1994 to 1998. This period corresponds to rising economic activity in South Africa. Then unemployment stabilizes somewhat around the NAIRU from 1998:Q2 to 2001:Q1. From 2001:Q2 the unemployment rate rises markedly way above the NAIRU and stays high for approximately three years. During this period the economy was weak, with a negative output gap. And from 2004:Q3 until the recent financial crisis the economy was at an expansionary stage and hence it created more jobs. Consequently, unemployment decreased below the NAIRU for about five years. The financial crisis pushed the South African economy into recession in 2009, which at the same juncture exerted pressure on unemployment, which rose back to the level of the NAIRU and has remained stable ever since. The persistent high level of unemployment at the end of the sample closely mimics weak economic activity in South Africa since the financial crisis.

Figure 4. Cyclical Components



4.2 Cyclical Components

In order to get some perspective on the Phillips curve in South Africa, in figure 4 we show the relationship between the cyclical components of inflation (inflation gap) and unemployment (unemployment gap) inferred from the model. According to equation (4), the Phillips-curve relationship involves the cyclical components of inflation and unemployment and not necessarily the levels as shown in figure 1. From figure 4 we can see the changing nature of the relationship between the two cycles for the period 1994–2014. There is evidence of a negative relationship between the two variables at the beginning of the sample, that is, from 1994:Q4 to 1998:Q1. The relationship becomes ambiguous from 1998:Q2 to 2001:Q3, which in turn is followed by a positive relationship between the two cyclical components. This result suggests that during this period of 1998:Q2–2001:Q3, the South African economy experienced stagflation whereby high cyclical inflation was explained by other factors than cyclical unemployment (see equation (4)). Note that the domestic currency (rand) depreciated significantly in 1998:Q2–2001:Q3.

That may explain a rise in inflation while at the same time the economy was subdued. The negative relationship between the two variables reemerges in 2004 and remains in existence but weakens somewhat towards the end the sample.

Importantly, the inflation gap seems more persistent than the unemployment gap. In addition, the inflation gap follows a long cycle, with a large amplitude from 2001 to 2008. Even if the 2001–02 cycle displays a large amplitude of 6.85 percent, it is short and lasts only a year. However, the 2002–08 cycle is both long and with a large amplitude, reaching a minimum of –4.63 percent. This cycle lasts four years and peaks in 2008:Q3 at 6.55 percent. From this finding it seems that the management of the inflation cycle was somewhat ineffective in reducing its length and/or its amplitude. The next section provides in-depth analysis of the dynamic nature of the Phillips curve, but also the persistence of the inflation cycle.

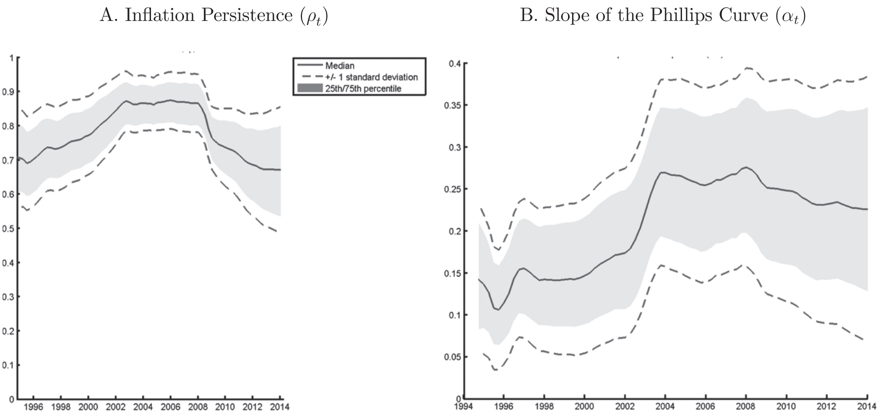
4.3 Parameters Estimates

In this part, we present the results of the parameters estimates. As discussed in the model section, we fixed the bounded parameters during the estimation except the bounded parameters of inflation target a_{π^*}, b_{π^*}, and the natural rate of unemployment bounds a_{u^n}, b_{u^n}, which have been estimated.

We set $a_\rho = 0, b_\rho = 1$ in line with our interpretation of ρ as the degree of inflation expectations anchoring to the SARB inflation target. The slope of the Phillips-curve bounds are set as $a_\alpha = 0, b_\alpha = 1$. We draw 250,000 observations from the posterior distributions and discard the first 100,000 observations before computing the statistics.

The posterior estimates indicate that the inflation target bounds are slightly higher than the official SARB inflation target bounds. The estimate of the posterior median of the lower bound of the inflation target is $a_{\pi^*} = 3.25$, whereas the posterior median of the upper bound is estimated to be $b_{\pi^*} = 6.41$. The posterior median of the natural rate of unemployment lower and upper bounds are $a_{u^n} = 17.50$ and $b_{u^n} = 27.31$, respectively. The unemployment gap is very persistent, with estimated posterior means of the AR(2) coefficients $\varphi_1 = 0.86$ and $\varphi_2 = 0.007$. We present in the appendix

Figure 5. Inflation Persistence and Slope Parameters Estimates



(table 1) some descriptive statistics of the posterior distributions and priors of the estimated constant parameters.

In a particular version of the model, we conduct the estimation without imposing the boundedness restrictions on the unobserved state variables and coefficients (u_t^n, π_t^*, ρ_t, α_t). However, then in some periods the estimated values of ρ_t and α_t lead to model instability. This highlights the importance of the restrictions imposed on ρ_t and α_t in this time-varying framework. Chan, Koop, and Potter (2016) find similar results for the U.S. economy. They find that bounding the random-walk processes (of the states), as in our case, yields better out-of-sample forecasts of inflation and unemployment compared with the unbounded case.

In figure 5 we present the smoothed estimates of ρ_t and α_t whereby the filtering is based on the full sample information. These two parameters are critical for the dynamics of inflation in the Phillips curve specified in equation (4). The parameter ρ_t captures the impact of inflation expectations on inflation, whereas the slope α_t captures the degree of the response of inflation to excess demand factors and the extent of the short-run tradeoff faced by policymakers. The more anchored inflation expectations are to the central bank target level π_t^*, the smaller ρ_t and the smaller the effect of inflation expectations on inflation. As shown in figure 5, our smoothed estimates suggest that the persistence parameter ρ_t is time varying and has increased from 1994 to 2001, remained constant from 2001 to 2008, and eventually declined after around 2008. This indicates that inflation expectations in South Africa have been relatively more anchored and stable since 2008.[16] Note, however, that the SARB's implicit target (π_t^*) estimates are consistent with financial market analysts' inflation expectations. Thus, this result would suggest that financial analysts' inflation expectations have been relatively more anchored than those of the price setters (unions and businesses) as found in Kabundi, Schaling, and Some (2015). Moreover, the financial crisis in 2008 followed by the recession in 2009 in South Africa open the question of whether this behavior of inflation expectations is the result of "good policy or good luck."

This can be formalized as follows. Lagging equation (1) by one period and substituting in (2), we get

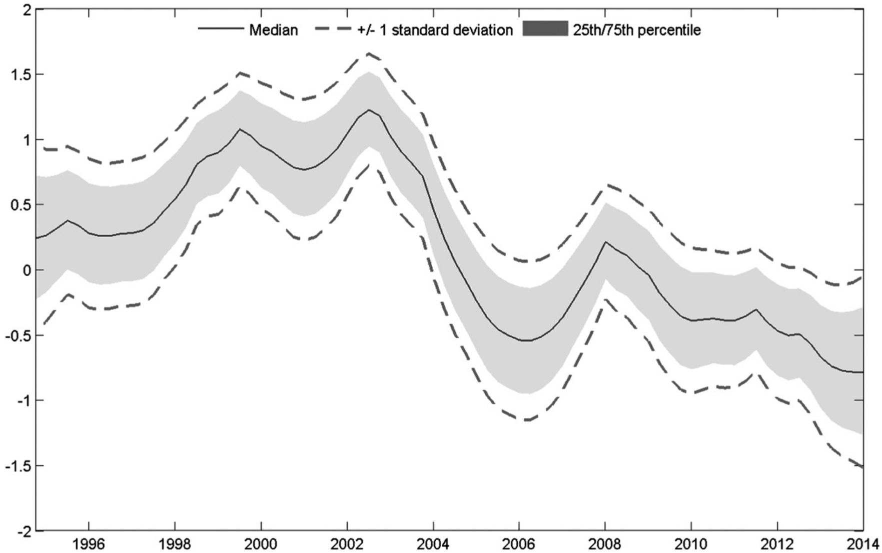
$$\pi_t^e = \frac{\rho_t}{1-\rho_t L}\left(\varepsilon_{t-1} - \alpha_{t-1}\left(u_{t-1} - u_{t-1}^n\right)\right) + \frac{1-\rho_t}{1-\rho_t L}\pi_t^*. \qquad (12)$$

From this moving-average presentation of inflation expectations it can be seen that these are a weighted average of, on the one hand, the central bank's inflation target and, on the other hand, past cyclical unemployment and past supply shocks. The hypothesis of good policy (better anchoring of expectations) is supported by the data. We have seen a decline in ρ_t after 2008. However, independently of this development, figure 6 shows that there has also been a decrease in the conditional volatility of the inflation residual term around 2008 associated with a global decline in energy and food prices. This means that the variance of inflation has fallen. Thus the drop in inflation expectations was not only driven by good policy but also by good luck.

However, at lower levels, the slope parameter α_t has increased from 1994 to the mid-2000s and since then exhibits a slight downward trend. The decline in the slope parameter means that inflation

[16] Matheson and Stavrev (2013) find that during the 1970s inflation expectations became more backward looking and volatile. As the Federal Reserve began moving towards inflation targeting in the early 1980s, long-run inflation expectations began drifting downward and eventually settled at a lower level (around 2 percent) around 2000.

Figure 6. Inflation Residual Conditional Volatility ($\log(h_t)$)



has become less responsive to demand-side factors. In the context of our model, this suggests that the importance of unemployment as a driver of the inflation process has decreased. The implications for disinflation policy is that the sacrifice ratio (SR), which captures the increase in unemployment above the natural rate due to each percentage-point decline in inflation, is becoming higher.

However, it can also be shown that under an optimal inflation reduction policy (which balances inflation and unemployment costs) the SR decreases with better anchoring of inflation expectations. That suggests that the central bank should focus on anchoring of expectations which can be done by means of communicating a faster disinflation policy. More specific, Schaling and Hoeberichts (2010) show that in the context of a two-period version of the model of this paper (with rational private-sector learning) better anchoring (a lower value of ρ) positively depends on the prior probability that the private sector attaches to a quick disinflation. This result is consistent with Cogley, Primiceri, and Sargent (2010), who show that the persistence of the inflation gap (defined as the difference between inflation and the Federal Reserve's long-run target for inflation) fell after the Volcker disinflation.

There is a link between an increase in credibility, i.e., a decrease in ρ, and the flattening of the Phillips curve, i.e., a decrease in α. With high credibility, inflation tends to be constant around the implicit target, which implies that unemployment becomes uncorrelated with inflation. Then unemployment only reacts to the demand (supply) shock, while inflation remains relatively constant.[17]

5. Conclusion

In this paper we have estimated a Phillips curve for South Africa using a bounded random-walk model. Inflation persistence, the slope of the Phillips curve, the natural rate of unemployment, and the central bank's inflation target band were modeled as time-varying parameters and variable, respectively.

This framework has enabled us to answer several questions central to the conduct of monetary policy, such as the evolution of the slope of the Phillips curve, the faith in the IT regime as proxied by the evolving level of the inflation persistence, and the perceived inflation target (band).

Our results indicate that the slope of the Phillips curve has flattened since the mid-2000s—particularly after the Great Recession—which is in line with the findings in most advanced countries.

We find that our estimated inflation target band from 3.25 to 6.41 percent is slightly higher than the official SARB target bounds of 3–6 percent band per the inflation-targeting regime that has been in existence in South Africa since 2000. This suggests that monetary policy has been relatively loose and that there has been little attempt to hit the lower bound of the official band.

Related to the band is the central bank's perceived inflation target. The estimated target is relatively constant, with a mean of 5.12 percent. As it lies closer to the upper bound than the lower

[17] The flattening of the Phillips curve could also be due to forces of globalization. Trade and financial integration put downward pressure on domestic prices by increasing competition and productivity in such a way that domestic firms have less power in setting prices. Furthermore, with trade integration, domestic demand is more easily satisfied by imported goods, thereby making inflation less responsive to excess demand from the domestic country.

bound of the official target band, this may be indicative of some accommodation of cost-push shocks.

With respect to the faith in the IT regime as measured by the degree to the extent of which inflation expectations are anchored to the target, our results indicate that the inflation persistence has increased from 1994 to 2001, remained constant from 2001 to 2008, and eventually increased around 2008. This pattern is different from that of advanced countries where expectations have become better anchored relatively early in the IT regime. Moreover, we find that the increased stability of inflation expectations after 2008—which coincides with the Great Financial Crisis—is not only a result of good policy but also of "good luck."

Appendix

Figure 7. Atheoretical and Phillips Curve NAIRU

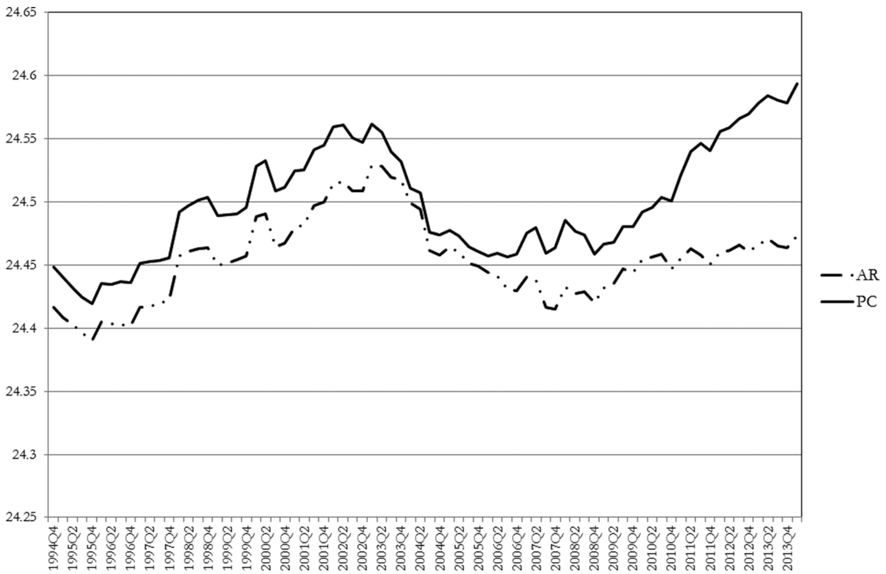


Table 1. Posterior Distribution Statistics and Priors

Parameter	Mean	Std.	Min.	Max.	Priors
φ_1	0.86	0.1	0.35	1.28	1.50
φ_2	0.01	0.1	−0.41	0.54	−0.50
σ_u^2	1.11	0.15	0.64	2.21	9.00
σ_h^2	0.10	0.02	0.042	0.28	1.80
$\sigma_{u^n}^2$	0.01	0.002	0.004	0.03	0.18
$\sigma_{\pi^*}^2$	0.02	0.005	0.008	0.07	0.36
σ_ρ^2	0.002	0.001	0.001	0.007	0.04
σ_α^2	0.002	0.001	0.001	0.006	0.04
a_{u^n}	17.50	1.45	15	20	20
a_{π^*}	3.25	0.14	2.98	3.50	4.50
b_{π^*}	6.41	0.40	5.50	7.03	6.00
b_{u^n}	27.40	1.42	25	30	26

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