# Bank Lending in Times of Large Bank Reserves*

Antoine Martin,[a] James McAndrews, and David Skeie[b]
[a]Federal Reserve Bank of New York
[b]Mays Business School, Texas A&M University

Reserves held by the U.S. banking system rose from under $50 billion before 2008 to $2.8 trillion by 2014. Some economists argue that such a large quantity of reserves could lead to overly expansive bank lending as the economy recovers, regardless of the Federal Reserve's interest rate policy. In contrast, we show that the amount of bank reserves has no effect on bank lending in a frictionless model of the current banking system, in which interest is paid on reserves and there are no binding reserve requirements. Moreover, we find that with balance sheet costs, large reserve balances may instead be contractionary.

JEL Codes: G21, E42, E43, E51.

*This paper is a revision of Federal Reserve Bank of New York Staff Report No. 497, May 2011 (revised June 2013), and was previously circulated with the title "A Note on Bank Lending in Times of Large Bank Reserves." The authors thank Todd Keister for valuable conversations that contributed to this paper; John Cochrane, Doug Diamond, Huberto Ennis, Marvin Goodfriend, Ellis Tallman, Steve Williamson, Steve Wolman, and the discussants of the paper, Morten Bech, Jagjit Chadha, Lou Crandall, Oreste Tristani, and Larry Wall, for helpful suggestions and conversations; the editor John Williams and two anonymous referees for very helpful comments; participants at the 2011 IBEFA/ASSA meetings, the 2013 Federal Reserve System Committee Meeting on Financial Structure and Regulation, the ECB conference on "The Post-Crisis Design of the Operational Framework for the Implementation of Monetary Policy," the Swiss National Bank conference on Policy Challenges and Developments in Monetary Economics, and at seminars at the Bundesbank, the Federal Reserve Bank of New York, the Federal Reserve Board, and the Swedish Riksbank for helpful comments; and Sha Lu and particularly Ali Palida for outstanding research assistance. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of New York or the Federal Reserve System. Any errors are the responsibility of the authors. Author e-mails: antoine.martin@ny.frb.org; jamesjmcandrews@gmail.com; dskeie@mays.tamu.edu.

## 1. Introduction

The amount of reserves held by the U.S. banking system rose from under $50 billion before 2008 to $2.8 trillion by 2014. This significant increase is important because some economists and financial market participants claim that large levels of bank reserves will lead to overly expansive bank lending.[1] Despite such concerns, little formal analysis has been conducted to show such an effect under the current banking system. In contrast, other commentators on the economy claim that the large level of reserves held in the banking system is evidence of a lack of bank lending.

In this paper, we present a basic model of the current U.S. banking system, in which interest is paid on bank reserves and there are no binding reserve requirements. We find that, absent any frictions, lending is unaffected by the amount of reserves in the banking system. The key determinant of bank lending is the difference between the return on loans and the opportunity cost of making a loan. We show that this difference does not depend on the quantity of reserves. Moreover, when we introduce frictions, in the form of a cost related to the size of a bank's balance sheet, increases in reserves may actually reduce bank lending and lead to a decrease in prices.

The current banking system in the United States and worldwide no longer resembles the traditional textbook model of fractional reserve banking. Historically, the quantity of reserves supplied by a central bank determines the amount of bank loans. Through the "money multiplier," banks expand loans to equal the amount of reserves divided by the reserve requirement. However, in many countries, reserve requirements have been reduced either to zero or to such small levels that they are no longer binding.[2]

[1]In speeches, former Federal Reserve Bank of Philadelphia President Charles Plosser expressed concern about the eventual need "to restrain the huge volume of excess reserves from flowing out of the banking system" (Plosser 2011), and former Federal Reserve Bank of Dallas President Richard Fisher cautioned about "excess reserves waiting to be converted to bank loans" (Fisher 2009). Meltzer (2010) expresses similar concerns.

[2]Bennett and Peristiani (2002) show that reserve requirements have been largely avoided in the United States since the 1980s by sweep accounts, and that the remaining reserve requirements are largely met by vault cash that banks hold

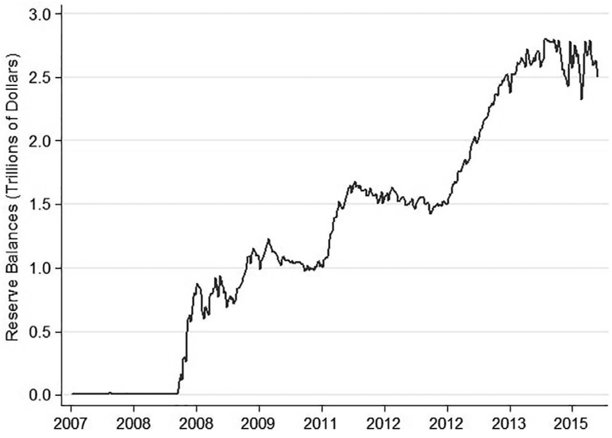
Starting in the late 1980s, the Federal Reserve supplied the quantity of reserves needed to maintain its policy target—the federal funds rate—which is the interest rate at which banks lend reserves to each other in the interbank market. The Federal Reserve did not target the amount of reserves, the quantity of deposits or loans on banks' balance sheets, or broad measures of the money supply. In that regime, the federal funds rate represents a bank's alternative return on assets and hence is the required marginal return on bank lending. Banks expand their balance sheets so long as the marginal cost of funding is less than the marginal return on bank lending, abstracting from credit and liquidity risk. The federal funds rate sets the level of the required marginal return.

From 2007 through 2014, the Federal Reserve greatly expanded the scope of its tools to address the financial crisis and a severe recession. Bank reserves increased rapidly during the financial crisis as the Federal Reserve provided unprecedented unsterilized lending through several facilities after the bankruptcy of Lehman Brothers. Reserves increased much further during the weak economic recovery as the Federal Reserve purchased Treasury securities, agency mortgage-backed securities, and agency debt as part of the large-scale asset purchases (LSAPs), also known as "quantitative easing," or "QE." Altogether, between September 2008 and mid-2014, bank reserves grew from under $50 billion to $2.8 trillion, as illustrated in figure 1. To allow the Federal Reserve to continue targeting its policy rate even with large reserves outstanding, Congress accelerated previously granted authority for the Federal Reserve to pay interest on reserves in the Emergency Economic Stabilization Act of 2008. The Federal Reserve began paying interest on reserves on October 9, 2008. Paying interest on reserves allows the Federal Reserve to choose the required return on banks' reserves independent of the quantity of reserves in the banking system.[3]

at branches and automated teller machines. As of mid-2008, required reserves were $71 billion, just 0.6 percent of total bank assets, and vault cash satisfied $43 billion of these requirements. Carpenter and Demiralp (2012) show empirically that the money multiplier does not hold using data from 1990–2007.

[3]For details and complementary analysis of interest on reserves as a monetary policy tool, see Ennis (2014), Ennis and Keister (2008), Keister, Martin, and McAndrews (2008), and Keister and McAndrews (2009). For details and analysis of additional new Federal Reserve monetary policy tools, including overnight

**Figure 1. Large Quantity of Reserves in the Banking System**




**Source:** Federal Reserve statistical release H.4.1: Factors Affecting Reserve Balances.

**Notes:** Frequency: biweekly. Reserve balances with Federal Reserve Banks are the difference between "total factors supplying reserve funds" and "total factors, other than reserve balances, absorbing reserve funds." This item includes balances at the Federal Reserve of all depository institutions that are used to satisfy reserve requirements and balances held in excess of balance requirements. It excludes reserves held in the form of cash in bank vaults, and excludes service-related deposits.

We introduce a new framework in which the role of fiat reserves that pay interest can be studied in a general equilibrium banking economy with a closed system of bank payments and central bank reserves. We include banking, corporate, and retail sectors, which transact in competitive markets for bonds, deposits, loans, and goods. Our benchmark model shows that, without frictions, bank lending quantities and interest rates are invariant to the level of reserves chosen by the central bank. Banks lend up to the point where the marginal return on lending equals the return on holding

---

reverse repurchases (reverse "repos") and the term deposit facility (TDF), see Martin et al. (2013), which expands upon the modeling framework in this paper.

reserves, which is equal to the interest rate on reserves set by the central bank. This provides an indifference result for the quantity of reserves. In particular, while the size of banks' balance sheets expands with increases in reserves, all else equal, the lending decision for a bank is determined by the same marginal return condition as with the former method of monetary policy implementation. A loan is made at the margin if its return exceeds the marginal opportunity cost of reserves, whether that is the federal funds rate as in the prior regime or the rate of interest on reserves as in the current regime. We also demonstrate that the quantity of reserves held in the banking system in the absence of binding reserve requirements or significant currency withdrawals is determined in the United States solely by the Federal Reserve. Aggregate bank reserves are independent of and provide no measure of the availability of bank credit or banks' willingness to lend.

We also study costs related to the size of a bank's balance sheet to examine whether the level of reserves affects bank lending under this friction. The concern that banks may face balance sheet costs has been raised by market observers.[4] Banks may have costs that are increasing in the size of their balance sheets because of agency costs or regulatory requirements for capital or leverage ratios. During the recent crisis, banks worked to reduce the size of their balance sheets and were slow to raise equity capital, suggesting an increase in balance sheet costs.[5] Our analysis shows that, with such increasing costs, large quantities of reserves may, surprisingly, have a contractionary effect on bank lending. Large balance sheet costs create a wedge between bank returns paid on deposits and returns received on assets. When returns paid on deposits cannot fall enough in the face of increasing balance sheet costs because of a lower zero bound, increases in reserves can partially crowd out lending and additionally cause disinflation.

The paper proceeds with the model presented in section 2. Section 3 gives results for the benchmark case with no frictions

[4]For example, Wrightson ICAP (2008) expressed the concern that excess reserves could "clog up bank balance sheets" (see also Wrightson ICAP 2009). Ennis and Wolman (2015), however, study the distribution of reserves through 2011 and do not find evidence for such an effect among U.S. domestic banks.

[5]See Martin et al. (2013), which provides a theoretical microfoundation for increasing marginal bank balance sheet costs owing to costly equity requirements that are constrained efficient to overcome banks' moral hazard.

and the cases with balance sheet costs. Section 4 concludes. Formal statements of each proposition and proofs are contained in the appendix.

## 2. Model

We consider a competitive economy with household, firm, and banking sectors, a central bank, and a government. At date 0, the government issues nominal bonds ($B$) that can be held by households ($B^H$), banks ($B^B$), or the central bank ($B^{CB}$):

$$B = B^H + B^B + B^{CB}. \tag{1}$$

The central bank has an inelastic demand for bonds, which are purchased by issuing reserves ($M$) that can only be held by banks:

$$B^{CB} = M. \tag{2}$$

Households are endowed with an amount of wealth $w$ of real goods.[6] The nominal price of goods in terms of the numeraire, reserves, is normalized at date 0 to be 1. Nominal wealth ($W$) can be held in deposits at banks ($D$), in government bonds ($B^H$), and in storage ($S$),

$$W = D + B^H + S. \tag{3}$$

Banks offer deposits ($D$) to households and make loans ($L$) to firms. Firms use the loans to purchase goods from households. These goods serve as input for the firms' investment. At date 1, firms produce output with a marginal real return $r(L)$ on the volume of loans ($L$). Firms sell their output to households at the date 1 price level of goods. Given our normalization of the price at date 0, this price is equal to the gross level of inflation ($\Pi$), i.e., the relative price of goods between dates 0 and 1. We define firms' marginal nominal return on the production and sale of their output as

$$R(L) \equiv \Pi r(L). \tag{4}$$

[6]We use uppercase letters to denote nominal amounts and lowercase letters to denote real amounts throughout the paper.

Firms pay a return ($R^L$) on the lending from banks, banks pay a return ($R^D$) on deposits, the government pays a return ($R^B$) on bonds, and the central bank pays a return ($R^M$) on reserves (for an interest rate on reserves of $R^M - 1$). The government, central bank, banks, firms, and households are competitive price takers in all markets, which include the markets for bonds, deposits, loans, and goods. For simplicity, we abstract from credit risk, liquidity risk, and risk aversion.[7]

Next, we can write the optimization problems faced by firms, households, and banks. For simplicity, we model each of these sectors as a representative price-taking entity. A firm chooses loans, sells output for revenue ($\int_L R(\hat{L})d\hat{L}$), and repays lending at a return ($R^L$) in order to maximize profit.[8] The firm's problem is

$$\max_L \int_L R(\hat{L})d\hat{L} - R^L L. \tag{5}$$

A household chooses how many deposits and bonds to hold, which, after paying a lump-sum tax ($T$), are used to purchase goods. Households keep any remaining wealth in storage, in order to maximize real consumption, given as

[7]During the financial crisis up through September 2008, there was less than \$100 billion in reserves in the banking system. At several points, banks appear to have had a demand for reserves for precautionary reasons that may have affected interest rate spreads for liquidity reasons (see Ashcraft, McAndrews, and Skeie 2011). However, the current paper focuses on the time period starting in late 2009 and beyond, when reserves ranged in the several hundreds of billions of dollars. This level was determined by the Federal Reserve supply for the purchase of assets rather than by bank demand. The ample supply of reserves has easily satisfied any potential liquidity demand for reserves. For analysis of banking fragility in related nominal contracting frameworks, see Allen, Carletti, and Gale (2011), Diamond and Rajan (2006), Martin (2006), and Skeie (2004, 2008); for studies of central bank interest rate policy within these frameworks, see Diamond and Rajan (2009) and Freixas, Martin, and Skeie (2011).

[8]For consistency of terminology, we refer to the (marginal, nominal) return on *lending* as the nominal return $R^L$ that a bank receives from a firm for loans $L$, where the return $R^L$ on the average of loans $L$ is equal to that on the margin since $R^L$ is a competitive price; the (marginal, nominal) return on *loans* as the marginal nominal return $R(L)$ a firm receives from sales of output goods produced from loans $L$; and the (marginal) *real* return on *loans* as the marginal real return $r(L)$ in the form of output goods that a firm produces from the investment of input goods purchased with loans $L$.

$$\tfrac{1}{\Pi}(R^D D + R^B B^H - T + S). \tag{6}$$

Substituting for deposits ($D = W - B^H - S$) from the household's budget constraint, equation (3), the problem can be written as

$$\max_{B^H, S} \tfrac{1}{\Pi}[R^D(W - B^H - S) + R^B B^H - T + S]. \tag{7}$$

A bank receives deposits and must choose how many loans to finance ($L$), as well as how many reserves ($M$) and how many bonds ($B^B$) to hold, in order to maximize profits. The bank's problem is

$$\max_{L,M,B^B} R^L L + R^M M + R^B B^B - R^D D - \int_D C(\hat{D})d\hat{D}, \tag{8}$$

where $C(D)$, the marginal nominal balance sheet cost, is defined as the product of the marginal real cost associated with a balance sheet of size $D, c(D)$, multiplied by the level of inflation, $\Pi$,

$$C(D) \equiv \Pi c(D).$$

The bank's balance sheet requires that

$$D = L + M + B^B, \tag{9}$$

so we can write

$$\begin{aligned} \max_{L,M,B^B} & R^L L + R^M M + R^B B^B - R^D(L + M + B^B) \\ & - \int_{L+M+B^B} C(\hat{D})d\hat{D}. \end{aligned} \tag{10}$$

The date 0 budget constraints for households, the central bank, and banks given by equations (3), (2), and (9), respectively, together imply that household wealth is divided among loans, storage, and government bonds,

$$W = L + S + B.$$

For a given amount of government bonds, $B$, maximum lending occurs when there is no storage, which we denote by

$$\bar{L} \equiv W - B.$$

We take as exogenous the government's choice of the quantity of bonds,

$$B = \bar{B},$$

and the central bank's choice of the quantity of reserves and return on reserves,

$$M = \bar{M},$$
$$R^M = \bar{R}^M,$$

respectively. The central bank remits its net revenue $(R^B B^{CB} - R^M M)$ to the government, and the government sets the lump-sum tax $(T)$ to repay its debt:

$$T = R^B B - (R^B B^{CB} - R^M M). \tag{11}$$

We make the following assumptions on exogenous parameters and functions:

(A1): $r(L) > 1, r^{'}(L) < 0, r^{''}(L) > 0, r(0) = \infty, \lim_{L\to\infty} r(L) = 1$
(A2): $0 < \bar{M} < \bar{B} < W$
(A3): $c(D) \geq 0, c(0) = 0, c^{'}(D) \geq 0, c^{'}(0) > 0$ if $c(D) > 0, c(\bar{M}) < \infty$.

Assumption (A1) states that the firm's technology is more productive than storage, along with standard Inada conditions. Assumption (A2) considers, for simplicity, monetary and fiscal policy parameters that are strictly within the feasible limit of the economy. Assumption (A3) states that when balance sheet costs are positive, these costs are increasing in the size of the balance sheet.

Letting $\mathbb{R} = (\Pi, R^M, R^L, R^D, R^B)$ and $\mathbb{Q} = (S, M, L, D, B^{CB}, B^H, B^B)$, we define an equilibrium as prices $\mathbb{R} > 0$ and quantities $\mathbb{Q} \geq 0$ such that markets clear at $\mathbb{Q}$ given individual optimizations at $\mathbb{R}$.

The first-order conditions for the firm, household, and bank are

$$L[R^L - R(L)] = 0, \tag{12}$$

$$B^R(R^B - R^D) = 0, \tag{13}$$

$$S(\Pi - R^D) = 0, \tag{14}$$

$$L\left[R^L - R^D - C(D)\right] = 0, \tag{15}$$

$$M\left[R^M - R^D - C(D)\right] = 0, \tag{16}$$

$$B^B\left[R^B - R^D - C(D)\right] = 0. \tag{17}$$

We focus on interior solutions. Since households can invest in both government bonds and deposits, they must have the same return for any interior solution, so we can write $R^D = R^B$. Since $M$ and $L$ are strictly positive, firms borrow loans to the point where their first-order condition binds, $R(L) = R^L$. Lending financed by banks have a return equal to the return paid on reserves, $R^L = R^M$.

## 3. Results

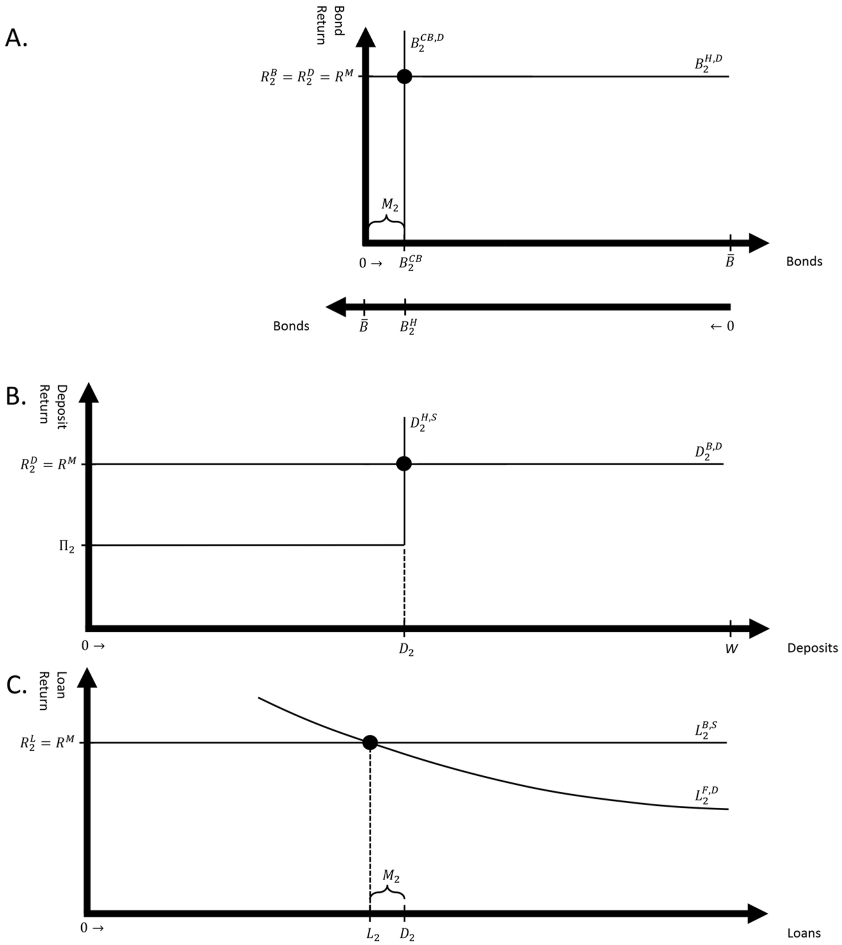
### 3.1 Benchmark Case

We first consider the benchmark case with no balance sheet costs, $c(D) = 0$. In equilibrium, a firm's return on a marginal loan, $r(L)$, and hence the quantity of loans financed, $L$, is independent of the quantity of reserves, $M$. This provides our first basic result.

PROPOSITION 1. *In the benchmark case with no balance sheet costs, there exists an equilibrium that is unique up to the allocation of bonds between households and banks. The quantity ($L$) of and return ($R^L$) on bank lending are independent of the quantity of reserves ($M$) issued by the central bank. Market returns ($R^M, R^L, R^D, R^B$) are equal to the return on reserves set by the central bank ($\bar{R}^M$) and are greater than inflation* ($\Pi$).

Figures 2 and 3 illustrate the effects of minimal and moderate levels of reserves, respectively, on the equilibrium in the benchmark case with no balance sheet costs. For simplicity, we focus on the case where banks do not hold any bonds, $B^B = 0$. Panel A in each figure shows the available bonds, $\bar{B}$, in the government bond market that must be split between the central bank and households. The central bank's demand for bonds, represented in figure 2 by $B_2^{CB,D}$, is perfectly inelastic and corresponds to the quantity purchased with the level of reserves $M_2$, where subscripts in the figures correspond to the figure numbers 2–5. Households purchase all the bonds $B_2^H$ not bought by the central bank.

**Figure 2. Benchmark Model with Minimal Reserves and No Balance Sheet Costs**




**Notes:** Superscripts "CB," "H," "B," and "F" denote "central bank," "household," "bank," and "firm," respectively; superscripts "S" and "D" denote "supply" and "demand," respectively. Subscript numbers correspond to the figure in which a particular supply or demand curve, or an equilibrium quantity, rate, or locus of points, is first determined.

**A. Bond Market.** Central bank bond holdings $B^{CB}$ increase rightward on the x-axis of the bond market graph. Household bond holdings $B^H$ increase leftward on the x-axis below the graph. The central bank has a perfectly price-inelastic demand for bonds $B_2^{CB,D}$, issuing $M_2$ reserves to buy $B_2^{CB}$ out of total government bond supply $\overline{B}$. The excess bond supply available to households, $\overline{B} - B_2^{CB}$, is perfectly price inelastic. The locus of points corresponding to a price-taking household's equilibrium demand for bonds, $B_2^{H,D}$, is perfectly elastic at the level of the return on bonds, $R^B$, that is equal to the equilibrium return on deposits, $R_2^D$. This reflects that a household is indifferent between bonds and deposits. Households' aggregate bond purchase is labeled $B_2^H$.

*(continued)*

**(Notes for figure 2, continued):**

**B. Deposit Market.** The locus of households' aggregate supply of deposits consistent with market clearing conditions for an equilibrium, $D_2^S$, is perfectly price inelastic at the quantity of deposits $D_2$, which is equal to household wealth $W$ (indicated on the x-axis) not held in bonds: $D_2 = W - B_2^H$. A bank's demand for deposits $D_2^{B,D}$ is perfectly elastic at $R_2^D = R^M$, reflecting an indifference to holding additional reserves funded by deposits.

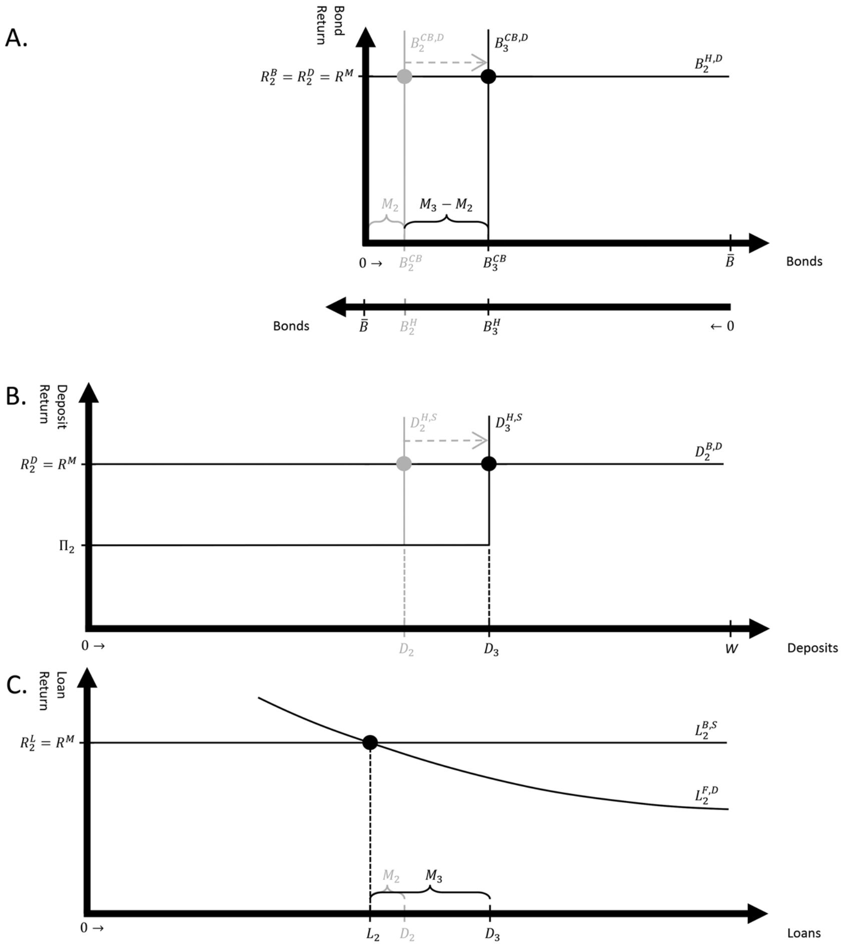
**C. Loan Market.** Firms have a downward-sloping demand curve for loans $L_2^{F,D}$, reflecting a price-taking firm's decreasing marginal real return on investment. A bank's supply of loans $L_2^{B,S}$ is perfectly elastic at the level of the return on lending that is equal to the return on reserves: $R_2^L = R^M$. This reflects that a price-taking bank is indifferent between holding loans and reserves. The equilibrium quantity of loans, equal to a bank's deposits minus reserves, $L_2 = D_2 - M_2$, determines the level of gross inflation, $\Pi_2 = \frac{R^M}{r(L_2)}$, which is shown in panels A and B.

Panel B represents the deposit market. Since there is no storage held, deposits, $D_2$, are determined by the difference between the wealth of households and their bond holdings. The vertical line $D_2^{H,S}$ represents the locus of the aggregate supply of deposits by households consistent with market clearing conditions for an equilibrium. Note that from equations (13) and (16), the supply of deposits by an individual household is perfectly elastic at the interest rate paid on reserves by the central bank. That said, the quantity of deposits supplied in equilibrium will be the same for any level of $\bar{R}^M$. This is because households' wealth is divided between deposits and bond holdings. And, as noted above, households' aggregate bond holdings are determined by the difference between the total stock of bonds and bonds purchased by the central bank. This also implies that households' aggregate deposits increase with the level of reserves issued by the central bank.

A bank has a perfectly elastic demand curve for deposits because any additional deposit gives the bank an additional reserve asset, which pays $R^M$. Panel C shows that in the loan market, firms' loan demand, $L_2^{F,D}$, determined by (12), is decreasing in the return on lending, $R^L$, and reflects the fact that $r(L)$ is decreasing in $L$. A bank's supply of lending, $L_2^{B,S}$, is perfectly elastic at the return on reserves, $\bar{R}^M$, which is the bank's opportunity cost for holding loans.

The quantity of banks' loans is independent of the supply of reserves and only depends on the return paid on reserves, highlighted

**Figure 3. Moderate Level of Reserves and No Balance Sheet Costs**




**Notes:**

**A. Bond Market.** An increase in the central bank inelastic demand for bonds to $B_3^{CB,D}$ increases reserves to $M_3$ and decreases households' bond holdings by the increased amount of reserves to $B_3^H$.

**B. Deposit Market.** Households' aggregate equilibrium supply of deposits $D^{H,S}$ remains inelastic and increases by the amount of households' decrease in bond holdings.

**C. Loan Market.** With no balance sheet costs, deposits increase exactly by the amount of the increase in reserves, showing that the return on lending $R^L$, the quantity of loans $L$, and hence inflation $\Pi$, are independent of the quantity of reserves $M$.

by the result that in equilibrium $R(L) = R^L = R^M$. This is an especially robust relationship that holds throughout the paper, even when balance sheet cost frictions are added in the next section. These equalities allow for substituting $R^M$ for $R(L)$ in equation (15), which rearranged shows that inflation is determined throughout the paper as

$$\Pi = \frac{R^M}{r(L)}. \tag{18}$$

This expression for inflation shows why there is no household storage when there are no balance sheet costs. As stated by the households' first-order condition (14), there is no storage in equilibrium when the nominal return on deposits $R^D$ is greater than inflation $\Pi$, or equivalently when the real return on deposits $\frac{R^D}{\Pi}$ is greater than one, the return on storage. In the case of no balance sheet costs, the real return on deposits, $\frac{R^D}{\Pi}$, is equivalent to $\frac{R^M}{\Pi}$, which by equation (18) equals the marginal real return on firm production, $r(L)$, greater than one. Equation (18) also highlights that in this model, inflation is always below the return on reserves $R^M$ chosen by the central bank, which can be see in figures 2 and 3.

In figure 2, with minimal reserves, equilibrium loans are equal to nearly the full quantity of deposits. In figure 3, with a moderate level of reserves, loans comprise a smaller share of banks' assets. Instead, the size of banks' balance sheets increases to fund both their loans to firms and the reserves issued by the central bank.

Figure 3 also demonstrates that the quantity of reserves held in the banking system is determined solely by the central bank's quantity of bond purchases. The level of bank reserves is independent of, and unaffected by, banks' supply of loans to firms. This means that whether bank reserves are high or low gives no indication of the amount of bank lending that is occurring. Equivalently, the amount of bank lending has no implication for the quantity of bank reserves held by banks.

### *3.2 Balance Sheet Costs*

Next, we consider the case of positive bank balance sheet costs. This is an important and natural friction to consider, since market participants have raised concern that banks' balance sheets may be too large (Wrightson ICAP 2008, 2009). Bank balance sheet costs may incorporate the costs of capital requirements and the shadow cost of potentially binding capital ratios.[9]

If $c(D) > 0$, then banks will reduce the size of their balance sheets by not holding bonds, $B^B = 0$. Households are at a corner solution of holding the government bonds not held by the central bank. A positive balance sheet cost for banks $c(D) > 0$ does not necessarily affect the amount of bank lending, and $R^L = R^M$ still holds for moderate balance sheet costs and reserve quantities. Instead, the return on deposits paid by banks is reduced: $R^D = R^M - C(D) < R^M = R(L)$. Banks' return on marginal lending, $R^L$, is not equal to banks' marginal funding costs, $R^D$, but rather is equal to the return on reserves, $R^M$, the alternative assets in which banks can invest.

PROPOSITION 2. *For moderate balance sheet costs, $c(D)$, and reserve levels ($\bar{M}$), the return ($R^L$) on lending by banks equals the return on reserves ($\bar{R}^M$). These returns are greater than the returns on deposits and bonds, which are equal ($R^D = R^B$), and which in turn remain above inflation ($\bar{\Pi}$). The amount of bank-sector lending ($L$) is independent of the amount of reserves ($\bar{M}$).*

Households' supply of deposits and a bank's demand for deposits are endogenous, and hence the size of banks' balance sheets is endogenous. The government bond return and the deposit return are both determined in equilibrium according to households' first-order conditions. Because households always hold bonds and deposits in equilibrium, their two returns must be equal to make the household indifferent between holding the two assets. When there are positive balance sheet costs, the deposit return falls below the bank's return on its assets (the return on reserves, which equals the lending return)

[9] We do not explicitly model bank capital, which is implicitly incorporated in banks' balance sheet liabilities ($D$). As a result, bank capital, which may need to be raised during times of distress to support continued or increased bank balance sheet size because of bank capital and leverage ratio requirements, may be an important part of a bank's balance sheet costs, $c(D)$. Carlson, Shan, and Warusawitharana (2011) argue that higher capital ratios may support greater loan growth, particularly in times of distress, an argument for which they find evidence during the recent financial crisis. The reluctance of many banks to raise capital during the crisis indicates that capital may be particularly costly to raise in times of distress.

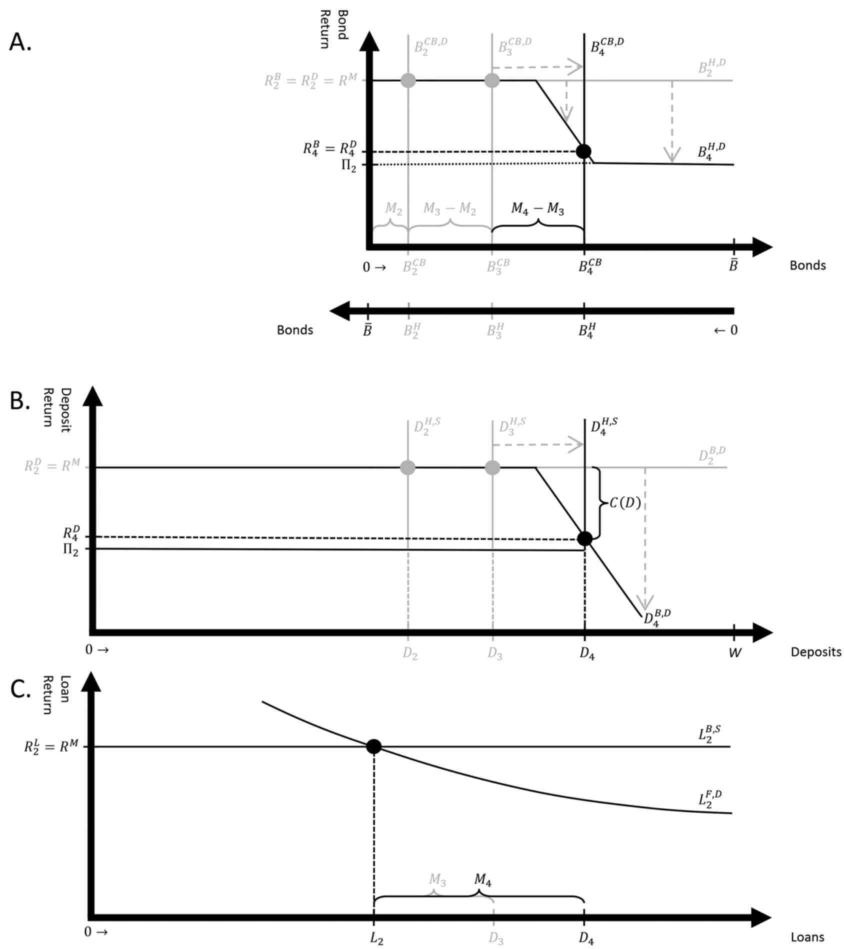
in order for the bank to be willing to hold a marginal deposit and a marginal asset. Thus, the government bond return falls below the banks' other asset returns, and banks prefer then to hold reserves and loans but zero bonds.

The invariance result of moderate balance sheet costs and reserves on bank lending is illustrated in figure 4. The equilibrium returns on deposits and government bonds are equal to and below the return on reserves: $R^D = R^B < \bar{R}^M$. The decrease in the return on deposits absorbs the balance sheet cost. Banks do not incur the balance sheet cost in their borrowing rates and do not pass the cost on through higher lending rates. Households receive the surplus from the banking sector at the margin. Households are willing to absorb the balance sheet costs as long as the marginal real return on deposits, $\frac{R^D}{\Pi}$, is greater than the return on storage, equal to one. In contrast, since banks operate competitively with a zero marginal-profit condition, they are not willing to absorb losses at the margin.

In panel A, the locus of points corresponding to households' equilibrium aggregate demand for bonds, $B_4^{H,D}$, becomes downward sloping in the region corresponding to positive balance sheet costs. The downward slope follows that of the bank's demand curve for deposits. The decrease in deposit returns decreases households' reservation rate for bonds, since the bond return must be equal to deposit return in equilibrium. However, as long as the return on deposits remains above inflation, the real return on deposits is greater than the return on storage. The locus of points corresponding to households' aggregate supply of deposits, $D_4^{H,S}$, remains vertical in panel B, equal to the share of wealth that is not held in bonds. The quantity of bank lending is unchanged from the benchmark case of zero balance sheet costs.

Finally, for large enough reserves and balance sheet costs, the net real deposit rate hits a zero real lower bound that leads to a reduction in real bank lending and disinflation. This occurs when the nominal deposit return, as given by $R^D = R(L) - C(D)$, falls to the nominal lower bound given by households' option to store goods at the nominal level of inflation instead of holding deposits. At this point, according to households' first-order condition (14), the real return on deposits is equal to that of storage, $\frac{R^D}{\Pi} = 1$. The nominal

**Figure 4. Moderate Level of Reserves and Moderate Balance Sheet Costs**




**Notes:**

**A. Bond Market.** For implied deposit quantities in the region where banks have positive balance sheet costs, a household's bond demand $B^{H,D}$ slopes downward from left to right. $B_4^{H,D}$ corresponds to the bank's downward-sloping demand for deposits $D_4^{B,D}$ in panel B. This again reflects a household's indifference between holding bonds or deposits, which in equilibrium must have equal returns that are determined in the deposit market according to the bank's deposit demand.

**B. Deposit Market.** The locus of points corresponding to households' aggregate inelastic supply of deposits $D_4^{H,S}$ increases by the decreased amount of household bonds. In the region of positive balance sheet costs, a bank's marginal balance sheet cost $C(D)$ increases with balance sheet size $D$ and lowers the bank's demand for deposits $D^{B,D}$ below $R^M$ by $C(D)$, pushing the balance sheet cost onto depositors. As a result, when reserves increase to $M_4$, households' bonds decrease to $B_4^H$, and deposits increase to $D_4$, which decreases the return on deposits and bonds to $R_4^D = R_4^B = R^M - C(D_4)$.

*(continued)*

**(Notes for figure 4, continued):**

**C. Loan Market.** With a moderate level of reserves and bank balance sheet costs, loans, lending returns, and inflation again remain unchanged. Deposits can increase with reserves because deposit returns are able to absorb the balance sheet costs.

return on deposits cannot fall below the level of inflation, because that would imply a real return on deposits below one: $\frac{R^D}{\Pi} < 1$. Households would prefer only to store goods.

Together, these constraints imply that bank lending will be reduced to a level such that the net marginal real rate of return on loans equals the marginal real bank balance sheet cost:

$$r(L) - 1 = c(D). \tag{19}$$

Lending in the economy can increase only to the point that the marginal real productivity of loans over that of the opportunity cost of storage equals the marginal real balance sheet cost, which is the real banking cost of intermediating loans. When reserves, $M$, are so large that (19) holds, reserves partially crowd out bank lending. This crowding-out effect is independent of the central bank's interest rate policy $(R^M - 1)$, as seen by the condition for no crowding out to occur, which is that the real return on deposits is greater than one: $\frac{R^D}{\Pi} = r(L) - c(D) > 1$.

PROPOSITION 3. *For a large enough supply of reserves ($M$) and balance sheet costs, $c(D)$, the return on deposits ($R^D$) and bonds ($R^B$) decreases to equal the level of inflation ($\Pi$). Bank lending ($L$) and inflation ($\Pi$) are decreasing in the quantity of reserves ($M$).*

The volume of loans continues to be determined according to $R(L) = R^M$. The nominal return on loans remains constant, since by $R(L) = \Pi r(L)$, the lower level of inflation offsets the higher marginal real return on the lower level of loans. Regardless of how high balance sheet costs are, a bank is always indifferent between holding marginally more loans or reserves, and so the returns are equal. The bank chooses its optimal amount of reserves according to a demand curve for reserves. The central bank chooses a quantity of reserves to supply, which is a point on the bank's demand curve and hence satisfies the bank's optimal demand. We have endogenized the lower bound on deposit returns by including the household's option to store goods. This implies that in equilibrium, the net real rate of return on deposits, $\frac{R^D}{\Pi} - 1$, cannot fall below zero (or equivalently that the real return on deposits of $\frac{R^D}{\Pi}$ cannot fall below one). Without the availability of storage, the quantity of lending would not be affected by the quantity of reserves or the size of balance sheet costs.

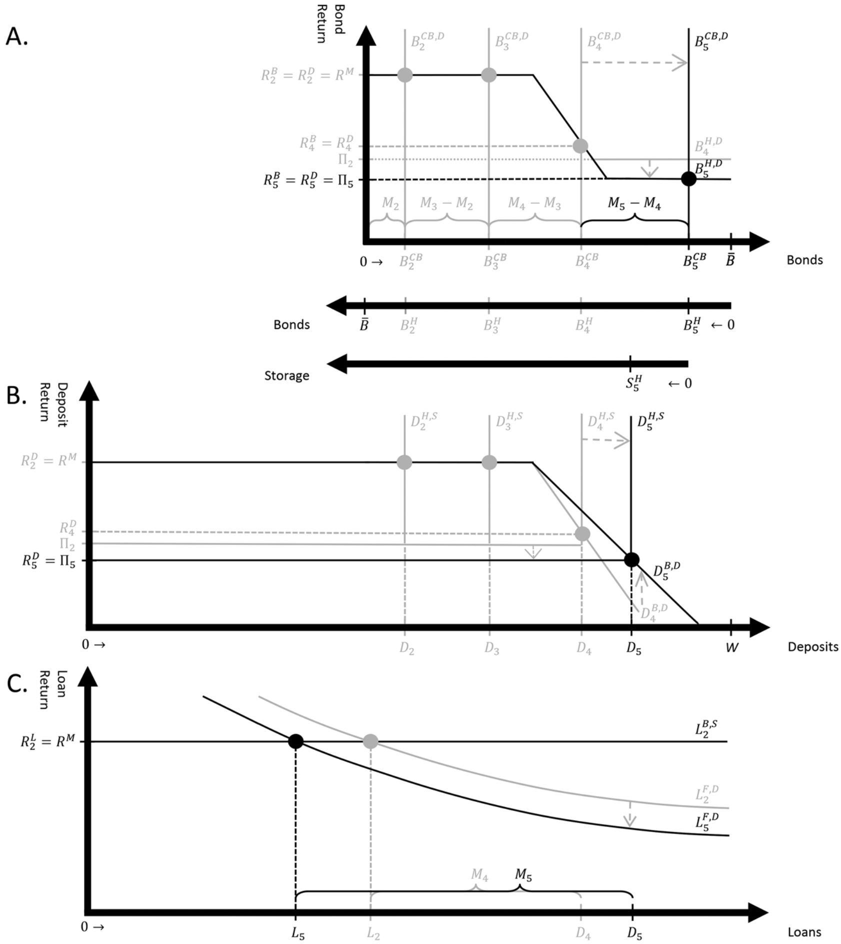
Bank loans are equal to deposits held in excess of reserves, $L = D - M$. As shown in figure 5, when bank balance sheet costs and reserves are large enough that $R^D$ is at the lower bound, at the margin a unit of reserves increases balance sheet costs by $c(D)$. Constraint (19) requires a corresponding increase in the real return on loans and hence a reduction in lending. This decrease is held in storage by households. In sum, an additional unit of reserves purchases a unit of bonds for the central bank from the households. Households hold a fraction of the unit of wealth as deposits and the remaining fraction as storage. Since banks can fund only a fraction of the additional unit of reserves with the additional fraction of deposits, banks must decrease lending by the remaining fraction.

The return on government bonds, $R^B$, also falls along with the return on deposits, $R^D$, to the lower nominal bound of the inflation level, $\Pi$. The return on deposits and bonds is below the return on reserves by the balance sheet cost wedge of $1 + c(D)$: $R^D = \frac{R^M}{1+c(D)} < R^M$.

In this case of large reserves crowding out bank lending, large reserves have the further effect of creating disinflation. Decreased lending raises the marginal real return on loans, which requires inflation to be lower following the determination $\Pi = \frac{R^M}{r(L)}$ than what it would be without large balance sheet costs and large reserves. The decrease in inflation from fewer loans is partially but not fully offset by the increase in marginal real balance sheet costs from a larger balance sheet in determining the lower equilibrium return on deposits: $R^D = R^M - \Pi c(D)$. This shows that inflation can also be expressed as $\Pi = \frac{R^M}{1+c(D)}$, where $\frac{\partial \Pi}{\partial M} < 0$.

Disinflation also partially offsets the increase in real marginal balance sheet costs that accompany increased deposits, as shown by $R^D = R^M - \Pi c(D)$, since lower inflation partially mutes the increase in nominal marginal balance sheet costs. Without this effect, once at

## Figure 5. Large Level of Reserves and Large Balance Sheet Costs




**Notes:**

**A. Bond Market.** Once a household's downward-sloping bond demand $B^{H,D}$ falls to a low enough household-bond quantity $B^H$ such that it receives an equilibrium bond return that falls to the level of inflation, $R^B = \Pi$, the household's bond demand becomes perfectly price elastic at the inflation level for any smaller quantities of household bonds. This price elasticity, as also illustrated in figure 4, reflects that households prefer to hold storage $S$ for a real return of one rather than bonds for a real return of less than one, which would occur for nominal returns of $R^B < \Pi$. When the central bank demand for bonds $B^{CB,D}$ increases enough to intersect with the elastic segment of household bond demand, households begin to hold positive amounts of storage. Storage is measured on the second x-axis below the bond market graph from right to left. The zero value of the storage x-axis lines up with the quantity of bonds held by households, $B_2^H$, in order to indicate that the amount of storage held is equal to household wealth, shown by $W$ on the deposit x-axis, that is not held in bonds or deposits.

(*continued*)

**(Notes for figure 5, continued):**

**B. Deposit Market.** Households' supply of deposits $D^{H,S}$ is perfectly elastic at $R^D = \Pi$ for deposit quantities below the amount of household wealth not held in bonds or storage in equilibrium, $W - B^H - S$, as also illustrated in figures 2–4. This elasticity reflects, similarly to that of household bond demand, the preference to hold storage rather than deposits that pay a real return less than one. Hence, the increase in reserves push nominal deposit and bond returns down to the inflation level, $R_5^D = R_5^B = \Pi_5$, with their real returns hitting the real zero rate lower bound. Since households replace the decrease in their bond holdings in part with storage, the increase in household deposits is less than the increase in reserves by the amount of storage: $\Delta D = \Delta M - \Delta S$.

**C. Loan Market.** With reserves increasing by more than deposits, loans must decrease with the increase in reserves by an amount equal to the increase in storage: $\Delta L = \Delta D - \Delta M = -\Delta S$. A lower quantity of loans raises their marginal real return, which requires inflation to decrease to $\Pi_5 = \frac{R^M}{r(L_5)}$, as shown in panels A and B. Hence, for large enough balance sheet costs, increases in reserves crowd out bank lending and decrease inflation. The decrease in inflation shifts firms' nominal loan demand leftward to $L_5^{F,D}$, such that the nominal return on marginal loans $\Pi_5 r(L_5)$ equates to the unchanged return on lending $R(L_2)$ and return on reserves $R^M$. At the zero lower bound, decreasing inflation allows deposits to expand partially with increasing reserves, so loans do not fall by the entire amount of the increase in reserves. Deposits partially increase with reserves because the perfectly elastic segments of household bond demand in panel A and deposit supply in panel B shift with the decrease in inflation down to $B_5^{H,D}$ and $D_5^{H,S}$, respectively. In addition, while the return on a bank's demand for any additional deposits would otherwise fall below the level of inflation from increasing marginal real balance sheet costs, the decrease in inflation partially offsets this movement downward along $D_4^{B,D}$. This disinflation rotates the downward-sloping segment of the bank's deposit demand upward, pivoting around the kink in $D^{H,D}$, to $D_5^D$, which allows for slightly higher deposit returns, $R^D = R^M - \Pi c(D)$, and a further partial increase in deposits with reserves. (For simplicity, the corresponding upward pivot of $B^{H,D}$ with the fall in inflation in panel A is not shown.)

the zero lower bound, increases in reserves would lead households' decrease in bonds to a one-for-one increase in storage, no increase in deposits, and a one-for-one decrease in loans, for an entire crowding out of loans from additional reserves.

In this case of disinflation, the inflation rate may still be positive, in which case the deposit rate remains positive: $R^D - 1 = \Pi - 1 > 0$. However, actual deflation may also occur, in which the inflation rate $\Pi - 1$ falls below zero. Nominal deposit and bond rates, $R^D - 1$ and $R^B - 1$, respectively, would be negative. The real deposit and bond rates, $\frac{R^D}{\Pi} - 1$ and $\frac{R^D}{\Pi} - 1$, respectively, would remain equal to zero.

### *3.3 Discussion*

We can discuss several potential extensions that lie outside the formal model, including the effect of macroeconomic shocks on bank lending, bank heterogeneity, and historical regimes for reserves.

To start, we examine how shifts in parameters can affect bank lending. First, we consider an increase in loan demand driven by a productivity shock. We compare the effect of an increase in the marginal real return on firms' investment up to $\tilde{r}(\cdot) > r(\cdot)$ for the cases of minimal versus large sizes of reserves and balance sheet costs. With minimal reserves, an increase in productivity leads to a decrease in inflation since $R(\cdot) = \Pi r(\cdot) = R^M$. The marginal nominal return of bank lending is unchanged and there is no change in lending.

With large reserves and balance sheet costs, an increase in real productivity to $\tilde{r}(\cdot) > r(\cdot)$, for a given level of loans $L$, increases the left-hand side of equation (19). There is an increase in equilibrium loans to $\tilde{L}$, which moderates the equilibrium increase in productivity to $\tilde{r}(\tilde{L})$. There is an increase in deposits, $\tilde{D} - D = \tilde{L} - L$, and in bank balance sheet costs $c(\tilde{D})$, to the point that equation (19) holds: $\tilde{r}(\tilde{L}) - 1 = c(\tilde{D})$. The increase in loans is supported by a decrease in household storage of $S - \tilde{S} = \tilde{L} - L$. This shows that an increase in loan demand driven by a positive real productivity shock leads to an increase in bank lending. However, the increase in the equilibrium marginal real return on loans to $\tilde{r}(\tilde{L})$ is complemented by a decrease in inflation to $\tilde{\Pi}$, because firms' marginal nominal return on loans, $\Pi\tilde{r}(\tilde{L})$, is tied to the return on lending, $R^L$, and return on reserves, $R^M$: $\Pi\tilde{r}(\tilde{L}) = R^L = R^M$. Again, we find overall that $\tilde{R}(\tilde{L}) = R^L$; the marginal nominal return on loans is unchanged.

Next, we consider an increase in loan demand that is driven by an increase in households' demand. We examine the effect of a increase in household wealth up to $\tilde{W} > W$, when there is a low or moderate size of reserves and balance sheet costs. The increase in wealth leads to an increase in households' supply of deposits and an increase in inflation, which shifts out firms' demand for loans, lowering firms' real return on investment. The increase in equilibrium deposits, loans, and inflation is given by $(\tilde{D} - D) = (\tilde{L} - L) = (\tilde{W} - W)$ and $(\tilde{\Pi} - \Pi) = \left(\frac{R^L}{r(\tilde{L})} - \frac{R^L}{r(L)}\right)$.

By using comparative statics, the model in essence allows for analyzing an instantaneous adjustment of deposits and loans regardless of the level of bank reserves. However, in practice, a lower velocity of money is required for a banking system with a higher level of reserves than for one with a lower level of reserves. The banking sector lends out of the quantity of reserves it holds, $M$, to firms that buy goods from households, who deposit the reserves in the banking system. The reserves have to turn over $\frac{\tilde{L}-L}{M}$ times for an increase in deposits up to $\tilde{D}$ and in loans up to $\tilde{L}$. Outside of the model, if there is heterogeneity among banks, it may take some time or cost for the adjustment process of banks that have suddenly increased lending opportunities to attract deposits or interbank loans. A higher quantity of reserves requires a lower velocity of money and may lead to a slightly faster increase in lending in response to a sudden increase in loan demand. Hence, the level of reserves could affect the speed at which equilibrium levels of lending would adjust to shocks in the economy. For either driver of increased loan demand above, we see that faster adjustment cost speeds that may result from larger reserve levels produce more efficient outcomes.

These adjustment effects may also provide insight into the consideration of the extreme heterogeneity of the banking sector in the United States. We model a representative bank that makes a representative type of loan to firms. In reality, banks in the United States vary tremendously in many features, including bank size, sources of deposits, and focus of lending (for instance, see Janicki and Prescott 2006). For example, banks provide commercial and industrial loans, real estate loans, and consumer loans. While aggregate reserves in the banking system are fixed by the Federal Reserve, the distribution of reserves among banks is not fixed and may depend on bank size, deposit sources, and lending focus. Outside of the model, we can consider that such variation among banks may lead to different speeds of adjustment to changes in bank borrowing and lending. For example, banks that have greater access to wholesale deposits can increase or decrease borrowing and hence lending faster than banks that rely more on retail deposits. However, we do not expect that variation among bank types or the speed of adjustments of bank borrowing and lending would lead to a significant change in our equilibrium results.

We can also use the model to compare the current regime of interest on reserves with past regimes. Historically, central banks used a reserve requirement ratio in order to create a demand for reserves on which they did not pay interest and to control the amount of bank loans through the money multiplier. For a reserve requirement ratio of $\rho$, the money multiplier is $\frac{1}{\rho}$. With a supply of reserves $(M)$ as chosen by the central bank, and under a binding money multiplier constraint, the banking sector could hold a maximum amount of deposits equal to $D = \frac{M}{\rho}$ and provide a maximum amount of loans equal to $L = \left(\frac{1-\rho}{\rho}\right) M$. Over time, most central banks have either eliminated reserve requirements entirely or have allowed banks to largely avoid them, such as through sweep accounts in the United States. Our model of bank lending, with interest on reserves and no meaningful reserve requirement, shows that the money multiplier is no longer relevant. Banks take deposits and lend to the point that the marginal return on loans $R(L)$ equals the return $R^M$ paid by the central bank on reserves, the banks' alternative asset.

In past regimes that did not pay interest on reserves, reserve requirements were considered to impose a "tax" on banks. This tax is the return that banks had to forgo by holding required reserves that paid no return, equal to $\rho DR(L)$. In comparison, under a policy of interest on reserves, banks no longer face the tax on required reserves. However, with a large quantity of reserves in the banking system, banks face the potential additional balance sheet costs from large levels of reserves, equal to $\int_D C(\hat{D})d\hat{D} - \int_L C(\hat{L})d\hat{L}$. Relative to the implicit tax on the modest level of required reserves that did not receive interest in past regimes, the balance sheet costs from reserves that do receive interest in the current regime would be smaller in times of low levels of reserves but would likely be much greater in times of high levels of reserves.

## 4. Conclusion

Perhaps because of its novelty, the large quantity of reserves in the banking system has generated a great amount of concern and debate. However, there is little analysis of how reserves affect bank lending when interest is paid on reserves. This paper presents a model of the current U.S. banking system that includes interest on reserves and no binding reserve requirements. The exercise is important because of expressed concerns that large reserves could lead to excessive lending by banks, despite little formal analysis of the issue.

We develop a complete yet parsimonious framework by fully specifying a general equilibrium economy with several competitive sectors and a closed system of reserves and payments within the banking system. We study households' supply of deposits and demand for bonds and consumption goods; firms' demand for loans and supply of consumption goods; and banks' supply of loans and demand for deposits, bonds, and reserves. While we consider a representative competitive price-taking bank, it would be interesting in future research to consider banks that are not fully price taking, such as banks that may have some monopoly power on deposits and loans.

We show that without frictions, the amount of lending is independent of the amount of reserves in the banking system. We also demonstrate that the quantity of reserves is determined by the Federal Reserve and does not provide any measure of the willingness of banks to lend. We have kept our model simple and elementary in order to illustrate that the key determinant of bank lending is not fundamentally affected by the quantity of reserves. This point has been obscured by the traditional textbook model of the money multiplier, which, while simple, is not an elementary model. Rather, that model assumes that a particular constraint—namely, the money multiplier—is always binding.

Our conclusion is likely to hold in more sophisticated models. While we cannot exclude the possibility that a more complicated model would overturn this result, economists concerned that large reserves will generate excessive lending should articulate precisely which frictions in a banking model will necessarily lead to this result. In contrast to such concerns, we study a friction under which the quantity of reserves could crowd out bank lending and lead to a decrease in inflation. Banks may have increasing costs in the size of their balance sheets because of agency costs or regulatory requirements on capital or leverage. Under such a friction, the effect of large reserves is contractionary rather than expansionary.

## Appendix. Proofs

*Proof of Proposition 1*

We will show that if $c(D) = 0$, then there exists an equilibrium $(\mathbb{Q}, \mathbb{R})$ where $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$, which is unique up to the allocation of bonds between households and banks.

In any equilibrium, equations (1), (3), and (2) must be satisfied. The central bank's choice of reserves supply and the return on these reserves requires that $R^M = \bar{R}^M$ and $M = \bar{M}$. We first show that there does exist an equilibrium with $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$. Consider $R^M = R^L = R^D = R^B = \bar{R}^M$. We have banks indifferent between holding bonds, reserves, and loans, and households indifferent between holding deposits and bonds. Consider $\Pi$ such that $r(\bar{L}) = \frac{R^L}{\Pi}$. By (A1) such a $\Pi$ exists and $\Pi < R^L$. Now consider $D = \bar{L} + \bar{M}$, $B^B = 0$, $B^{CB} = \bar{M}$, $B^H = \bar{B} - \bar{M}$, $L = \bar{L}$, and $S = 0$. Clearly these quantities satisfy individual optimizations at the given prices, are non-negative given (A2), and clear the market. Thus, this is an equilibrium at $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$.

To see how $\Pi$ is determined, note that in equilibrium, real investment by firms plus real storage by households $s$ must be equal to the difference between real wealth $w$ and the government's choice of real bonds $b$ to issue. This difference is a fixed amount, given the quantity of reserves $M$ issued by the central bank, the real balance sheet cost function $c(D)$, and the real marginal return on firm investment $r(L)$. $\Pi$ must adjust so that the solution to the firms' problem, given by (5), is exactly that amount of real investment by firms.

To show uniqueness we argue that $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$ must hold in any equilibrium, and that $L = \bar{L}$ and $S = 0$ in any equilibrium. This will imply that all equilibria are unique up to the allocation of bonds between households and firms since in equilibrium $M = \bar{M}$. Since $r(L) > 1$, we must have $R^L > \Pi$ in any equilibrium; otherwise, firms' first-order conditions could never be satisfied. (A1) also requires that $L > 0$, which in turn implies that $R^M = R^L \geq R^B$ since $\bar{M} > 0$. Also, we must have $R^M = R^L = R^D$, for inequality would imply that banks would demand either zero or infinite quantities of deposits. Market clearing in the bond market then requires that $R^M = R^L = R^D = R^B$. Since we always must have $R^M = \bar{R}^M$, we have that $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$ in any equilibrium. Now $R^D = R^B > \Pi$ directly implies that $S = 0$, which in turn implies that $L = \bar{L}$ since households must expend their entire wealth. In sum, we have that any potential equilibrium must have $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$, $L = \bar{L}$, and $S = 0$. Thus, the equilibrium is unique up to the allocation of bonds between households and firms.

*Proof of Proposition 2*

We will show that if $c(D) > 0$ and $c(\bar{M} + \bar{L}) \leq r(\bar{L}) - 1$, then there exists a unique equilibrium $(\mathbb{Q}, \mathbb{R})$ where $L = \bar{L}$ and $R^M = R^L = \bar{R}^M > R^D = R^B \geq \Pi$.

Because of (A3), (12), and (15) we must have $R^L > R^D$. (A1) requires $L > 0$ and (A2) requires $M > 0$ in equilibrium, thus we must have $R^M = \bar{R}^M = R^L$. Once again, market clearing in the bond market then requires that $R^D = R^B \geq \Pi$. In sum, we have that $R^M = \bar{R}^M = R^L > R^D = R^B \geq \Pi$. Now, we show that there is an equilibrium with $L = \bar{L}$. We first find an $R^D$ and $\Pi$ such that $R^D = R^L - \Pi c(\bar{M} + \bar{L})$ and $\Pi = \frac{R^L}{r(\bar{L})}$; i.e., consumption of $\bar{L}$ must be optimal for both banks and firms. As (A3) guarantees that $\bar{L} > 0$, we have that $\Pi > 0$. Thus, $R^D = R^L - \left(\frac{R^L}{r(\bar{L})}\right)(c(\bar{M} + \bar{L})) < R^L$. Furthermore, $c(\bar{M} + \bar{L}) \leq r(\bar{L}) - 1$ implies that $R^D \geq \frac{R^L}{r(\bar{L})} = \Pi$. Finally, setting $R^D = R^B$, we have an equilibrium where $\mathbb{Q} = (\bar{M}, \bar{L}, \bar{L} + \bar{M}, \bar{M}, B - \bar{M}, 0)$. To see that this is unique, consider a potential equilibrium loan quantity $L' \neq \bar{L}$. Clearly, $L' < \bar{L}$, but this implies that $S > 0$, since $R^M = R^L = \bar{R}^M > R^B$ implies that banks will not hold bonds and households need to expend all of their wealth. $S > 0$ implies that $R^D = R^B = \Pi$. However, if $c(\bar{M} + \bar{L}) \leq r(\bar{L}) - 1$, then $c(\bar{M} + L') < r(L') - 1$, which implies that $R^D > \Pi$ for $L'$ to be optimal loan consumption for both banks and firms. Thus, $L'$ cannot be an equilibrium, and any potential equilibrium must have $L = \bar{L}$. Clearly, if $L = \bar{L}$ in equilibrium, then the only quantity vector that would clear the market is $\mathbb{Q}$. Thus, the equilibrium quantity vector is unique.

*Proof of Proposition 3*

We will show that if $c(\bar{M} + \bar{L}) > r(\bar{L}) - 1$, then there exists a unique equilibrium $(\mathbb{Q}, \mathbb{R})$, where $L < \bar{L}$, $R^M = R^L = \bar{R}^M > R^D = R^B = \Pi$, $\frac{\delta L}{\delta M} < 0$, and $\frac{\delta \Pi}{\delta M} < 0$.

Consider $L$ such that $c(\bar{M}+L)=r(L)-1$. Such an $L$ exists and is greater than zero by (A3). Since $c(\bar{M}+\bar{L})>r(\bar{L})-1$, $L<\bar{L}$. This $L$ is optimally demanded by both banks and firms when $R^D=\Pi$. Since $R^D=R^L-\left(\frac{R^L}{r(L)}\right)(c(\bar{M}+L))<R^L$ for $L>0$, we must have $R^M=\bar{R}^M=R^L>R^D=R^B=\Pi$. Now consider $\mathbb{Q}=(\bar{M},L,L+\bar{M},\bar{M},\bar{B}-\bar{M},W-(\bar{B}-\bar{M})-L)$. $\mathbb{Q}$ obviously clears the market at $R^M=\bar{R}^M=R^L>R^D=R^B=\Pi$. To see that $\mathbb{Q}$ is a unique quantity vector, it suffices to show that $L$ is the only potential equilibrium loan quantity, for then market clearing would imply all other quantities would have to equate with $\mathbb{Q}$. Consider some $L^{'}\neq L$. $L^{'}>L$ would imply that $\Pi>R^D$ for $L^{'}$ to be optimal for both banks and firms, so $L^{'}>L$ cannot be an equilibrium. $L^{'}<L$ implies that $\Pi<R^D$ for $L^{'}$ to be optimal for both firms and banks. But this would imply that $S=0$, and $L^{'}$ would not clear the market since $L^{'}<L<\bar{L}$. So $L^{'}$ cannot be an equilibrium loan quantity, and the only potential equilibrium loan quantity is $L$. Through implicit differentiation, we have $\frac{\delta L}{\delta M}=\frac{c^{'}(D)}{r^{'}(L)-c^{'}(D)}<0$ by (A1). Similarly, we have $\frac{\delta\Pi}{\delta M}=\frac{[-R^L(c^{'}(D))(1+\frac{\delta L}{\delta M})]}{[1+c(D)]^2}$. Clearly, $\left|\frac{\delta L}{\delta M}\right|<1$, so we have that $\frac{\delta\Pi}{\delta M}<0$.

## References


Allen, F., E. Carletti, and D. Gale. 2011. "Money, Financial Stability and Efficiency." Working Paper.

Ashcraft, A., J. McAndrews, and D. Skeie. 2011. "Precautionary Reserves and the Interbank Market." *Journal of Money, Credit and Banking* 43 (s2): 311–48.

Bennett, P., and S. Peristiani. 2002. "Are U.S. Reserve Requirements Still Binding?" *Economic Policy Review* (Federal Reserve Bank of New York) 8 (1).

Carlson, M., H. Shan, and M. Warusawitharana. 2011. "Capital Ratios and Bank Lending: A Matched Bank Approach." Finance and Economics Discussion Paper No. 2011-34, Board of Governors of the Federal Reserve System.

Carpenter, S., and S. Demiralp. 2012. "Money, Reserves, and the Transmission of Monetary Policy: Does the Money Multiplier Exist?" *Journal of Macroeconomics* 34 (1): 59–75.

Diamond, D. W., and R. G. Rajan. 2006. “Money in a Theory of Banking.” *American Economic Review* 96 (1): 30–53.

———. 2009. “Illiquidity and Interest Rate Policy.” NBER Working Paper No. 15197.

Ennis, H. M. 2014. “A Simple General Equilibrium Model of Large Excess Reserves.” Working Paper No. 14-14, Federal Reserve Bank of Richmond.

Ennis, H. M., and T. Keister. 2008. “Understanding Monetary Policy Implementation.” *Economic Quarterly* (Federal Reserve Bank of Richmond) (Summer): 235–63.

Ennis, H. M., and A. L. Wolman. 2015. “Large Excess Reserves in the United States: A View from the Cross-Section of Banks.” *International Journal of Central Banking* 11 (1): 251–89.

Fisher, R. W. 2009. “Excerpts from Remarks Before the Texas Christian University Business Network of Dallas.” September 29.

Freixas, X., A. Martin, and D. Skeie. 2011. “Bank Liquidity, Interbank Markets, and Monetary Policy.” *Review of Financial Studies* 24 (8): 2656–92.

Janicki, H. P., and E. S. Prescott. 2006. “Changes in the Size Distribution of U.S. Banks: 1960–2005.” *Economic Quarterly* (Federal Reserve Bank of Richmond) 92 (4): 291–316.

Keister, T., A. Martin, and J. McAndrews. 2008. “Divorcing Money from Monetary Policy.” *Economic Policy Review* (Federal Reserve Bank of New York) 14 (2).

Keister, T., and J. McAndrews. 2009. “Why Are Banks Holding So Many Excess Reserves?” *Current Issues in Economics and Finance* (Federal Reserve Bank of New York) 15 (December).

Martin, A. 2006. “Liquidity Provision vs. Deposit Insurance: Preventing Bank Panics without Moral Hazard.” *Economic Theory* 28 (1): 197–211.

Martin, A., J. McAndrews, A. Palida, and D. Skeie. 2013. “Federal Reserve Tools for Managing Rates and Reserves.” Staff Report No. 642, Federal Reserve Bank of New York.

Meltzer, A. H. 2010. “The Fed’s Anti-Inflation Exit Strategy Will Fail.” *Wall Street Journal,* January 27. Available at http://www.wsj.com/articles/SB10001424052748704375604575023632319560448.

Plosser, C. I. 2011. “Exit.” Speech at the Shadow Open Market Committee Symposium, New York, New York, March 25.

Skeie, D. 2004. "Money and Modern Bank Runs." Dissertation, Princeton University.

———. 2008. "Banking with Nominal Deposits and Inside Money." *Journal of Financial Intermediation* 17 (4): 562–84.

Wrightson ICAP. 2008. *Money Market Observer*, October 6.

———. 2009. *Money Market Observer,* January 12.