QE 1 vs. 2 vs. 3. . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool*

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We introduce large-scale asset purchases (LSAPs) as a monetary policy tool within a macroeconomic model. We allow for purchases of both long-term government bonds and securities with some private risks. We argue that LSAPs should be thought of as central bank intermediation that can affect the economy to the extent there exist limits to arbitrage in private intermediation. We then build a model with limits to arbitrage in banking that vary countercyclically and where the frictions are greater for private securities than for government bonds. We use the framework to study the impact of LSAPs that have the broad features of the different quantitative easing (QE) programs the Federal Reserve pursued over the course of the crisis. We find that (i) LSAPs work in the model in a way mostly consistent with the evidence; (ii) purchases of securities with some private risk have stronger effects than purchases of government bonds; (iii) the effects of the LSAPs depend heavily on whether the zero lower bound is binding. Our model does *not* rely on the central bank having a more efficient intermediation technology than the private sector: We assume the opposite.

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1. Introduction

For the last fifty years or so, the primary tool of monetary policy has been the federal funds rate. During the recent crisis, however, the Federal Reserve unveiled a variety of new policy measures never used before in its history. What forced its hand initially was the disruption of credit markets in the wake of the deterioration of the subprime mortgage market, which began in August of 2007. By December of 2008, however, a second factor came into play: The funds rate effectively reached its zero lower bound, implying that, despite the severity of the recession, the conventional option of reducing the funds rate was no longer available. Beyond managing expectations of the future path of the funds rate, the new unconventional measures afforded the Federal Reserve the only avenue for stimulating the economy.

Because of their dramatic impact on the size of the Federal Reserve's balance sheet, the most visible of the new policy measures have been large-scale asset purchases (LSAPs), known more generally as quantitative easing (QE). Shortly after the meltdown of the shadow banking system that followed the Lehman failure in September 2008, the Federal Reserve initiated what is now known as QE1: the purchase over time of a variety of high-grade securities, including agency mortgage-backed securities (AMBS), agency debt, and long-term government bonds, with AMBS ultimately accounting for the bulk of the purchases. It also set up a commercial paper lending facility, which effectively involved the purchase of commercial paper, since the Federal Reserve accepted these instruments as collateral for the loans made to the facility. In October 2010, the Federal Reserve announced a second wave of asset purchases (QE2), this time restricted to long-term government bonds and smaller in scale than QE1. Finally, in September 2011, the Federal Reserve embarked on a variation of QE, known as Operation Twist. This action was essentially a sterilized acquisition of long-term government bonds financed by selling some of its short-term bonds. Twist was extended in the summer of 2012 and the possibility of more LSAPs remains under discussion.

A lengthy empirical literature has emerged attempting to identify the effects of the LSAP programs on market interest rates and economic activity.[1] Though not without considerable controversy, a common theme of this research is that the LSAPs have indeed been effective in reducing various interest rates and interest rate spreads and, as a consequence, in stimulating economic activity. In addition, the weight of the evidence also suggests that QE1 was more effective in this regard than either QE2 or Operation Twist.

At the same time, given the descriptive nature of much of this empirical work, the precise mechanism through which LSAPs may have affected the economy remains an open question. So too is a theoretical account for why QE1 may have had different effects than the subsequent LSAP programs. In addition to being larger in scale, QE1 differed from the other LSAPs in several other important respects. First, the asset purchases involved securities with at least some degree of private payoff risk, whereas QE2 and Operation Twist were restricted to the acquisition of government bonds. In addition, QE1 was undertaken at the height of the crisis when financial markets and institutions were under maximum duress. By contrast, QE2 and Operation Twist were undertaken in periods of greater normalization of credit markets. Exactly which of these factors could account for differences in the impact of various LSAP programs has yet to be resolved.

The purpose of this paper is to develop a macroeconomic model that presents a unified approach to analyzing LSAPs as a monetary policy tool. A number of papers have analyzed specific types of LSAPs. For example, Curdia and Woodford (2011), Del Negro et al. (2011), Gertler and Karadi (2011), and Williamson (2012) have considered central bank purchases of imperfectly secured private claims, as in QE1. Others have considered purchases of long-term government bonds, such as Chen, Curdia, and Ferrero (2011) and Vayanos and Vila (2009). The mechanisms emphasized both within and across these strands of work have been somewhat different. Our goal is to present a single framework that can be used to analyze the impact of LSAPs across the variety that are used in practice.

[1] See, for example, Baumeister and Benati (2010), Chung et al. (2011), D'Amico and King (2010), Doh (2010), Gagnon et al. (2011), Gambacorta, Hoffman, and Peersman (2011), Hamilton and Wu (2010), Hancock and Passmore (2011), Krishmamurthy and Vissing-Jorgensen (2011), Stroebel and Taylor (2009), Williams (2011), and Wright (2011).

As in Gertler and Karadi (2011; GKa) and Gertler and Kiyotaki (2011; GKi), we start from the perspective that LSAPs reflect central bank intermediation. Like any private intermediary, the central bank funds asset acquisition by issuing interest-bearing short-term claims. In the early stages of QE1 the Federal Reserve raised funds by issuing short-term government debt it borrowed from the Treasury. Shortly thereafter, it made use of its recently acquired power to pay interest on reserves. It funded subsequent expansion of its balance sheet by issuing interest-bearing reserves, which can be thought of as overnight government debt. Seen from this vantage, it is clear that LSAPs can usefully affect real activity only to the extent there exist limits to arbitrage in private financial intermediation. If an extranormal return on a particular asset is present, one would expect private intermediaries to expand their balance sheets to eliminate this premium, so long as they do not face any constraints in borrowing. In this instance, as we will make clear, central bank intermediation of the asset is neutral: It does not affect asset prices and returns; it simply displaces private intermediation.

If, however, private intermediaries are constrained in their ability to borrow, LSAPs can matter. The advantage the central bank has is that it is able to obtain funds elastically by issuing riskless government debt. It is this advantage in borrowing over private intermediaries that introduces a role for central bank intermediation in reducing excess returns. In this regard, as GKa show, the net benefits from LSAPs can be positive even if the central bank is less efficient than the private sector in intermediating the assets, so long as this efficiency differential is not "too large." Further, these net benefits are likely to be increasing in a financial crisis, since in this instance limits to private arbitrage are likely to be unusually tight.

Along these lines, one can interpret QE1 as the Federal Reserve increasing central bank intermediation to offset the disruption of private intermediation brought about by the demise of the shadow banking system.[2] Indeed, the assets it purchased were held largely by the financial institutions that had devolved into distress. Further, given that various measures of credit spreads suggested that excess

[2]Here it is interesting to note that Ben Bernanke used the term "credit easing" to describe the first round of LSAPs. We think this is a more accurate term than quantitative easing. See, for example, Bernanke (2009).

returns were at a peak in the wake of the Lehman collapse, the expected gains from central bank intermediation were likely largest at this point. It is this kind of reasoning about the effect of QE1 that our model will capture.

We will also argue that a similar logic applies to the purchase of long-term government bonds. Absent limits to arbitrage in the private sector, central bank exchanges of short-term for long-term government debt should be neutral. To the extent that credit market frictions give rise to an extranormal term premium in the market for government bonds, there is scope for LSAPs to reduce long-term rates.[3] The way they reduce long-term rates is by reducing inefficiently large term premia. Of course, one should expect limits to arbitrage to be weaker in markets for government bonds than for private securities. We incorporate this feature in our model. The net effect is that a dollar purchase of government bonds has a weaker effect on excess returns than a dollar purchase of private sector assets. This accords with the conventional wisdom that the liquidity of the government bond market makes purchases of this asset less effective, everything else equal, than purchases of less liquid assets such as AMBS or commercial paper.

An alternative view of how LSAPs affect the economy stresses household asset demands that are less sensitive to returns than a standard frictionless model might predict, due to factors such as a "preferred habitat" for particular maturities.[4] Given these frictions in asset demand, changes in asset supplies in the private sector brought about by LSAPs affect prices and returns of competing assets. What matters, however, is the behavior of the marginal investors, which in reality are likely to be leveraged financial intermediaries.

Indeed, as table 1 shows, in 2008 leveraged financial institutions held significant fractions of the types of assets ultimately acquired under the various LSAP programs, including roughly 45 percent of the AMBS outstanding, 40 percent of the agency debt, and 16

[3]For financial institutions borrowing from the Federal Reserve using ten-year government bonds as collateral, there is a 4 percent haircut. One would expect that private lenders require a larger haircut on these bonds, suggesting at least some degree of friction in the market.

[4]See, for example, Chen, Curdia, and Ferrero (2011) and the references therein.

Table 1. Asset Holdings of Leverage and Non-Leveraged Institutions

	Domestic Net Assets (Billions, $)			%	
	All	**Leveraged**	**Non-Lev'd**	**Leveraged**	**Non-Lev'd**
Total Mortgages	**14336**	**6170**	**8166**	**43.0%**	**57.0%**
Agency MBS	3590	1567	2023	43.6%	56.4%
Treasuries and Agency Debt	**4195**	**1325**	**2870**	**31.6%**	**68.4%**
Treasuries	1876	312	1563	16.7%	83.3%

Source: Flow of Funds, December 2008.
Notes: Leveraged institutions include commercial banks, savings banks, credit unions, brokers and dealers, government-sponsored enterprises, and finance companies. Agency debt and agency MBS are not published separately for different financial intermediaries. We assume each are held proportionally to their combined holdings by leveraged and non-leveraged institutions.

percent of the government debt.[5] Thus, any characterization of how LSAPs affect the economy must take into account the behavior of these institutions. In this respect, even if household demands for long-maturity assets are "excessively inelastic," arbitrage by private intermediaries could render central bank purchases of long-term government bonds neutral. We clarify this point within our formal analysis.

Section 2 presents the key elements of our model. We derive a set of qualitative results regarding how LSAPs affect the economy. Section 3 adds the production sector and then characterizes the complete equilibrium. Section 4 then presents some numerical experiments to illustrate the impact of LSAPs. Here we emphasize the implications of purchases of securities with private risks versus long-term government bonds. We also consider the implications of the zero lower bound and also compare LSAPs with conventional interest rate adjustments. Concluding remarks are in section 6.

[5]Following Greenlaw et al. (2008), we define leveraged institutions as those intermediaries whose equity capital is only a small fraction of the assets they hold and whose liabilities consist mostly of short-term debt.

2. The Model: Key Ingredients

The framework is based on GKa. It is a reasonably standard New Keynesian model modified to allow for banks that transfer funds from households to non-financial firms, as well as to the government. An agency problem constrains the ability of banks to obtain funds from households. It ultimately makes the balance sheet of the banking sector a critical determinant of the cost of credit that borrowers face. One difference from GKa is that banks may intermediate the funding of long-term government bonds as well the funding of non-financial firms.

In addition, there is a central bank that can conduct monetary policy either by adjusting the short-term interest rate (so long as the zero lower bound is not binding) or by engaging in asset purchases. The central bank may purchase long-term government bonds as well as private securities.

In this section we characterize the distinctive elements of the model, which involve the behavior of households, banks, and the central bank. We defer a description of the production sector and complete equilibrium to the next section. For didactic reasons, we start with the case where banks intermediate all the funding of non-financial firms and long-term government bonds. This simple setup allows us to starkly illustrate some of the key results regarding the effects of central bank asset purchases. We then subsequently allow households to directly hold long-term securities subject to transaction costs and then draw out the implications of this more general setting.

In the interest of parsimony, we abstract from a number of the features present in conventional quantitative DSGE models that are not central to understanding the effects of central bank asset purchases (e.g, variable capital utilization, wage rigidity, price and wage indexation, etc.). However, we include two standard features—habit formation and flow investment adjustment—because they can be added at minimal cost of complexity and they substantially improve the model's quantitative performance.

Finally, we should make clear that we do not attempt to develop a model that can provide a comprehensive description of recent events. We do not include an explicit housing sector, nor do we try to model asset bubbles, etc. Rather, our goal is to formulate a macroeconomic model to help understand how LSAPs might work in a setting that has some of the key features of the current crisis.

2.1 Households

There is a continuum of identical households of measure unity. Each household consumes, saves, and supplies labor. Households save by lending funds to competitive financial intermediaries and possibly also by lending funds to the central bank.

Within each household there are two types of members: workers and bankers. Workers supply labor and return the wages they earn to the household. Each banker manages a financial intermediary and similarly transfers any earnings back to the household. The household thus effectively owns the intermediaries that its bankers manage. It deposits funds, however, in intermediaries that is does not own. Finally, within the family there is perfect consumption insurance. As will become clear, this simple form of heterogeneity within the family allows us to introduce financial intermediation in a way that maintains much of the tractability of a representative agent framework.

At any moment in time, the fraction $1 - f$ of the household members are workers and the fraction f are bankers. Over time an individual can switch between the two occupations. In particular, a banker this period stays a banker next period with probability σ, which is independent of history. The average survival time for a banker in any given period is thus $\frac{1}{1-\sigma}$. We introduce a finite horizon for bankers to insure that over time they do not retain earnings to the point where they can fund all investments from their own capital. Thus every period $(1 - \sigma)f$ bankers exit and pay out their retained earnings as dividends to their respective household. The bankers who exit become workers and are replaced by a similar number of workers randomly becoming bankers, keeping the relative proportion of each type fixed. The household, though, provides its new bankers with a small amount of startup funds equal to $\frac{X}{(1-\sigma)f}$ per new banker.

Let C_t be consumption and L_t family labor supply. Then the household's discounted utility u_t is given by

$$u_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \right], \qquad (1)$$

with $0 < \beta < 1$, $0 < h < 1$, and $\chi, \varphi > 0$. As in Woodford (2003), we consider the limit of the economy as it become cashless, and thus ignore the convenience yield to the household from real money balances.

Both intermediary deposits and government debt are one-period real bonds that pay the gross real return R_t from $t-1$ to t. In the equilibrium we consider, the instruments are both riskless and are thus perfect substitutes. Thus, we impose this condition from the outset. Thus let D_{ht} be the total quantity of short-term debt the household acquires, W_t be the real wage, Π_t payouts to the household from ownership of both non-financial and financial firms, and T_t lump-sum taxes. Then the household budget constraint is given by

$$C_t = W_t L_t + \Pi_t - X + T_t + R_t D_{ht-1} - D_{ht}, \tag{2}$$

where X is the total transfer the household gives to its members that enter banking at t. Finally, as will be clear later, it will not matter in our model whether households hold government debt directly or do so indirectly via financial intermediaries (that in turn issue deposits to households.)

The household's objective is to choose C_t, L_t, and D_{ht} to maximize (1) subject to (2). Let $u_{C_t t}$ denote the marginal utility of consumption. Then the first-order conditions for labor supply and consumption/saving are standard:

$$u_{C_t} W_t = \chi L_t^{\varphi} \tag{3}$$

$$E_t \Lambda_{t,t+1} R_{t+1} = 1 \tag{4}$$

with

$$\Lambda_{t,t+1} \equiv \beta \frac{u_{C_{t+1}}}{u_{C_t}}.$$

2.2 Banks

Banks lend funds obtained from households to non-financial firms and to the government. In addition to acting as specialists that assist in channeling funds from savers to investors, they engage in maturity transformation. They hold long-term assets and fund these assets with short-term liabilities (beyond their own equity capital).[6] In addition, financial intermediaries in this model are meant to capture the entire banking sector, i.e., investment banks as well as commercial banks.

Intermediaries fund two type of activities: First, they make loans to non-financial firms to finance capital. Let Z_t be the net period income flow to the bank from a loan that is financing a unit of capital, Q_t the market value of the security, δ the depreciation rate of a unit of capital, and ξ_t a random disturbance. Then the rate of return to the bank on the loan, R_{kt+1}, is given by

$$R_{kt+1} = \frac{Z_{t+1} + (1-\delta)Q_{t+1}}{Q_t}\xi_{t+1}. \tag{5}$$

The variables Z_t, Q_t, and ξ_t are determined in the general equilibrium of the model, as we show later.

In addition, banks hold long-term government bonds. Here we suppose that it is too costly for households to directly manage long-term bonds in their portfolios. As we noted earlier, we relax this assumption by permitting households to directly hold long-term securities subject to explicit transaction costs. For our benchmark model, however, banks intermediate all the funding of long-term bonds. We assume each bond is a perpetuity that pays one dollar per period indefinitely. Let q_t be the price of the bond and P_t the price level. Then the real rate of return on the bond R_{bt+1} is given by

$$R_{bt+1} = \frac{1/P_t + q_{t+1}}{q_t}. \tag{6}$$

The general equilibrium also determines P_t and q_t.

2.2.1 *The Bank's Maximization Problem*

Let n_t be the amount of equity capital—or net worth—that a banker/intermediary j has at the end of period t, d_t the deposits the intermediary obtains from households, s_t the quantity of financial

[6] In Gertler and Kiyotaki (2011), we consider a generalization of this framework that has banks manage liquidity risks (stemming from idiosyncratic shocks to firm investment opportunities) via an interbank market. In this setup, financial frictions may also affect the functioning of the interbank market.

claims on non-financial firms that the intermediary holds, and b_t the quantity of long-term government bonds. The intermediary balance sheet is then given by

$$Q_t s_t + q_t b_t = n_t + d_t. \tag{7}$$

Net worth is accumulated through retained earnings. It is thus the difference between the gross return on assets and the cost of liabilities:

$$n_t = R_{kt} Q_{t-1} s_{t-1} + R_{bt} q_{t-1} b_{t-1} - R_t d_{t-1}. \tag{8}$$

The banker's objective is to maximize the discounted stream of payouts back to the household, where the relevant discount rate is the household's intertemporal marginal rate of substitution, $\Lambda_{t,t+i}$. Under frictionless capital markets the timing of the payouts is irrelevant. To the extent the intermediary faces financial market frictions, it is optimal for the banker to retain earnings until exiting the industry. Accordingly, the banker's objective is to maximize expected terminal wealth, given by

$$V_t = E_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1} \Lambda_{t,t+i} n_{t+i}. \tag{9}$$

To motivate a limit on the bank's ability to obtain deposits, we introduce the following moral hazard/costly enforcement problem: At the beginning of the period, the banker can choose to divert funds from the assets it holds and transfer the proceeds to the household of which he or she is a member.[7] The cost to the banker is that the depositors can force the intermediary into bankruptcy and recover the remaining fraction of assets. However, it is too costly for the depositors to recover the funds that the banker diverted.

We assume that it is easier for the bank to divert funds from its holdings of private loans than from its holding of government bonds: In particular, it can divert the fraction θ of its private loan portfolio and the fraction $\Delta\theta$ with $0 \leq \Delta < 1$ from its government

[7] One way the banker may divert assets is to pay out large bonuses and dividends to the household.

bond portfolio. Here we are attempting to capture in a simple way that the bank's private loan portfolio is likely an easier target for bank malfeasance than its government bond portfolio, given that it is more difficult for depositors to monitor the performance of the latter than the former.[8]

Accordingly, for depositors to be willing to supply funds to the banker, the following incentive constraint must be satisfied:

$$V_t \geq \theta Q_t s_t + \Delta \theta q_t b_t. \tag{10}$$

The left side is what the banker would lose by diverting a fraction of assets. The right side is the gain from doing so.

The banker's maximization problem is to choose s_t, b_t, and d_t to maximize (9) subject to (7), (8), and (10).

2.2.2 Solution

Let λ_t be the Lagrange multiplier associated with the incentive constraint (10) and let $\widetilde{\Lambda}_{t,t+1}$ be the bank's "augmented" stochastic discount factor, equal to the product $\Lambda_{t,t+i}$ and the multiplier Ω_{t+1}:

$$\widetilde{\Lambda}_{t,t+1} \equiv \Lambda_{t,t+1} \cdot \Omega_{t+1}, \tag{11}$$

where the Ω_{t+1} reflects the shadow value of a unit of net worth to the bank at $t+1$, as we make clear shortly. Then we can characterize the solution as follows:

The expected excess returns on bank assets satisfy[9]

$$E_t \widetilde{\Lambda}_{t,t+1}(R_{kt+1} - R_{t+1}) = \frac{\lambda_t}{1+\lambda_t}\theta \tag{12}$$

$$E_t \widetilde{\Lambda}_{t,t+1}(R_{bt+1} - R_{t+1}) = \Delta \cdot \frac{\lambda_t}{1+\lambda_t}\theta. \tag{13}$$

[8]A more explicit approach to motivating weaker limits to arbitrage for long-term government bonds would be to allow for default risk on private securities in a way that enhances the agency friction. For parsimony, we stick with our simple friction as a way to motivate differential arbitrage limits (stemming from balance sheet constraints).

[9]Here we use the term "excess return" to refer to the difference between the discounted return and what its value would be under frictionless markets. This is different from the standard use in finance, where the term reflects the premium due to risk (within a frictionless market setup).

When the incentive constraint is not binding, the discounted excess returns are zero. With $\lambda_t = 0$, $\forall\ t$, financial markets are frictionless: Banks acquire assets to the point where the discounted return on each asset equals the discounted cost of deposits. Further, in this case Ω_{t+1} equals unity, so that for each asset the standard arbitrage condition under perfect markets arises: The expected product of the households' intertemporal marginal rate of substitution and the excess return equals zero.

When the incentive constraint is binding, however, limits to arbitrage emerge that lead to positive excess returns in equilibrium. The excess returns increase with how tightly the incentive constraint binds, as measured by the multiplier λ_t. Note that the excess return to capital implies that for a given riskless interest rate, the cost of capital is higher than it would otherwise be. As a consequence, investment and real activity will be lower than they would otherwise be in general equilibrium. Indeed, a financial crisis in the model will involve a sharp increase in the excess return to capital.

Notice also that the excess return on government bonds is smaller than the excess return on loans by the multiple $\Delta < 1$. This occurs because the proportion of funds a bank can divert from its bond portfolio is only the fraction Δ of the proportion it can divert from its loan portfolio. As a result, the incentive friction that limits arbitrage is weaker for government bonds than for loans by the factor Δ.

The limits to arbitrage stem from the following restriction that the incentive constraint places on the size of a bank's portfolio relative to its net worth:

$$\begin{aligned} Q_t s_t + \Delta q_t b_t &= \phi_t n_t \text{ if } \lambda_t > 0; \\ &< \phi_t n_t \text{ if } \lambda_t = 0 \end{aligned} \tag{14}$$

with

$$\phi_t = \frac{E_t \widetilde{\Lambda}_{t,t+1} R_{t+1}}{\theta - E_t \widetilde{\Lambda}_{t,t+1}(R_{kt+1} - R_{t+1})}. \tag{15}$$

The measure of assets that enters the bank's balance sheet constraint applies a weight of Δ to government bonds, reflecting the weaker constraint on arbitrage for this asset than for loans. As the bank expands this adjusted measure of assets by issuing deposits, its incentive to divert funds increases. The constraint (14) limits the portfolio size to the point where the bank's incentive to cheat is exactly balanced by the cost of losing the franchise value. In this respect the agency problem leads to an endogenous capital constraint.

Observe that ϕ_t is the maximum ratio of the adjusted measure of assets to net worth that the bank may hold without violating the incentive constraint. It depends inversely on θ; an increase in the bank's incentive to divert funds reduces the amount depositors are willing to lend. Conversely, an increase in the discounted excess return on assets, $E_t\widetilde{\Lambda}_{t,t+1}(R_{kt+1}-R_{t+1})$, or the discounted safe rate, $E_t\widetilde{\Lambda}_{t,t+1}R_{t+1}$, increases the franchise value of the bank, V_t, reducing the bank's incentive to divert funds. Depositors thus become willing to lend more, raising ϕ_t.

Finally, the weight Ω_{t+1} that augments the bank's discount factor is the marginal value of net worth averaged across exiting and continuing states:

$$\Omega_{t+1} = 1 - \sigma + \sigma\frac{\partial V_{t+1}}{\partial n_{t+1}} \tag{16}$$

with

$$\frac{\partial V_t}{\partial n_t} = E_t\widetilde{\Lambda}_{t,t+1}[(R_{kt+1} - R_{t+1})\phi_t + R_{t+1}].$$

With probability $1 - \sigma$, the bank exits and has a marginal value of net worth of unity since it simply transfers its retained earnings to the household. With probability σ, it continues and uses the net worth to expand its asset base. So long as the excess returns on assets are positive, the marginal value $\frac{\partial V_t}{\partial n_t}$ exceeds unity.

2.2.3 Aggregation

Let S_{pt} be the total quantity of loans that banks intermediate, B_{pt} the total number of government bonds they hold, and N_t their total net worth. Since neither component of the maximum adjusted leverage ratio ϕ_t depends on bank-specific factors, we can simply sum across the portfolio restriction on each individual bank (14) to obtain

$$Q_tS_{pt} + \Delta q_tB_{pt} \leq \phi_tN_t. \tag{17}$$

Equation (17) restricts the aggregate value of (adjusted) assets that the banking system can hold to be less than or equal to the multiple ϕ_t of total bank capital. When the constraint is binding, variation in N_t will induce fluctuations in overall asset demand by intermediaries. Indeed, in the general equilibrium of the model, a crisis will feature a sharp contraction in N_t.

Total net worth evolves as the sum of the retained earnings by the fraction σ of surviving bankers and the transfers that new bankers receive, X, as follows:

$$N_t = \sigma \left[(R_{kt} - R_t) \frac{Q_{t-1} S_{pt-1}}{N_{t-1}} + (R_{bt} - R_t) \frac{q_{t-1} B_{pt-1}}{N_{t-1}} + R_t \right] N_{t-1} + X. \tag{18}$$

The main sources of variation in N_t are fluctuations in the ex post return on loans R_{kt} and the ex post return on bonds R_{bt}. Further, the percentage impact of this return variation on N_t, in each case, is increasing in the bank's degree of leverage, reflected by the respective ratios of assets to net worth, $Q_{t-1}S_{pt-1}/N_{t-1}$ and $q_{t-1}B_{pt-1}/N_{t-1}$.

2.3 Central Bank Asset Purchases

As equations (12) and (13) suggest, if private intermediation is balance sheet constrained, excess returns on assets arise. If these constraints are particularly tight, as would be the case in a financial crisis, then excess returns will be unusually high, with negative consequences for the cost of capital and real activity. Within our model, large-scale asset purchases provide a way for the central bank to reduce excess returns and thus mitigate the consequences of a disruption of private intermediation.[10]

[10] We abstract from moral hazard considerations emphasized, for example, by Chari and Kehoe (2010) and Farhi and Tirole (2012). Gertler, Kiyotaki, and Queralto (2011) address this issue in a framework similar to the one here by allowing banks the options of issuing outside equity as well as deposits, where equity issuance is subject to agency costs. The possibility of LSAPs then reduces banks' incentives to hedge their portfolios. The precise degree is a quantitative issue. We would expect a similar outcome in the framework here but defer an explicit treatment to the future.

In particular, we now allow the central bank to purchase quantities of private loans S_{gt} and long-term government bonds B_{gt}. For each each type of security, it pays the respective market prices Q_t and q_t. However, when limits to arbitrage in the private market are operative, the central bank's acquisition of securities will have the effect of bidding up the prices on each of these instruments and down the excess returns. To finance these purchases, it issues riskless short-term debt D_{gt} that pays the safe market interest rate R_{t+1}. In particular, the central bank's balance sheet is given by

$$Q_t S_{gt} + q_t B_{gt} = D_{gt}, \tag{19}$$

where we assume that the central bank turns over any profits to the Treasury and receives transfers to cover any losses. For the time being we suppose that the central bank issues the short-term debt to households. Later we discuss an equivalent scenario where D_{gt} is interpretable as interest-bearing reserves (essentially overnight government debt) held by banks on account at the central bank.

As we discussed earlier, these kinds of asset purchases essentially involve substituting central bank intermediation for private intermediation. What gives the central bank an advantage in this situation is that, unlike private intermediaries, it is able to obtain funds elastically by issuing short-term liabilities. It is able to do so because within our framework the government can always commit credibly to honoring its debt. Accordingly, there is no agency conflict that inhibits the central bank from obtaining funds from the private sector. Put differently, in contrast to private financial intermediation, central bank intermediation is not balance sheet constrained.[11]

At the same time, we allow for the central bank being less efficient than the private sector at making loans. In particular, we assume the central bank pays an efficiency cost of τ_s per unit of private loans intermediated and τ_b per unit of government bonds. Accordingly, for asset purchases to produce welfare gains, the central bank's advantage in obtaining funds cannot be offset by its disadvantage in making loans. Its advantage in obtaining funds is greatest when

[11] As Wallace (1981) originally noted, for government financial policy to matter, it is important to identify what is special about government intermediation. Sargent and Wallace (1982) provide an early example of how credit policy could matter, based on a setting of limited participation in credit markets.

excess returns are large (i.e., when limits to private arbitrage are tight), as will be the case in a financial crises. As for its disadvantage in making loans: It is reasonable to suppose the relative efficiency cost of intermediating government bonds, τ_b, is small. For τ_s, it depends on the type of credit instrument. The types of "private loans" for which one might expect τ_s to be small include highly rated securitized assets such as agency mortgage-backed securities as opposed to commercial and industrial loans that involve extensive monitoring. Accordingly, it is the former type of instrument we have in mind in characterizing central bank purchases of private securities as opposed to the latter.

The way asset purchases affect the real economy is ultimately by affecting the price Q_t and (hence the) excess return on capital $E_t\widetilde{\Lambda}_{t,t+1}(R_{kt+1} - R_{t+1})$. Accordingly, let S_t and B_t be the total supplies of private loans and long-term government bonds, respectively. Then by definition,

$$\begin{aligned} S_t &= S_{pt} + S_{gt} \\ B_t &= B_{pt} + B_{gt}, \end{aligned} \tag{20}$$

where, as before, S_{pt} and B_{pt} are the total amounts that are privately intermediated. We combine these identities with the balance constraint on the banks to obtain the following relation for the total value of private securities intermediated, Q_tS_t:

$$Q_tS_t \leq \phi_tN_t + Q_tS_{gt} + \Delta(q_tB_{gt} - q_tB_t). \tag{21}$$

When the aggregate balance sheet constraint is not binding, asset prices and returns are determined by frictionless arbitrage. Asset purchases by the central bank of either private loans or long-term bonds are neutral. They simply lead to central bank intermediation displacing some private intermediation, without any effect on asset prices. To the extent central bank intermediation involves efficiency costs, further, asset purchases are clearly welfare reducing in this kind of environment.

This neutrality result disappears, however, if the constraint is binding. Given the total quantity of bank equity, an increase in the central bank's holding of either private securities or long-term government bonds raises the total demand for private securities.

Intuitively, with limits to arbitrage present on private credit flows, central bank intermediation expands overall asset demand and does not simply displace bank intermediation one for one. Further, given that asset supplies are relatively inelastic in the short run, the enhanced asset demand pushes up Q_t and pushes down the excess return on capital.

Equation (21) also reveals that it matters which asset the central bank acquires. In particular, purchases of government bonds will have a weaker effect on the demand for private assets than would the direct purchase of this asset by the factor $\Delta < 1$. Intuitively, the central bank acquiring government bonds frees up less bank capital than does the acquisition of a similar amount of private loans. It is effectively by freeing up intermediary capital that asset purchases are able to expand the overall demand for private assets. In the limiting case of frictionless arbitrage in the government bond market (i.e., $\Delta = 0$), bond purchases have no effect.

Purchases of either asset affect the excess returns of both due to the arbitrage relation implied by equations (12) and (13):

$$E_t\widetilde{\Lambda}_{t,t+1}(R_{bt+1} - R_{t+1}) = \Delta E_t\widetilde{\Lambda}_{t,t+1}(R_{kt+1} - R_{t+1}). \tag{22}$$

As we noted earlier, though, because limits to arbitrage are weaker for government bonds than for private securities, the excess return on the former is only the fraction Δ of the excess return on the latter. Thus, everything else equal, in the wake of an asset purchase, government bond yields should move by less than the yield on private securities. This should hold regardless of which asset the central bank purchases.

Finally, up to this point we have assumed that the central bank funds asset purchases by issuing short-term debt directly to households. An equivalent formulation has the central bank issue the debt directly to banks, which in turn fund this activity by issuing deposits to households. The short-term government debt that banks absorb, further, can take the form of interest-bearing reserves held on account at the central bank, as was the case in practice for the most part. Assuming that the agency friction does not apply to intermediating reserves, the bank will not be constrained in its funding of this asset. Thus, as in the baseline scenario, the central bank is able to elastically issue short-term liabilities to fund its asset purchases. It is straightforward to show that the equilibrium conditions in the scenario are identical to those in the baseline case. The identical balance sheet constraint on bank asset holdings applies.

Along these lines, it does not matter whether the central bank finances asset purchases by issuing short-term liabilities or by selling some of its holdings of short-term government debt, so long as its short-term assets and liabilities are in effect perfect substitutes. Thus for example, purchases of long-term government bonds financed by interest-bearing reserves, as occurred under QE2, are equivalent to purchases financed by selling holdings of short-term Treasury bills, so long as the Treasury bills and interest-bearing reserves are close substitutes. In either case, the central bank is expanding the amount of long-term government bonds funded by short-term government debt. Also, how the asset purchase works in either case depends on the same set of considerations: the extent of limits to arbitrage in private markets.

2.4 Allowing for Direct Household Securities Holdings

We now permit households to directly hold private securities and long-term government bonds. However, we introduce limits on household participation by assuming transaction costs. Absent these costs, households would engage in frictionless arbitrage of asset returns.

We suppose that for private securities a household faces a holding cost equal to the percentage $\frac{1}{2}\kappa(S_{ht}-\overline{S}_h)^2/S_{ht}$ of the value of the securities in its respective portfolio for $S_{ht} \geq \overline{S}_h$. Similarly, for government bonds there is a holding cost equal to the percentage $\frac{1}{2}\kappa(B_{ht}-\overline{B}_h)^2/B_{ht}$ of the total value of government bonds held for $B_{ht} \geq \overline{B}_h$. Accordingly, there is a certain amount of each asset that the household can hold costlessly. Going above these levels involves transaction costs which are increasing at the margin. We motivate this cost structure as capturing in a simple way limited participation in asset markets by households that leads to incomplete arbitrage.

Accordingly, the household budget constraint becomes

$$C_t + D_{ht} + Q_t[S_{ht} + \frac{1}{2}\kappa(S_{ht}-\overline{S}_h)^2] + q_t[B_{ht} + \frac{1}{2}\kappa(B_{ht}-\overline{B}_h)^2]$$
$$= W_tL_t + \Pi_t + T_t + R_tD_{ht-1} + R_{kt}S_{ht-1} + R_{bt}B_{ht-1}.$$

Resolving the household's optimization yields the same first-order conditions for labor supply and deposits as before. The choices for private securities and long-term government bonds are given by

$$S_{ht} = \overline{S}_h + \frac{E_t \Lambda_{t,t+1}(R_{kt+1} - R_{t+1})}{\kappa} \tag{23}$$
$$B_{ht} = \overline{B}_h + \frac{E_t \Lambda_{t,t+1}(R_{bt+1} - R_{t+1})}{\kappa}.$$

Demand for each asset above its frictionless capacity level is increasing in the excess return relative to the respective curvature parameter that governs the marginal transaction cost. Note that as marginal transaction costs go to zero, excess returns disappear: Households are able to engage in frictionless arbitrage of security returns. Conversely, as marginal transaction costs go to infinity, households' asset demands go to their respective frictionless capacity values, $\overline{S}_h$ and $\overline{B}_h$.

Overall, one can view the household asset demand structure as a parsimonious way to capture two important forms of heterogeneity that are absent from the model. First, in reality, a sizable fraction of non-financial firms are able to obtain funds by issuing securities directly to households on the open market and do not have to borrow directly from banks. These firms are typically large, well-established entities, in contrast to younger and smaller non-financial borrowers that typically require the kind of evaluation and monitoring services that banks offer. Second, households differ in their ability to manage a sophisticated portfolio: A limited supply of "sophisticated" households accordingly prevents frictionless arbitrage of security returns by the household sector. In practice, both forms of heterogeneity help explain why both private and government securities holdings are divided between households and banks. Our model provides a very simple way to account for this pattern of asset holdings that is meant to be a stand-in for a more explicit treatment.

With households directly participating in securities markets, the equilibrium conditions in the markets for private loans and government bonds now require

$$S_t = S_{pt} + S_{ht} + S_{gt} \tag{24}$$
$$B_t = B_{pt} + B_{ht} + B_{gt}.$$

To understand the implications for central bank asset purchases, note that with direct household participation in securities markets we can rewrite the aggregate bank portfolio constraint (21) as

$$Q_t(S_t - S_{ht}) \leq \phi_t N_t + Q_t S_{gt} + \Delta q_t [B_{gt} - (B_t - B_{ht})], \tag{25}$$

with S_{ht} and B_{ht} given by (23). The portfolio constraint is now a restriction on the total demand for securities net the quantity held by households.

In this general case, the effects of asset purchases on prices and excess returns depend on the responsiveness of households as well as bank portfolios to arbitrage opportunities. Consider first the case where the marginal transaction costs facing the household are infinity (i.e., $\kappa = \infty$). In this instance, a household holds the respective frictionless capacity value of each asset, $\overline{S}_h$ and $\overline{B}_h$, and is completely unresponsive to arbitrage opportunities. Here the analysis is very similar to the simple case of no direct household participation analyzed in section 2.2. If the portfolio constraint on banks is not binding, then, as before, banks adjust their asset holding to drive excess returns to zero. Even though households cannot absorb additional securities, they are willing to absorb deposits which do not involve transaction costs. Given that banks are free to arbitrage returns, central bank asset purchases are neutral. An increase in either S_{gt} or B_{gt} simply leads to a one-for-one reduction in private bank intermediation of the respective security without any impact on prices or returns.

If the portfolio constraint binds then, as in the simple case of section 2.2, asset purchases increase the net demand for private securities. The presence of inelastic household security demands further strengthens the effects of a given size purchase of either security. It does so by reducing the participation of the active traders in the market (in this case the banks). Because everything else equal, the purchases are larger relative to bank holdings of the respective asset, they will have a larger impact on prices and returns. These results are consistent with the fact that asset prices depend on asset supplies if household demand is relatively inelastic (e.g., for "preferred habitat" reasons.) We stress, however, that it is also key that arbitrage by the active traders in the market is limited. Absent the balance sheet constraint on banks, asset purchases would be neutral despite inelastic asset demands by households.

As household security demands become increasingly elastic (κ moves toward zero), the effects of central bank asset purchases weaken. As before, assuming total supplies of each asset are inelastic in the short run, central bank purchases of either security will place downward pressure on excess returns. A decline in excess returns, however, reduces households' security holdings, dampening the overall effect of the purchases on asset demands. Put differently, household asset demands move in a way that offsets the effect of central bank asset purchases. This offsetting effect becomes stronger as transaction costs become smaller. In the limiting case of zero transaction costs, of course, households are able to perfectly arbitrage and central bank asset purchases are neutral.

In sum, for central bank asset purchases to affect asset prices and returns, limits to arbitrage must be present for both households and banks.

2.5 Long-Term Bond Yields

We have argued that the effects of LSAPs are transmitted to the real economy via their impact on excess returns (relative to a frictionless benchmark). Popular discussions of LSAPs, however, emphasize the impact on long-term bonds rates and various credit spreads. The empirical literature has followed this direction by studying the effects of LSAPs on these variables. Of course, another relevant consideration in focusing on the behavior of these yields is that excess returns are not directly observable.

Within our model the government bond is a consol that pays a dollar in perpetuity. Let $R^n_{bt+1+i} \equiv R_{bt+1+i} \cdot \frac{P_{t+1+i}}{P_{t+i}}$ be the ex post gross nominal return on this security from $t+1+i$. Then we can express the nominal price $P_t q_t$ as the following discounted sum:

$$P_t q_t = \sum_{i=1}^{\infty} \frac{1}{E_t \Pi^i_{j=1} R^n_{bt+j}}. \tag{26}$$

To understand the impact of LSAPs on long-term bond yields, it is useful to define R^{n*}_{bt+j} as the ratio of nominal return in the absence of credit market frictions, everything else equal, and define

$\Psi_{t+j} = R^n_{bt+j}/R^{n*}_{bt+j}$ as the ratio of nominal return to its "frictionless value." We can express the discounted return as

$$P_t q_t = \sum_{i=1}^{\infty} \frac{1}{E_t \Pi_{j=1}^{i} \Psi_{t+j} R^{n*}_{bt+j}}, \tag{27}$$

where discount factors depend on the expected sequence of excess returns measured by Ψ_{t+j}. Finally, we compute the nominal (net) yield to maturity as the constant per-period nominal discount rate i^n_{bt} that yields the same nominal value as the consol, given the same sequence of coupon payments:

$$\sum_{s=1}^{\infty} \frac{1}{(1+i^n_{bt})^s} = \sum_{i=1}^{\infty} \frac{1}{E_t \Pi_{j=1}^{i} \Psi_{t+j} R^{n*}_{bt+j}}. \tag{28}$$

To a first order, we can decompose the movement in i^n_{bt} into terms reflecting the expected path of the frictionless nominal rate R^{n*}_{bt+j} and terms reflecting the excess return Ψ_{t+j}. As we saw in the previous section, LSAPs work by pushing down the component of i^n_{bt} due to expected excess returns that stem from limits to arbitrage. Absent these excess returns, LSAPs would have no effect on i^n_{bt}.

On the other hand, to the extent that long-term bond purchases are successful in pushing down excess returns, the overall impact on i^n_{bt} may be muted by an expected increase in the frictionless nominal rate. In particular, by pushing down excess returns, the LSAPs stimulate both real activity and inflation, leading to an expected future increase in short-term interest rates. It is the expected response of future short rates that dampens the overall responds of LSAPs on long-term yields.

We can similarly construct a yield to maturity for the private security. The main difference is that now the per-period payoff is the nominal dividend payment net depreciation, $[Z_{t+1} - \delta]\, P_{t+1}$. Finally, much of the evidence of LSAPs on returns is reported for securities of a given finite maturity, as opposed to consols or other kinds of infinitely lived assets. In the quantitative section we describe how we approximate the returns on shorter-maturity securities.

3. The Production Sector, Government, and Equilibrium

We now close the model by describing the non-financial production sector, government policy, and the general equilibrium.

3.1 Non-Financial Firms

There are three types of non-financial firms in the model: intermediate goods producers, capital producers, and monopolistically competitive retailers. The latter are in the model only to introduce nominal price rigidities. We describe each in turn.

3.1.1 Intermediate Goods Producers

Intermediate goods producers make output that they sell to retailers. They are competitive and earn zero profits in equilibrium. Each operates a constant returns to scale technology with capital and labor inputs. Let Y_t be output, A_t total factor productivity, L_t labor, and K_t capital. Then

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \tag{29}$$

Let P_{mt} be the relative price of intermediate goods. Then the firm's demand for labor is given by

$$W_t = P_{mt}(1-\alpha)\frac{Y_t}{L_t}. \tag{30}$$

It follows that we may express gross profits per unit of capital Z_t as follows:

$$Z_t = P_{mt}\alpha\frac{Y_t}{K_t}. \tag{31}$$

The acquisition of capital works as follows. At the end of any period t, the intermediate goods producer is left with a capital stock of $(1-\delta)K_t$. It then buys I_t units of new capital from capital producers. Its capital stock for $t+1$ is then given by

$$K_{t+1} = \xi_{t+1}[I_t + (1-\delta)K_t], \tag{32}$$

where ξ_t is a random disturbance that we refer to as a "capital quality" shock. Following the finance literature (e.g., Merton 1973), we introduce the capital quality shock as a simple way to introduce an exogenous source of variation in the return to capital.[12] It is best thought of as capturing some form of economic obsolescence, as opposed to physical depreciation.[13]

To finance the new capital, the firm must obtain funding from a bank.[14]

For each new unit of capital it acquires, it issues a state-contingent claim to the future stream of earnings from the unit: $\xi_{t+1}Z_{t+1}, (1-\delta)\xi_{t+1}\xi_{t+2}Z_{t+2}, (1-\delta)^2\xi_{t+1}\xi_{t+2}\xi_{t+3}Z_{t+3}$, etc. As we discussed earlier, banks are able to perfectly monitor firms and enforce contracts. As a result, through competition, the security the firm issues is perfectly state contingent, with producers earning zero profits state by state. In addition, the value of the security Q_t is equal to the market price of the capital underlying security. Finally, the period $t+1$ payoff is $(Z_{t+1} + (1-\delta)Q_{t+1})\xi_{t+1}$: the sum of gross profits and the value of the leftover capital multiplied by the capital quality shock, which corresponds to the definition of the rate of return in equation (5).

Before proceeding, it is worth emphasizing that the financial frictions that banks face in obtaining funds from depositors affect the

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[12] Other recent papers that make use of this kind of disturbance include Brunnermeier and Sannikov (2009), Gertler and Karadi (2011), and Gourio (2012).

[13] One way to motivate this disturbance is to assume that final output is a constant elasticity of substitution (CES) composite of a continuum of intermediate goods that are in turn produced by employing capital and labor in a Cobb-Douglas production technology. Suppose that, once capital is installed, capital is good specific and that each period a random fraction of goods become obsolete and are replaced by new goods. The capital used to produce the obsolete goods is now worthless and the capital for the new goods is not fully on line. The aggregate capital stock will then evolve according to equation (32).

[14] For simplicity only, we assume that all non-financial firms are homogenous in their access to credit. As emphasized by Bernanke, Gertler, and Gilchrist (1996) and Gertler and Gilchrist (1994), firms have heterogenous access to credit, ranging from those who must exclusively rely on bank credit to those that can meet their financing needs mainly from open-market credit. Both Bernanke and Gertler (1989) and Bernanke, Gertler, and Gilchrist (1999) present models which allow for firm heterogeneity in credit access. The latter shows that so long as there are output complementarities across firms with differential access to credit, a "heterogenous" firm model can produce cyclical dynamics similar to a "homogenous" firm model.

cost of capital to non-financial firms. As we saw in section 2.2, the capital constraints on banks limit the supply of funds they can intermediate, which raises loan rates. As we illustrate later, a financial crisis sharply tightens these capital constraints.

3.1.2 Capital Goods Producers

Capital producers make new capital using input of final output and subject to adjustment costs. They sell the new capital to firms at the price Q_t. Given that households own capital producers, the objective of a capital producer is to choose I_t to solve

$$\max E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_\tau^i I_\tau - \left[1 + f\left(\frac{I_\tau}{I_{\tau-1}} \right) \right] I_\tau \right\}. \tag{33}$$

From profit maximization, the price of capital goods is equal to the marginal cost of investment goods production as follows:

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - E_t \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right). \tag{34}$$

Profits (which arise only outside of steady state) are redistributed lump sum to households.

3.1.3 Retail Firms

Final output Y_t is a CES composite of a continuum of mass unity of differentiated retail firms that use intermediate output as the sole input. The final output composite is given by

$$Y_t = \left[\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{35}$$

where Y_{ft} is output by retailer f.

Retailers simply repackage intermediate output. It takes one unit of intermediate output to make a unit of retail output. The marginal cost is thus the relative intermediate output price P_{mt}. We introduce nominal rigidities following Calvo. In particular, each period, a firm is able to freely adjust its price with probability $1 - \gamma$. Accordingly, each firm chooses the reset price P_t^* to maximize expected discounted profits subject to the restriction on the adjustment frequency. Following standard arguments, the first-order necessary condition for this problem is given by

$$\sum_{i=0}^{\infty} \gamma^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} - \mu P_{mt+i} \right] Y_{ft+i} = 0, \tag{36}$$

with $\mu = \frac{1}{1-1/\varepsilon}$. From the law of large numbers, the following relation for the evolution of the price level emerges:

$$P_t = \left[(1-\gamma)(P_t^*)^{1-\varepsilon} + \gamma (P_{t-1})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{37}$$

3.2 Government Policy

Government expenditures are composed of government consumption, which we hold fixed at G, and the net interest payments from an exogenously fixed stock of long-term government debt, which we set at $\overline{B}$. Revenues consist of lump-sum taxes and the earnings from central bank intermediation net transaction costs. As discussed in section 2.3, central bank asset purchases are financed by short-term government debt. Given the central bank balance sheet (19), we can express the consolidated government budget constraint as

$$\begin{aligned} G + (R_{bt} - 1)\overline{B} = T_t &+ (R_{kt} - R_t - \tau_s) Q_{t-1} S_{gt-1} \\ &+ (R_{bt} - R_t - \tau_b) q_{t-1} B_{gt-1}. \end{aligned} \tag{38}$$

We suppose monetary policy is characterized by a simple Taylor rule. Let i_t be the net nominal interest rate, i the steady-state nominal rate, and Y_t^* the natural (flexible-price equilibrium) level of output. Then

$$i_t = i + \kappa_\pi \pi_t + \kappa_y (\log Y_t - \log Y_t^*) + \epsilon_t, \tag{39}$$

where ϵ_t is an exogenous shock to monetary policy, and where the link between nominal and real interest rates is given by the following Fisher relation:

$$1 + i_t = R_{t+1} \frac{P_{t+1}}{P_t}. \tag{40}$$

We suppose that the interest rate rule is sufficient to characterize monetary policy in normal times. In a crisis, however, we allow for large-scale asset purchases. In particular, we suppose that at the onset of a crisis, which we define loosely to mean a period where excess returns rise sharply, the central bank purchases the fraction φ_{st} of the outstanding stock of private securities and the fraction φ_{st} of the outstanding stock of long-term government bonds:

$$S_{gt} = \varphi_{st} S_t \tag{41}$$

$$B_{gt} = \varphi_{st} B_t,$$

where both φ_{st} and φ_{bt} obey second-order stationary stochastic processes. In the next section we clarify how the central bank intervenes in a crisis with asset purchases.

3.3 Resource Constraint and Equilibrium

Output is divided between consumption, investment, government consumption, and expenditures on central bank intermediation Φ_t. The economy-wide resource constraint is thus given by

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + G + \Phi_t \tag{42}$$

with $\Phi_t = \tau_s Q_{t-1} S_{gt-1} + \tau_g q_{t-1} B_{gt-1}$.

Finally, to close the model, we require market clearing in markets for private securities, long-term government bonds, and labor. The supply of private securities at the end of period t is given by the sum of newly acquired capital I_t and leftover capital $(1 - \delta) K_t$:

$$S_t = I_t + (1 - \delta) K_t. \tag{43}$$

The supply of long-term government bonds is fixed by the government:

$$B_t = \overline{B}. \tag{44}$$

Finally, the condition that labor demand equals labor supply requires that

$$(1 - \alpha)\frac{Y_t}{L_t} \cdot E_t u_{Ct} = \frac{1}{P_{mt}} \chi L_t^{\varphi}, \tag{45}$$

where the inverse of the price of intermediate goods $\frac{1}{P_{mt}}$ is effectively the retail goods price markup. As we show, this markup can rise in a crisis, enhancing the contraction in employment.

We note that because of Walras's Law, once the market for goods, labor, and long-term securities has cleared, the market for riskless short-term debt will be cleared automatically. As we discussed, the short-term government debt may either be directly held by households or instead by banks, which in turn issue deposits to households. In the latter case, one can interpret the debt as interest-bearing reserves. This completes the description of the model.

4. Model Analysis

Our goal here is to provide concrete numerical examples to illustrate the qualitative insights about the effects of LSAPs developed in section 2.

4.1 Calibration

Table 2 lists the choice of parameter values for our baseline model. Overall there are twenty parameters. Twelve are conventional. Eight $(\sigma, \theta, \Delta, X, \overline{K}^h, \overline{B}^h, \kappa, \overline{B})$ are specific to our model.[15]

We begin with the conventional parameters. For the depreciation rate δ, the capital share α, the elasticity of substitution between goods, ε, and the government expenditure share, we choose standard values. For the discount factor β we assign a quarterly value of 0.995, which implies a steady-state short-term interest rate of 2 percent. Following the literature on the zero lower bound (ZLB), we choose a low steady-state real rate to increase the likelihood that the ZLB is binding in the crisis experiment that we study. For the other conventional parameters we use estimates from Primiceri, Schaumburg, and Tambalotti (2006) to obtain values. These parameters include the habit parameter, h; the inverse elasticity of investment to the price of capital, η_i; the relative utility weight on labor, χ; the Frisch

[15]We simply fix the parameters which reflect the efficiency costs of central bank intermediation, τ_b and τ_s, equal to zero since under reasonable values they do not affect model dynamics. They will matter for welfare calculations, but we do not do these here. See Gertler and Karadi (2011) for an analysis.

Table 2. Parameters

Households		
β	0.995	Discount rate
h	0.815	Habit parameter
χ	3.482	Relative utility weight of labor
B/Y	0.450	Steady-state Treasury supply
$\overline{K}^h/K$	0.500	Proportion of direct capital holdings of the households
$\overline{B}^h/B$	0.750	Proportion of long-term Treasury holdings of the households
κ	1.000	Portfolio adjustment cost
φ	0.276	Inverse Frisch elasticity of labor supply
Financial Intermediaries		
θ	0.345	Fraction of capital that can be diverted
Δ	0.500	Proportional advantage in seizure rate of government debt
X	0.0062	Transfer to the entering bankers
σ	0.972	Survival rate of the bankers
Intermediate Goods Firms		
α	0.330	Capital share
δ	0.025	Depreciation rate
Capital-Producing Firms		
η_i	1.728	Inverse elasticity of net investment to the price of capital
Retail Firms		
ε	4.167	Elasticity of substitution
γ	0.779	Probability of keeping the price constant
Government		
$\frac{G}{Y}$	0.200	Steady-state proportion of government expenditures
κ_π	1.500	Inflation coefficient in the Taylor rule
κ_X	−0.125	Markup coefficient in the Taylor rule

elasticity of labor supply, φ^{-1}; and the price rigidity parameter, γ. Since the policy rule the authors estimate is somewhat non-standard, we use instead the conventional Taylor rule parameters of 1.5 for the feedback coefficient on inflation, κ_π, and 0.5 for the output-gap coefficient, κ_y. For simplicity, we use minus the price markup as a proxy for the output gap.

Our choice of the financial sector parameters is meant to be suggestive. We choose a survival probability $\sigma = 0.975$ that implies an expected horizon of ten years for bankers. We set $\overline{K}^h$ so that in steady state, households hold half the quantity of private securities, and $\overline{B}^h$ so that households hold three-quarters of the outstanding stock of long-term government debt. We choose values for θ, Δ, and X to hit the following targets: a steady-state excess return on government bonds of 50 basis points, a steady-state excess return on private securities of 100 basis points, and a steady-state leverage ratio for banks of six. We base the steady-state target for the excess return on bonds on estimates of the term premium by Ludvigson and Ng (2009) using pre-crisis data. For private securities we use information on the pre-2007 spreads between mortgage rates and government bonds and between BAA corporate versus government bonds, in conjunction with the evidence on the term premium. The steady-state leverage ratio is trickier to calibrate. For investment banks and commercial banks, which were at the center of the crisis, leverage ratios (assets to equity) were extraordinarily high—typically in the range of fifteen to twenty for the former and eight to ten for the latter. However, everything else equal, our model overstates the risk from asset-price fluctuations that banks face since they essentially hold equity claims: In practice, creditors share more of the risk with banks. Accordingly, we compensate by assuming a steady-state leverage ratio that is roughly half the average across banking institutions. The reduced leverage ratio dampens the impact of asset-price fluctuations on bank net worth. We choose the household portfolio adjustment cost parameter κ so that the model approximates the evidence on the impact of effect of QE2 on both real activity and the ten-year bond rate. It will turn out that $\kappa = 1$ reasonably satisfies this requirement. Finally, $\overline{B}$ is set to have the ratio of the stock of long-term government bonds to (steady-state) output equal to its pre-crisis value of approximately 0.45.

Finally, to be consistent with much of the evidence of LSAPs, within our simulations we report the behavior of yields to maturity of ten-year bond rates on securities that have equivalent value to the respective infinite horizon claims in the model. In the case of the long-term government bond, we consider a ten-year equivalent government debt that has an identical price to the consol in the baseline model but a slightly different payoff structure: For the first forty quarters it yields a coupon payment identical to the consol (i.e., unity per period). For the quarter after, there is a "principal" payment equal to the nominal steady-state price of the consol $q_{ss}^n = 1/(R^n - 1)$, where R^n is the steady-state nominal interest rate. The nominal yield to maturity on the ten-year government bond i_{bt}^* is accordingly

$$P_t q_t = \sum_{s=1}^{40} \frac{1}{(1 + i_{bt}^*)^s} + \frac{q_{ss}^n}{(1 + i_{bt}^*)^{40}}, \tag{46}$$

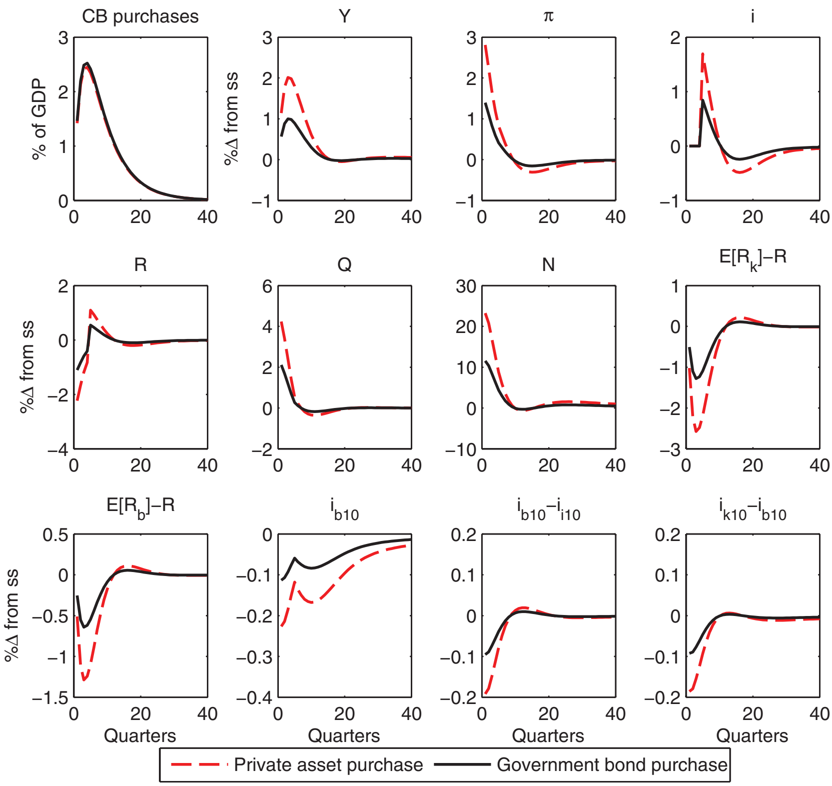
where $P_t q_t$ is the nominal price of the bond (equal to the price of the equivalent consol) and q_{ss}^n is the terminal payment of the bond. The yield to maturity on the analogous ten-year private security is given by

$$P_t Q_t = E_t \sum_{s=1}^{40} \frac{(Z_{t+s} - \delta_{t+s})P_{t+s}}{(1 + i_{kt}^*)^s} + \frac{P_{t+40} Q_{ss}}{(1 + i_{kt}^*)^{40}}. \tag{47}$$

4.2 Model Simulations

We begin with several simulations designed to illustrate how LSAPs affect real activity and inflation in our model economy. We start with a basic exercise that compares how similar-size purchases of private versus government securities affect the economy. We then compare the government bond purchase program under QE2 to a conventional interest rate policy. That is, we find the interest rate policy that provides roughly equivalent stimulus to the bond purchase program. Next, we analyze how the strength of the impact of LSAPs is affected by whether the zero lower bound is binding. We then explore the implications of household participation in long-term securities markets. Finally, we consider a financial crisis within the model that has some of the key features of the one that the global economy had in late 2008/early 2009. We then explore the effects of

Figure 1. Private and Government Asset Purchase Shocks



Note: Purchases are calibrated to a peak effect of 2.5 percent of GDP and interest rates are kept unchanged for four periods.

asset purchase programs similar in nature to what occurred under QE1.

In figure 1, the panels report the responses of the model economy to an LSAP similar in size to QE2. The solid line reports the response to purchases of long-term government bonds, as actually occurred under QE2. The dotted lines report the effects of a similar-size purchase of private securities. The purchases were phased in over roughly a six-month period and the cumulative total equaled approximately 2.5 percent of GDP. By this, we are approximating the size of purchases in terms of "ten-year equivalents" that incorporates their maturity structure.[16] Following Chen, Curdia, and Ferrero (2011), we suppose that the purchases are kept at the peak for two years and are gradually phased out, though we approximate this pattern with a second-order autoregressive process. We also follow these authors by appealing to evidence from the federal funds futures market which suggested that the funds rate was likely to remain fixed for a year after the purchases were initiated. Accordingly, in the experiment we keep the funds rate fixed for the first four quarters and then let it revert to the Taylor rule described earlier. Finally, we fix the portfolio adjustment cost parameter κ to ensure that the QE2 experiment produces a reduction in the ten-year government bond rate i_{b10} of 12 basis points, which is within the range of estimates in Williams (2011) and elsewhere. We then explore how the rest of the model economy responds to our QE2 simulation before going on to consider a variety of other experiments.

As the figure shows, the decline in long-term rates produces a peak increase in output of 1 percent, which is closely in line with the time-series estimates of Gambacorta, Hoffman, and Peersman (2011). There is also an increase in inflation and asset prices, which is consistent with the event-study evidence for QE2.[17] Overall, the response of the standard macroeconomic variable mirrors the response to a conventional monetary policy easing. We stress, though, that LSAPs work ultimately by reducing excess returns. Underlying the drop in the long-term bond rate is a decline in the current and expected sequence of one-period excess returns $E[R_b] - R$, as panel 9 in figure 1 shows (the first panel in the bottom row). We can isolate the component of the drop in the ten-year government bond rate that is due to a decline in excess returns by examining the spread between i_{b10} and the yield to maturity on the ten-year "risk-free" swap rate i_{i10} (i.e., the rate on a security that pays the short rate each quarter for ten years that would be

[16]$600 billion translates into $374 billion in ten-year equivalents based on the planned maturity breakdown of the Federal Reserve Bank of New York. See www.newyorkfed.org/markets/lttreas_faq.html.

[17]The counterfactual jump in inflation in the initial period occurs because we have abstracted from the usual frictions that tend to smooth inflation such as wage rigidity and so on. However, averaging over the first five years, the increase in inflation is roughly 17 basis points, which is in line with the event-study evidence for QE2 (see, for example, Krishnamurthy and Vissing-Jorgensen 2011.)

priced by the household if it were allowed to hold it). As the figure indicates, the lion's share of the drop in the ten-year government bond rate—roughly 10 of the overall 12-basis-point drop—is due to a decline in excess returns.

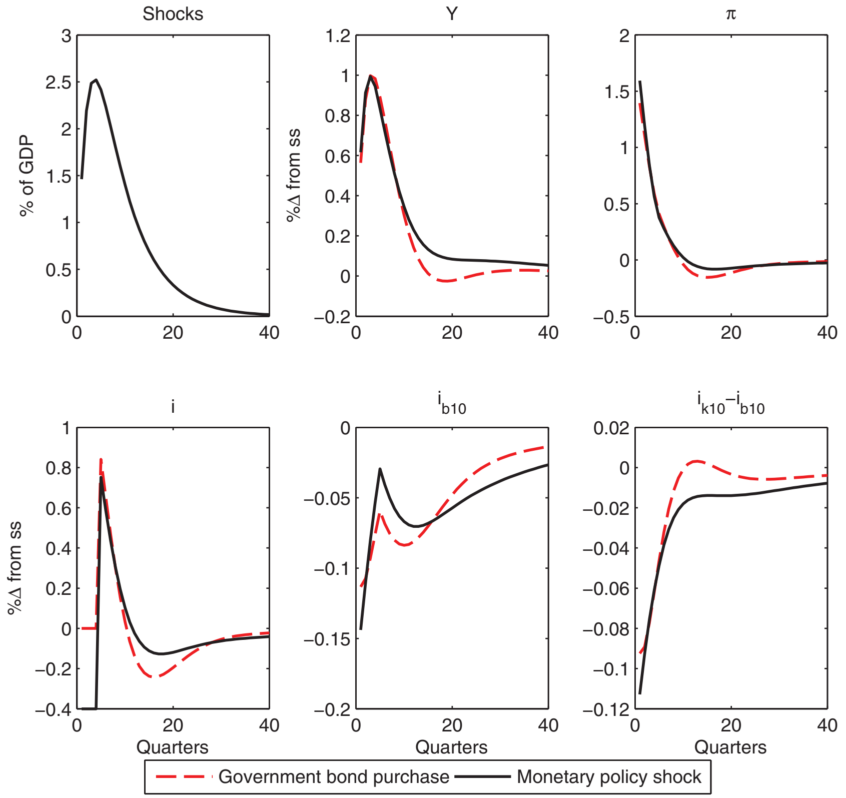
The LSAP also reduces the yield to maturity on the private security $i_{k_{10}}$ by nearly double the drop in the rate on government bonds (which can be seen by noting that the spread $i_{k10} - i_{b10}$ declines 10 basis points on impact).[18] The decline in $i_{k_{10}}$ is key for the transmission of the LSAP to the real economy. It causes the asset price Q_t to increase, which in turn stimulates investment spending. As discussed in section 2.3, i_{k10} moves proportionately more than i_{b10} in response to the LSAP because for banks—the marginal traders in the securities markets—agency costs of financing private securities are proportionately greater than those for government bonds (see equation (22)).

Finally, figure 1 shows that an equivalent-size purchase of the private security has roughly double the effect on long-term bond yields and the rest of the economy. This kind of differential effect is consistent with the rough evidence.[19] As we emphasized in section 2.3, a central bank purchase of the private security relaxes banks' balance sheet constraint proportionately more than a similar-size purchase of government bonds, which enhances the market demand for securities. The exact difference depends on the assumption about the

[18]The relative effect of government bond purchases on private security yields is an open question. Krishnamurthy and Vissing-Jorgensen (2011) present careful event-study evidence from the QE2 period suggesting that default-corrected corporate yields moved one for one with government bond yields, using credit default swap (CDS) spreads to correct for default risk. The issue here is how reliable CDS spreads are as pure measures of default risk if CDS providers themselves are financially constrained. It is also true that AMBS rates moved by less than government bond yields during this episode. At this time, however, the Federal Reserve was effectively the dominant holder of AMBS.

[19]For example, Gagnon et. al (2011) show that the announcement effect of the initial wave of QE1 in December 2008 led to a reduction in long-term government bond rates of 67 basis points versus estimates of roughly 15 basis points for QE2 (see Williams 2011). After we adjust for the fact that this first phase of QE1 involved about one and a half times the amount of purchases done under QE2, we still get a larger drop in long rates from QE1, which involved mostly AMBS purchases, than from the government bond purchases under QE2: approximately 44 basis points versus 15 basis points. Of course, there is a huge amount of noise underlying these estimates. Other studies similarly find stronger effects of QE1 (adjusting for size).

Figure 2. Monetary and Government Bond Purchase Shocks



Notes: The government bond purchase shock is calibrated to a peak effect of 2.5 percent of GDP with interest rates kept unchanged for four periods. The monetary policy shock reduces the nominal interest rate by 40 basis points below its steady-state value for four periods.

strength of the agency friction that inhibits arbitrage in each case. But so long as the friction is greater for private securities than for government bonds, purchases of the former will have stronger effects than purchases of the latter.

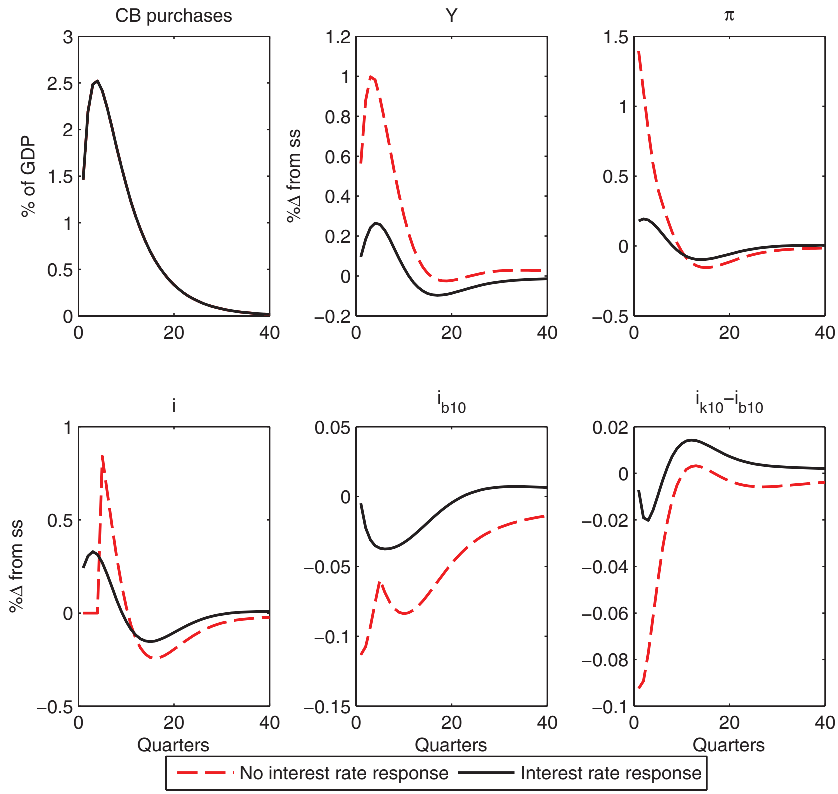
Figure 2 identifies the conventional interest policy that provides stimulus equivalent to the government bond LSAP portrayed in figure 1. In particular, we suppose that the central bank reduces the short-term interest rate in the first period and then keeps it fixed at this level for a total of four quarters, the length of time it was fixed in the QE2 experiment. Then, as in the latter case, we allow the interest rate to revert to the Taylor rule. As the figure shows, the conventional monetary policy that corresponds to our approximation of QE2 is a 40-basis-point reduction in the short rate for a period of a year.[20] Interestingly, this result comes close to the rule of thumb that many in the Federal Reserve use: A reduction in long rates from monetary stimulus corresponds to a threefold reduction in short rates. It is also interesting to observe from comparing the inflation responses that the inflation generated by the LSAP is nearly identical to that created by the "equivalent" monetary policy. This occurs of course because inflation under the LSAP is ultimately the product of the stimulus provided from interest rate reduction, as is the case with interest rate policy. There is no independent effect of the size of the balance sheet on inflation. Of course, were the central bank to take significant losses on its balance sheet that could only be financed by exorbitant money creation, matters would change. But at least currently, this does not appear to be a danger facing the Federal Reserve's balance sheet.

Figure 3 addresses the issue of how the zero lower bound affects the impact of the LSAP. We compare our baseline scenario where short rates are expected to be fixed for a year with one where they adjust immediately. As the figure shows, the immediate adjustment of short rates offsets more than 80 percent of the effect of the LSAP on output. The rise in short rates generated by the Taylor rule is roughly 30 basis points for the first year, which, as implied by figure 2, mostly offsets the stimulus from the LSAP. Thus it makes sense to use LSAPs only in situations where short rates are expected to remain fixed for a considerable period of time.

In figure 4 we illustrate the implications of imperfectly elastic household demands for long-term securities. We consider an alternative to our baseline where households only hold short-term securities and banks hold the entire stock of long-term private securities and

[20]Though we do not report the results here, our estimate of the interest policy that approximates QE1 would be a reduction in the short rate of 240 basis points for a period of a year. This estimate comes from the fact that QE1 was roughly three times the size of QE2 and, within our model, a dollar purchase of the private security has twice the effect on long rates as a similar purchase of government bonds.

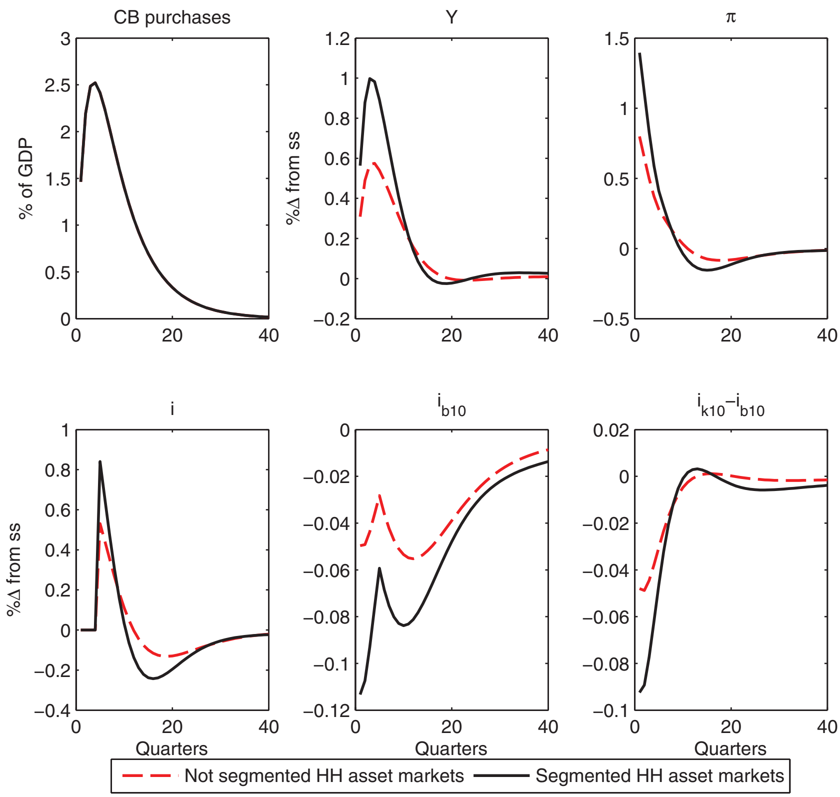
Figure 3. Government Bond Purchase Shocks with and without Interest Rate Responses



Note: The bond policy is calibrated to a peak effect of 2.5 percent of GDP. The interest rate is kept unchanged for four periods.

government bonds. The impact of LSAPs is clearly weaker in this alternative scenario: The long-term bond rate drops only about 5 basis points as opposed to 12 in the baseline and the increase in output is only about half the baseline case. Intuitively, in the baseline, imperfectly elastic household demands work to "segment" asset markets. In effect, the pool of actively traded securities is reduced: A given change in central purchases then has a greater proportionate effect in the market for actively traded securities, causing asset prices and returns to respond accordingly. This impact of friction

Figure 4. Government Bond Purchase Shocks with and without Segmented Household Asset Markets



Note: Purchases are calibrated to a peak effect of 2.5 percent of GDP and interest rates are kept unchanged for four periods.

in household asset demands on the strength of LSAPs accords well with conventional wisdom. We stress, however, that it is also critical that the marginal traders—i.e., the banks—also face limits to arbitrage. As we showed in section 2, absent these limits, LSAPs are neutral even with imperfectly elastic household demand.

We now explore how LSAPs work in the context of a financial crisis that has some key features of the one that recently occurred in the wake of the Lehman Brothers collapse. In particular, we use the model to illustrate how an LSAP program with features similar to

QE1 may have worked to moderate the downturn. One distinguishing feature of the policy experiments we perform is that the number of periods the zero lower bound binds is endogenous, in contrast to the earlier experiments where the short rate was held fixed for a given number of periods.

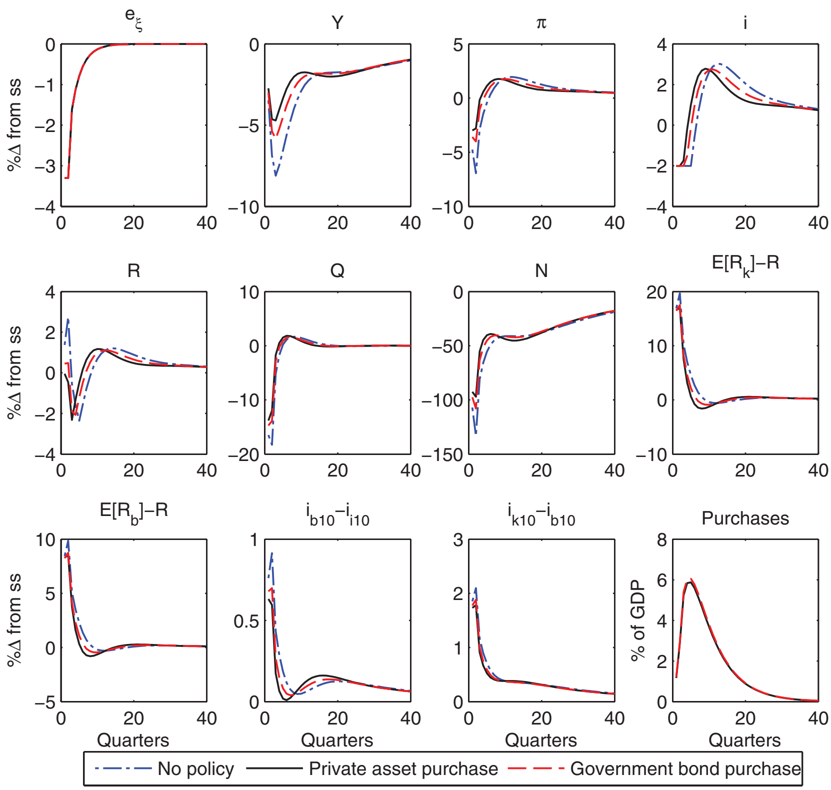
We stress that we are not attempting a complete description of the crisis: The model is too simple for this. We are, however, able to create a recession where financial intermediation is disrupted in a way that raises the cost of credit, which in turn amplifies the downturn. Further, the downturn is sufficiently sharp to push the economy to the point where the zero lower bound on the nominal interest rate is binding. In this way we capture the two features of the crisis that are most relevant in our view for the role of LSAPs.

The initiating shock for the crisis is a decline in capital quality.[21] The reduction in the value of bank assets forces a fire sale of bank assets to satisfy the balance sheet constraint. Asset prices decline and excess returns rise, which depresses real activity. The process is amplified as the asset fire sale and decline in real activity further weaken bank balance sheets. We suppose the the shock obeys a first-order autoregressive process with coefficient 0.7 (so that it mostly dies out after a year and a half). Then, to produce a sharp output contraction in the first two quarters (as occurred after the Lehman collapse), we hit the model economy with two consecutive unanticipated negative capital quality shocks: The first is a 3.3 percent decline and the second is just large enough to offset the mean reversion in the variable for one period.

We also suppose that an LSAP program involving the purchase of the private security is initiated in the wake of the shock. The sequencing of the purchases as well as the overall size at the peak is meant to be similar to QE1. In December 2008 the initial purchase program was announced that was equal to roughly half the eventual size. The announcement of the second stage of the program came in March 2009. In the meantime a number of temporary complementary measures, including the commercial paper funding facility, were set up in 2008:Q4. Accordingly, we assume that the policy is

[21] In Gertler and Karadi (2011), we show that simply beliefs that asset values will fall in the future can generate a crisis similar to that generated by a decline in capital quality. How to tie down beliefs, however, is an issue.

Figure 5. Crisis Experiment

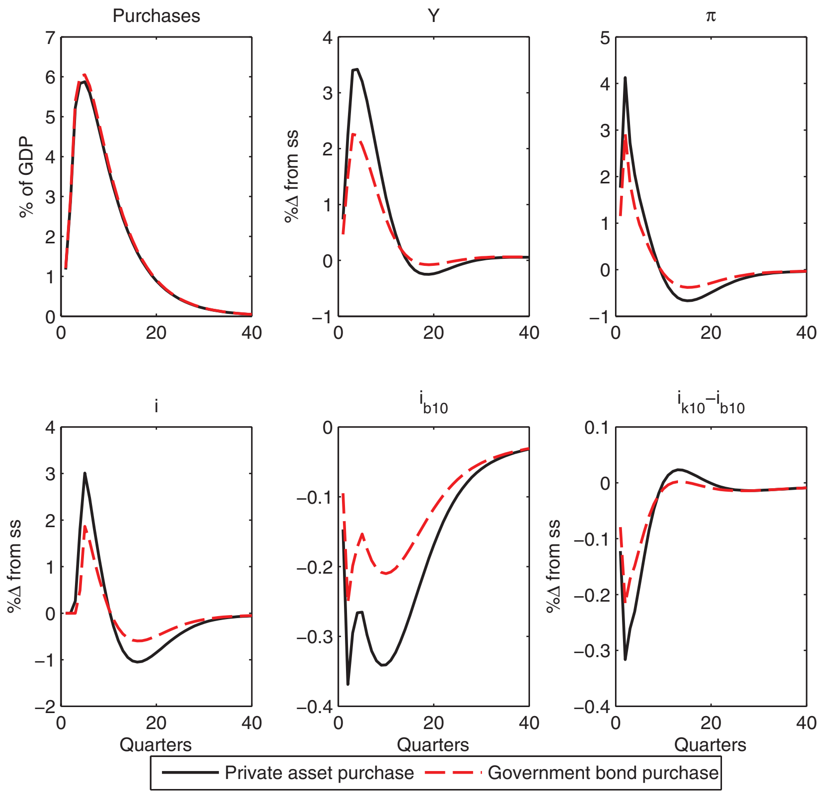


Note: Reactions to two consecutive unexpected capital quality shocks with gradual private and government asset purchases with the zero lower bound.

phased in as a sequence of three unexpected shocks, with the first shock accounting for one-quarter of the total cumulative effect and the next two the remaining three-quarters, divided evenly among each. To capture the cumulative buildup, the shocks obey second-order autoregressive processes with the first lag coefficient equal to 1.5 and the second equal to −0.55. The peak of the LSAP is 6 percent of GDP, consistent with the evidence on ten-year equivalent purchases.

Figure 5 illustrates the crisis experiment under three different scenarios: (i) no central bank response, (ii) an LSAP similar to QE1,

Figure 6. Effects of Private and Government Asset Purchases Following the Crisis Experiment



Note: The figures plot the differences from a no-policy-response case.

and (iii) an LSAP similar in timing and magnitude to QE1, except that the central bank purchases government bonds instead of the private security. Figure 6 reports how much difference the LSAPs made to the response of the model economy under both scenarios (ii) and (iii).

Under each case there is a sharp drop in output and an associated increase in credit spreads. The latter serves to propagate the downturn. The "QE1" LSAP moderates the decline substantially. The output drop is roughly 3.5 percent lower relative to the case without central bank intervention. The policy works by reducing both interest rate and interest rate spreads by amounts in accord with the evidence. It reduces the long-term bond rate roughly 30 basis points for a period of more than two years and reduces the yield on the long-term private security by roughly 55 basis points, also for a sustained period. Underlying this reduction in rates is a reduction in one-period excess returns on both the private security and government bonds.

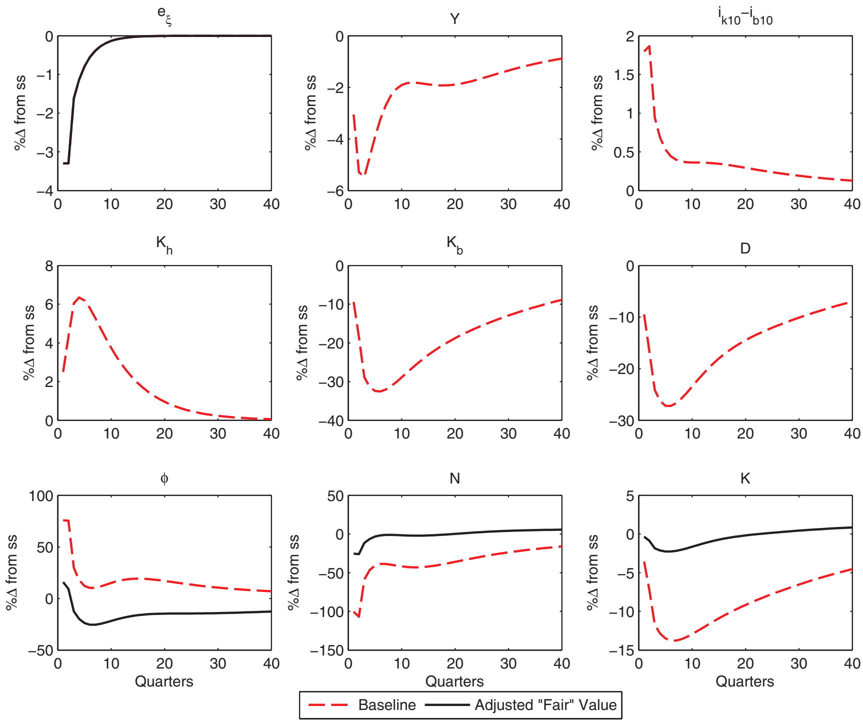
Substituting long-term government bond purchases for private sector purchases weakens the effect of the LSAP by a factor of roughly one-third. This differential is smaller than in the "non-crisis" experiments we did earlier. The reason is that the horizon over which the short-term interest rate is constant is endogenous in this case, whereas it was fixed earlier. The purchases of private securities shortens the period of time the economy remains at the zero lower bound by a quarter, weakening the overall impact. The overall effect, however, remains stronger than if government bonds were purchased instead.

We stress that the crisis experiment focuses on the immediate period around the Lehman collapse and the QE1 intervention. The model is not set up to capture the duration of the crisis. Factors such as overhang of the housing stock, household indebtedness, and global spillovers of economic distress that have contributed to delaying the recovery are not considered. Thus, one should interpret these exercises as an attempt to tease out the nature and strength of the transmission mechanisms for LSAPs, and not an attempt to provide a complete historical account of the recession.

Finally, as a check on the model, we note that a recent paper by Adrian, Colla, and Shin (2012) documents four facts any macroeconomic model of financial crises should have to explain: (i) a disruption in the flow of bank credit, (ii) a sharp increase in credit spreads, (iii) a rise in open-market credit relative to bank credit (i.e., bond financing relative to bank loans), and (iv) procyclical bank leverage ratios. The authors then conclude that most macro models of financial crises can capture (i) and (ii) but not (iii) and (iv).

Here we demonstrate that our model can account for all four facts. Figure 7 repeats the same experiment as in figure 6, this time portraying the response of the bank versus open-market credit flows and the bank leverage ratio. The decline in asset quality produces a sharp contraction in the net worth of banks (see the second panel in

Figure 7. Crisis Experiment with Adjusted "Fair" Book Values and Market Values



the bottom row). The leads to a contraction in bank loans (see the second panel in the middle row) and a rise in the spread between the loan rate and government bond rate (see the third panel in the top row). Thus, the model captures facts (i) and (ii). It also captures fact (iii): New bond issues increase as the tightening of bank lending induces firms to substitute to open-market credit (see the first panel in the middle row).

Finally, the model is able to match the cyclical behavior of bank leverage ratios, assuming the leverage ratios from the model simulations are constructed the same way they are in the data. In particular, in the data, bank equity is computed as the difference between assets and liabilities, where assets are measured using "fair-value" accounting, which in practice is a mixture of book-value and market-value accounting. Fair-value accounting, further, uses market prices when available, but during a liquidity disruption where trade may be disrupted, it instead uses a "normal" value, which is effectively a smoothed value. Thus, bank equity as measured in the data is less procyclical than true market values would suggest.[22] Since leverage is procyclical and measured equity is relatively acyclical, measured bank leverage ratios are thus procyclical. By contrast, equity in the model is in terms of market value, which is highly procyclical, leading to a countercyclical leverage ratio (see the first panel in the third row). However, when bank equity in the model is measured the same way as it is in the data, then the model leverage ratio becomes procyclical, as the first two panels of the bottom row indicate.

5. Concluding Remarks

A popular view of LSAPs—known more broadly as "quantitative easing" —is that they reflect money creation. We instead argue that LSAPs should be seen as central bank intermediation. Just like private intermediaries, the Federal Reserve has financed its asset purchases with variable interest-bearing liabilities and not money per se. The difference, of course, is that the Federal Reserve's liabilities are effectively government debt. Thus the Federal Reserve is able to obtain funds elastically in a way that private intermediaries facing financial market frictions are not. As we have shown, it is because of these limits to arbitrage in private intermediation that LSAPs can be effective. It is also worth emphasizing that effectiveness of LSAPs within our model is not due to the central bank being more efficient at holding assets than the private sector: In fact, we assume the opposite.

[22]The fair-value adjustment we use assumes that the "fair-value" assets of the banks are a weighted average of market value and book value. The weight on market value is 0.25 and on book value 0.75. In reality, 50 percent of assets follow fair-value accounting (while the other 50 percent is on book-value accounting; see Securities and Exchange Commission 2008) and we have further halved this number to reflect the fact that even fair value does not mean mark-to-market during fire sales (as fair value is a price on which the bank would be willing to sell). The book value of capital, furthermore, is assumed to disregard the effect of capital quality shock, and is only influenced by the observed drop in the investment. Book and market value of deposits are the same, and book value of net worth is calculated as a difference between book value of assets and deposits.

While the details of transmission differ, as with conventional monetary policy, LSAPs stimulate the economy by reducing credit costs. Thus, as we have shown, the transmission to real output and inflation is very similar to that occurring under conventional policy. Unlike conventional policy, however, LSAPs are an option when the zero lower bound is binding. In addition, as we have shown, LSAPs are actually most effective in this situation: Holding constant the size of the purchase and the type of the security, an LSAP leads to a larger reduction in long rates the longer is the horizon over which short-term rates are expected not to rise.

The framework we presented was designed to provide a unified way to think about the various LSAP programs that the Federal Reserve has pursued over the course of the recent crisis. We think that it may also be useful for analyzing new programs under consideration, as well as some LSAPs pursued by other central banks.

For example, under consideration at the Federal Reserve is "sterilized" QE, which basically involves lengthening the maturity of the liabilities issued to fund asset purchases from overnight to up to six months. In addition, investors other than banks can hold these liabilities. With sterilized QE, our interpretation of LSAPs as central bank intermediation, if anything, becomes more obvious. Again, key to the effectiveness of these types of LSAPs are limits on private intermediaries' ability to fund the same long-term securities by issuing liabilities of the same (short-term) maturity as the central bank.

Finally, though the details differ, the recent long-term refinancing operations (LTROs) undertaken by the European Central Bank (ECB) have a similar flavor to the LSAPs we have been analyzing. Under the LTROs, the ECB does not directly purchase assets, but it does so indirectly by accepting the assets as collateral for loans to participating banks. In particular, it provides three-year variable-rate credit to banks for loans collateralized by assets it deems acceptable, including certain government bonds, certain asset-backed securities, and even certain types of bank loans. The haircuts on the collateral vary according to the risk class. As with LSAPs, for LTROs to be effective, private intermediaries must be limited in their ability to perform the same type of arbitrage as the central bank. We leave for future research, however, working out the modifications of the model needed to precisely capture LTROs.

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