Central Bank Communication and Multiple Equilibria*

Kozo Ueda
Institute for Monetary and Economic Studies, Bank of Japan

In this paper, we construct a simple model for communication between a central bank and money-market traders. It is demonstrated that there are multiple equilibria. In one equilibrium, traders truthfully reveal their own information, and by learning this, the central bank can make better forecasts. Another equilibrium is a "dog-chasing-its-tail" equilibrium described by Blinder (1998). Traders mimic the central bank's forecast, so the central bank simply observes its own forecast from traders. The latter equilibrium is socially worse, as inflation variability becomes larger. As policy implications, we find that too-high transparency of central banks is bad because it yields the "dog-chasing-its-tail" equilibrium, and central banks should conduct continuous monitoring or emphasize that their forecasts are conditional because doing so eliminates the "dog-chasing-its-tail" equilibrium. We also consider the possibility of the existence of an optimal degree of transparency.

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1. Introduction

Monetary policy transparency plays an essential role in conducting monetary policy. It not only serves legitimacy but also enhances the effectiveness of monetary policy. High transparency helps anchor

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inflation expectations (e.g., Faust and Svensson 2001) and influences the path of future interest rates (e.g., Woodford 2005). For these reasons, central banks around the globe have greatly increased monetary policy transparency in recent years.

However, too-high transparency is costly, and no central bank discloses all the information it has. This may partly reflect the fear of losing credibility (e.g., Mishkin 2007) or of political pressure in a federal state like Canada (Chant 2003). From a different perspective, and in a theoretical manner, Morris and Shin (2002) illustrate that a coordination problem makes too-high transparency harmful. They argue that public information disclosed by central banks is focal, which plays an excessive coordination role among private agents. Furthermore, as Blinder (1998) points out, too-high transparency prevents the efficient feedback from private agents to the central bank. Blinder (1998) calls this the "dog-chasing-its-tail" problem. He argues that if a central bank tries to "please the markets" too much, the markets stop functioning and the central bank only observes its own image in the markets. However, Blinder (1998) does not specify the meaning of "please the markets."

In this paper, to formulate Blinder's (1998) idea, we construct a simple model for communication between a central bank and private agents.[1] Communications are made in both directions: from the central bank to private agents and vice versa. As private agents, we consider an infinite number of money-market traders in the financial market. In order to finance their portfolios, hedge their positions, and square liquidity imbalances, the traders take position in relation to short-term interest rate expectations. This paper therefore assumes that the money-market traders aim to forecast interest rates set by a central bank as accurately as possible. The central bank aims to stabilize inflation by accurately forecasting an incoming shock. Following Grossman and Stiglitz (1976, 1980) and Hellwig and Veldkamp (2009), we assume that traders must pay an information acquisition cost to gather private information.

By constructing a simple model and solving it analytically, we demonstrate that there are multiple equilibria regarding the action of

[1] Woodford (1994), Bernanke and Woodford (1997), Morris and Shin (2005), and Gosselin, Lotz, and Wyplosz (2008) also formulate Blinder's idea, as is discussed below.

the central bank and the traders. In one equilibrium, traders truthfully reveal their own information, and by learning this, the central bank can make a better forecast. This equilibrium represents the good-coordination outcome as in Hellwig (2005). Transparency helps traders accurately forecast the interest rate set by the central bank, and owing to traders' truthful revelation, the central bank can also make a better forecast in setting the interest rate.

What is notable in this paper is the existence of another equilibrium. This equilibrium corresponds to the bad-coordination outcome as in Morris and Shin (2002) and more closely to the "dog-chasing-its-tail" equilibrium described by Blinder (1998). At the equilibrium, traders mimic the forecast disclosed by the central bank. The central bank therefore cannot improve its own forecast through communication and cannot optimally set the interest rate to stabilize inflation. The reason for the existence of the "dog-chasing-its-tail" equilibrium is as follows. Suppose all other traders mimic the central bank's forecast. Since there are an infinite number of traders in the market, even if one trader truthfully reveals its information, the central bank can obtain little information by communicating with the traders. This makes the central bank set an interest rate close to its original forecast. Therefore, by mimicking the central bank, the trader can make a good forecast of the central bank's action, yielding the "dog-chasing-its-tail" equilibrium.

Our analyses yield several policy implications. First, too-high transparency of central banks is bad because it yields the "dog-chasing-its-tail" equilibrium. Higher transparency helps traders forecast the central bank's action more accurately. However, if traders reveal their information truthfully, it makes the central bank's actions uncertain because truthfully revealed information by other traders is uncertain. Therefore, high transparency increases the incentive to mimic the central bank's forecast, which yields the "dog-chasing-its-tail" equilibrium. This implies that pleasing the markets may be good in the short run but detrimental in the long run because it prevents the markets from functioning, which induces instability in the economy. Second, the cost of transparency depends positively on the information acquisition cost and negatively on traders' forecast precision and the central bank's forecast precision after disclosure. In this respect, for central banks, it is important to evaluate traders' ability to judge whether the "dog-chasing-its-tail" equilibrium is likely to arise. Furthermore, a central bank should conduct continuous monitoring or emphasize that its forecasts are conditional because this eliminates the "dog-chasing-its-tail" equilibrium by making a central bank's disclosed information less valuable for the guidance of future monetary policy. Third, we consider the possibility that there exists an optimal degree of transparency. It is the highest level of transparency that eliminates the "dog-chasing-its-tail" equilibrium.

There is a sizable amount of literature on central bank transparency. Morris and Shin's (2002) paper has attracted much attention and has been challenged, for example, by Angeletos and Pavan (2004), Hellwig (2005), Svensson (2006), Demertzis and Viegi (2008), and Hellwig and Veldkamp (2009). Svensson (2006) argues that the coordination induced by high transparency is good, provided that the public signal is more precise than the private signal. In reply to Svensson (2006), Morris, Shin, and Tong (2006) argue that if the public signal is correlated with the private signal, then quantitative evaluation supports Morris and Shin (2002). Morris and Shin (2005) and Gosselin, Lotz, and Wyplosz (2008) formulate Blinder's idea in the context of central bank communication. With a simple theoretical model, they demonstrate the risk of informational inefficiency. According to their model, transparency prevents the central bank from obtaining potentially valuable market information and reduces the precision of its forecasts. However, in Morris and Shin (2005) and Gosselin, Lotz, and Wyplosz (2008), private agents' disclosed information is still valuable, while in our "dog-chasing-its-tail" equilibrium, a central bank simply observes its own forecast, so private agents' disclosed information has no value. Blinder's idea is also partly formulated by Woodford (1994) and Bernanke and Woodford (1997), who point out the risk of indeterminacy arising from using not a structural model but private forecasts.

As the communication counterpart of a social planner, we focus on money-market traders in the financial market. All the above literature looks at private firms that engage in capital investment (Angeletos and Pavan 2004), price-setting (Hellwig 2005, Demertzis and Viegi 2008, and Gosselin, Lotz, and Wyplosz 2008), or zero-sum trades (Morris and Shin 2002, 2005) and regards these firms as consumers ultimately, so the sum of their utility equals social welfare, which a social planner aims to maximize. We admit that it is very important to study central bank communication with consumers, particularly in terms of anchoring inflation expectations (Demertzis and Viegi 2008). However, regarding central bank communication, traders in the financial market should not be neglected. Traders are keen observers of central bank disclosure and also providers of information on financial market conditions. Further, these traders account for only a fraction of the total population, so the sum of the traders' utility does not represent social welfare. The "dog-chasing-its-tail" problem raised by Blinder (1998) seems to be related to traders in the financial market. In this paper, we consider more explicitly what can happen in central bank communication with money-market traders in the financial market. In doing so, we assume that a central bank's action is focal in that the interest rate set by the central bank directly affects a trader's loss. A coordination motive appears not explicitly but implicitly. In contrast, in Morris and Shin (2002, 2005), a coordination motive is introduced as an assumption. A common feature is, however, that a central bank's disclosed information is focal due to the coordination motive.

The existence of multiple equilibria we demonstrate in this paper differs from Morris and Shin (2002) and Hellwig (2005), where the equilibrium is unique. Morris and Shin (2002) and Hellwig (2005) disagree as to whether transparency yields better or worse coordination, but because of multiple equilibria, our paper integrates both the bad-coordination outcome and the good-coordination outcome. In our model, whether there are multiple equilibria (in particular, the "dog-chasing-its-tail" equilibrium) depends crucially on various parameters such as those of the information acquisition cost, forecast precision, and transparency. In particular, the assumption of the information acquisition cost is essential to yield multiple equilibria and differs from Morris and Shin (2002) and Hellwig (2005). Another modeling difference between our model and that of Morris and Shin (2002) and Hellwig (2005) is that we incorporate money-market traders in the financial market. In Angeletos and Pavan (2004) and Demertzis and Viegi (2008), the possibility of multiple equilibria is demonstrated, but their characteristics are very different from ours. In particular, the "dog-chasing-its-tail" equilibrium in Blinder (1998) does not appear.

The coexistence of the good and the bad equilibria provides us with implications on the optimal degree of transparency. In this paper, we examine how the central bank's and traders' utility change depending on the transparency parameter and ask whether there is an optimal degree of transparency. Regarding the optimal degree of transparency, the above papers (with the exception of Angeletos and Pavan 2004) assume either full transparency or perfect secrecy. However, this assumption is unrealistic and may lead to an extreme conclusion that supports perfect secrecy. Angeletos and Pavan (2004) argue that there is a transparency parameter, but it is merely a central bank's forecast accuracy that is not controllable. In this paper, we explicitly introduce the measure of central bank transparency that is controllable, continuous, and representative of the clearness of central bank disclosure.[2] We then examine how the central bank's and traders' utility change depending on the transparency parameter and ask whether there is an optimal degree of transparency.

The structure of this paper is as follows. Section 2 introduces our model. Section 3 solves the model and shows that there arise multiple equilibria. Section 4 concludes the paper.

2. Model

We assume that inflation π is given by

$$\pi = i + u, \tag{1}$$

where i is an interest rate set by a central bank (hereafter CB) and u is a shock. Neither CB nor money-market traders (hereafter T) know the amount of the shock u in advance. By setting an appropriate interest rate, CB aims to minimize inflation variability,

$$L^{CB} = \pi^2, \tag{2}$$

which results in ultimately minimizing the loss of consumers.

[2] There have been relatively few papers on the optimal extent of transparency to date. Furthermore, their measures of transparency are not necessarily controllable and representative of the clearness of central bank disclosure. For example, Morris and Shin (2007) investigate the optimal number of signals to be disclosed. Walsh (2007) and Cornand and Heinemann (2008) investigate how many private agents should be informed by a central bank. Myatt and Wallace (2008) interpret the correlation between public signal and pure private signal as the measure of publicity.

As to CB's counterpart, we consider a sufficiently large number of money-market traders. The traders take positions in relation to short-term interest rate expectations, to finance their portfolios, to hedge their positions, and to square liquidity imbalances, so they aim to forecast i set by CB as accurately as possible. In other words, trader j's loss function is written as

$$L_j^T = \left(i_j^T - i\right)^2, \tag{3}$$

where i_j^T is trader j's forecast about i. Two remarks are worth making. First, CB's action is, by construction, focal because the interest rate set by CB directly affects T's loss. Instead, a coordination motive is not explicit in the above setup. However, as will become clear, traders have a coordination motive because CB's action is influenced by the average of traders' forecasts. Second, we explicitly model money-market traders who are different from consumers, so the sum of the traders' utility does not represent social welfare.

In the communication between CB and T, we consider the following four steps. At step 1, CB receives some information about the shock u—that is, u_1^{CB}—and discloses an imperfectly transparent forecast about the interest rate i—that is, i^{CB}. CB does not fully commit to the disclosed interest rate at step 1 because CB gathers new information by step 3, at which CB sets the actual interest rate i. In other words, CB's disclosed forecast is conditional. At step 2, T gathers information and, using its information set, tries to forecast the interest rate set by CB at step 3. As in Hellwig and Veldkamp (2009), we assume that trader j can gather its own information about the shock u_j^T by paying a fixed cost, $cE(u^2)$. Public information i^{CB} disclosed by CB is free. This assumption is motivated by Grossman and Stiglitz (1976, 1980), who argue that when there is a cost of information, a price system cannot perfectly convey information to the public. In our model, as we will soon see, this assumption yields an incentive for T to mimic CB's disclosed information.[3] At step 3, CB observes the average of T's forecasts i^T and also receives new

[3]The appendix considers whether multiple equilibria arise without introducing fixed costs. It reveals that if there are a limited number of money-market traders but CB cannot observe each trader's strategy, then similar multiple equilibria can arise.

information about the shock u_2^{CB}. Using CB's information set, CB refines its forecast about the shock and sets the optimal interest rate i so as to stabilize inflation. Finally, at step 4, the inflation rate π is realized, by which the true amount of the shock u is known to everyone.

Forecast precisions are formulated in a standard way following Morris and Shin (2002, 2005). CB and T cannot observe an incoming u but observe u_1^{CB}, u_j^T, and u_2^{CB} given by

$$\begin{aligned}
u_1^{CB} &= u + e_1^{CB}, & E\left(e_1^{CB^2}\right)/E(u^2) &= 1/\alpha_1^{CB},\\
u_j^T &= u + e_j^T + e^T, & E\left(e_j^{T^2}\right)/E(u^2) &= 1/\alpha_j \text{ and}\\
& & E\left(e^{T^2}\right)/E(u^2) &= 1/\alpha^T,\\
u_2^{CB} &= u + e_2^{CB}, & E\left(e_2^{CB^2}\right)/E(u^2) &= 1/\alpha_2^{CB}, \qquad (4)
\end{aligned}$$

where α_1^{CB} and α_2^{CB} represent CB's forecast precisions at step 1 and at step 3, respectively. At step 2, trader j encounters idiosyncratic uncertainty e_j^T and aggregate uncertainty e^T, and the inverse of its amplitude is given by α_j^T and α^T. It is convenient to transform these parameters into

$$x_1^{CB} = \frac{\alpha_1^{CB}}{1+\alpha_1^{CB}},\ x_2^{CB} = \frac{\alpha_2^{CB}}{1+\alpha_1^{CB}+\alpha_2^{CB}},\ x_j^T = \frac{\alpha_j^T}{1+\alpha_j^T},$$
$$x^T = \frac{\alpha^T}{1+\alpha^T}. \qquad (5)$$

Moreover, different from the earlier literature discussed in the introduction, we explicitly introduce a continuous transparency parameter τ so that it is controllable, continuous, and representative of the clearness of CB's disclosure. CB's disclosure is described as

$$i^{CB} = -E\left(u\middle|u_1^{CB}\right) + v, \quad E(v^2) = E\left[\left(u - E\left(u\middle|u_1^{CB}\right)\right)^2\right]/\tau. \qquad (6)$$

In our model, it is optimal for CB to set an interest rate so as to cancel a coming shock, so if CB is perfectly transparent, the error term v becomes zero. In this sense, the variance of v represents the obscurity of CB's disclosure. Note, however, that the variance of the error term v depends on CB's forecast precision before disclosure x_1^{CB}, which cannot be easily controlled by CB, although early literature such as Angeletos and Pavan (2004) interprets CB's forecast precision as the measure of transparency. In order to draw policy implications using a controllable measure of transparency, we focus on a continuous transparency parameter τ as the measure of transparency.[4] The variance of v is decreasing with the transparency parameter τ as well as CB's forecast precision. As τ becomes larger, the variance of v becomes smaller, which suggests higher transparency.

3. Equilibrium

This section solves the above model and shows that multiple equilibria can exist. The solution of this model is derived by answering the following two questions. The first question is how T discloses its own forecast about the interest rate at step 2. More precisely, we ask if T tries to make the best guess about the shock u and truthfully reveals its own forecast, or if T mimics CB's forecast made at step 1. The second question is how CB evaluates the shock u and sets the optimal interest rate i. More precisely, we ask how CB refines its forecast by combining its own information and T's forecast, and if CB can learn anything valuable from T's forecasts.

Since the model is linear quadratic, the actions of CB and T are written in the following linear form:

$$\begin{aligned} &\text{Step 2} \quad i_j^T = E\left(i \middle| I_j^T\right) = a_1^j i^{CB} + a_2^j u_j^T, \\ &\text{Step 3} \quad E(u|I^{CB}) = -i = b_1 i^T + b_2 u_2^{CB} + b_3 u_1^{CB} + b_4 v. \end{aligned} \tag{7}$$

Here we use the fact that trader j's information set at step 2 is $I_j^T = \{i^{CB}, u_j^T\}$ and that CB's information set at step 3 is $I^{CB} = \{i^T, u_2^{CB}, u_1^{CB}, v\}$. The coefficients a and b are obtained from

$$\begin{aligned} \left(a_1^j, a_2^j\right) &= \arg\min E\left[i - E\left(i \middle| I_j^T\right)\right]^2 \quad \text{given } \left(a_1^k, a_2^k, b_1, b_2, b_3, b_4\right) \\ (b_1, b_2, b_3, b_4) &= \arg\min E[u - E(u|I^{CB})]^2 \quad \text{given } (a_1, a_2), \end{aligned} \tag{8}$$

where $k \neq j$.

[4]As we will show later in figure 1, CB's forecast precision before disclosure x_1^{CB} appears to be irrelevant to the condition of multiple equilibria. Therefore, in fact, our results appear to hold, even though we focus on ν.

The above problem can be solved analytically, which leads to our first proposition. Hereafter, we limit our attention to the equilibrium where all traders have the same strategy.

PROPOSITION 1. *There exist multiple equilibria, "M" and "R," when fixed costs c satisfies*

$$c > F\big(x_1^{CB}, x_2^{CB}, x^T, x_i^T, \tau\big). \tag{9}$$

Otherwise, only equilibrium "R" exists. In equilibrium "M" (or "dog-chasing-its-tail"), T mimics CB's forecast. In equilibrium "R," T truthfully reveals its own forecast.

Proof. We can show that there can exist two equilibria, where the coefficients a and b are respectively written as

$$\begin{aligned}
a_1 &= \left\{1 - x_1^{CB} + \frac{1-x_1^{CB}}{\tau x_1^{CB}} + \left(\frac{1}{\tau}\frac{1-x_1^{CB}}{x_1^{CB}} + 1\right)\right.\\
&\quad \left.\times\left(\frac{1-x_i^T}{x_i^T} + \frac{1-x^T}{x^T}\right)\right\}^{-1}\left(\frac{1-x_i^T}{x_i^T} + \frac{1-x^T}{x^T}\right),\\
a_2 &= -1 + a_1\left(\frac{1}{\tau}\frac{1-x_1^{CB}}{x_1^{CB}} + 1\right),\\
b_1 &= \frac{1}{a_2}\frac{\big(1-x_1^{CB}\big)\big(1-x_2^{CB}\big)}{\big(1-x_1^{CB}\big)\big(1-x_2^{CB}\big) + \frac{1-x^T}{x^T}}, \quad b_2 = x_2^{CB}(1 - a_2 b_1),\\
b_3 &= x_1^{CB}\big(1 - a_2 b_1 - x_2^{CB} + x_2^{CB} a_2 b_1 + a_1 b_1\big), \quad b_4 = -a_1 b_1,
\end{aligned} \tag{10}$$

or

$$\begin{aligned}
a_1 &= \left(\frac{1-x_1^{CB}}{\tau x_1^{CB}} + 1\right)^{-1}, \quad a_2 = 0,\\
b_1 &= \text{arbitrary}, \;\; b_2 = x_2^{CB}, \;\; b_3 = x_1^{CB}\big(1 - x_2^{CB} + a_1 b_1\big), \;\; b_4 = -a_1 b_1.
\end{aligned} \tag{11}$$

In equation (11), a_2 is zero, which suggests that T does not reveal its own information and mimics CB's forecast. CB therefore sees its own image through communication with T. This is the "M" or "dog-chasing-its-tail" equilibrium. The parameter b_1 is arbitrary. This is because, in “M,” CB’s reaction function in equation (7) becomes redundant. Using equation (11), we can easily show that equation (7) is transformed into

$$\begin{aligned} E(u|I^{CB}) &= b_1 i^T + b_2 u_2^{CB} + b_3 u_1^{CB} + b_4 v \\ &= x_2^{CB} u_2^{CB} + x_1^{CB}(1 - x_2^{CB}) u_1^{CB}. \end{aligned} \tag{12}$$

In other words, in “M,” CB has to rely on its own information. On the other hand, in equation (10), a_2 is non-zero, which suggests that T truthfully reveals its own forecast.

We next examine the condition that no trader has an incentive to deviate from “M.” If we neglect fixed costs, and when all the other traders choose “M,” a trader j who deviates from “M” can minimize its expected loss by choosing

$$\begin{aligned} a_2^j &= \left(\frac{1}{x_i^T} + \frac{1 - x_1^{CB}}{\tau x_1^{CB} x_i^T} - x_1^{CB} + \frac{1 - x^T}{x^T}\left(1 + \frac{1 - x_1^{CB}}{\tau x_1^{CB}}\right)\right)^{-1} \\ &\quad \times \left(x_2^{CB} - x_1^{CB} x_2^{CB} + \frac{1 - x_1^{CB}}{\tau x_1^{CB}}\left(x_1^{CB} + x_2^{CB} - x_1^{CB} x_2^{CB}\right)\right), \\ a_1^j &= \frac{1}{x_1^{CB}}\left(a_2^j\left(\frac{1}{x_i^T} + \frac{1 - x^T}{x^T}\right) + x_1^{CB} + x_2^{CB} - x_1^{CB} x_2^{CB}\right). \end{aligned} \tag{13}$$

Comparing the expected loss including fixed costs in this case with the expected loss in “M” suggests that the equilibrium “M” exists when

$$\begin{aligned} c &> F\left(x_1^{CB}, x_2^{CB}, x^T, x_i^T, \tau\right) \\ &\equiv -x_1^{CB}\left\{\left(1 + \frac{1 - x_1^{CB}}{\tau x_1^{CB}}\right)^{-1} - 1 + x_2^{CB}\right\} \\ &\quad - 2x_1^{CB} x_2^{CB}\left\{\left(1 + \frac{1 - x_1^{CB}}{\tau x_1^{CB}}\right)^{-1} - 1 + x_2^{CB}\right\} - x_2^{CB^2} \\ &\quad - \frac{1 - x_1^{CB}}{\tau}\left(1 + \frac{1 - x_1^{CB}}{\tau x_1^{CB}}\right)^{-2} \end{aligned}$$

$$+\frac{1-x_1^{CB}}{x_1^{CB}}x_2^{CB2}+\frac{1-x_1^{CB}}{\tau}\left(\frac{x_1^{CB}+x_2^{CB}-x_1^{CB}x_2^{CB}}{x_1^{CB}}\right)^2$$

$$-\frac{(1-x_1^{CB})^2}{x_1^{CB}}\left(\frac{1}{x_i^T}+\frac{1-x^T}{x^T}\right)\left(x_2^{CB}+\frac{x_1^{CB}+x_2^{CB}-x_1^{CB}x_2^{CB}}{\tau x_1^{CB}}\right)^2$$

$$\times\left\{\frac{1}{x_i^T}-x_1^{CB}+\frac{1-x_1^{CB}}{x_1^{CB}}\frac{1}{x_i^T}+\frac{1-x^T}{x^T}\left(1+\frac{1-x_1^{CB}}{x_1^{CB}}\right)\right\}^{-1}. \tag{14}$$

In equilibrium "R," T truthfully reveals its own information, and by learning this, CB can make a better forecast. This equilibrium represents the good-coordination outcome as in Hellwig (2005). Transparency helps traders accurately forecast the interest rate set by the central bank, and due to traders' truthful revelation, the central bank can also make a better forecast in setting the interest rate.

On the other hand, the equilibrium "M" corresponds to the bad-coordination outcome as in Morris and Shin (2002) and more closely to the "dog-chasing-its-tail" equilibrium in Blinder (1998). In this equilibrium, T mimics CB's forecast. CB therefore cannot improve its own forecast through communication, preventing CB from optimally setting the interest rate to stabilize inflation. The reason for the existence of the "dog-chasing-its-tail" equilibrium is as follows. Suppose all other traders mimic CB's forecast. Since there are an infinite number of traders in the market, even if one T truthfully reveals its information, CB can obtain little information by communicating with T. This makes CB set an interest rate close to its original forecast. Therefore, by mimicking CB, T can make a good forecast of CB's action, yielding the "dog-chasing-its-tail" equilibrium.

The equilibrium "M" in our model is much closer to the "dog-chasing-its-tail" problem raised by Blinder (1998) than that of the earlier literature. For example, the problem pointed out by Woodford (1994) and Bernanke and Woodford (1997) is the risk of indeterminacy arising from using not a structural model but private forecasts. Their papers do not necessarily explain Blinder's argument that if a central bank tries to "please the markets" too much, the markets stop functioning and the central bank only observes its own image from the markets. In Morris and Shin (2005), transparency makes private agents' disclosed information less valuable to a central bank, but it is still valuable. On the other hand, our paper demonstrates that in the "dog-chasing-its-tail" equilibrium, CB simply observes its own forecast, so private agents' disclosed information has no value.

Morris and Shin (2002) and Hellwig (2005) disagree as to whether transparency yields better or worse coordination, and their disagreement is not easily resolved because the equilibrium is unique and their models are different. Our model helps resolve their disagreement because it integrates both the bad-coordination outcome and the good-coordination outcome in a unified framework. Compared with Morris and Shin (2002) and Hellwig (2005), there are two key features in our model that lead to multiple equilibria. First, we introduce the information acquisition cost. The appendix suggests that we need an alternative assumption to yield multiple equilibria without introducing the information acquisition cost. Second, as the communication counterpart of a social planner, we focus on money-market traders in the financial market, while Morris and Shin (2002) and Hellwig (2005) focus on private firms that engage in zero-sum trades and price setting, respectively.

As we noted in the previous section, CB's action is, by construction, focal because the interest rate set by CB directly affects T's loss. A coordination motive appears not explicitly but implicitly because CB's action is influenced by the average of traders' forecasts. This is understood by rearranging equation (3) using equation (7):

$$\begin{aligned} L_j^T &= \left(i_j^T - i\right)^2 \\ &= \left\{ i_j^T - (-b_1) \lim_{N\to\infty} \frac{1}{N} \sum_k i_k^T \right\}^2 + \dots. \end{aligned} \tag{15}$$

Omitted terms $(\dots)$ in the right-hand side of equation (15) are the function of the shock u.[5] From this, it is clear that trader j who

[5] We can show that even though T's loss depends on a fundamental described by inflation variability as well as T's forecast error, the same multiple equilibria can arise. This implies that this result is robust even though traders are not of the money market but are of long-term bond or equity markets. However, the following discussion of traders' welfare will be modified.

aims to forecast CB's action attempts to evaluate other traders' forecasts. This induces a coordination motive among agents. In this sense, T's loss function is similar to that in Morris and Shin (2002, 2005). However, a difference is that in Morris and Shin (2002, 2005), a coordination motive is introduced as an assumption. A common feature is, however, that CB's disclosed information is focal due to the coordination motive.

Which equilibrium is better for CB? The second proposition answers this question.

PROPOSITION 2. *For CB, "R" is better than "M."*

Proof. CB's loss, $L^{CB} = \pi^2$, is given by

$$\text{(M)}\; L^{CB(M)} = \left(1 - x_1^{CB}\right)\left(1 - x_2^{CB}\right),$$

$$\text{(R)}\; L^{CB(R)} = \left(\frac{1}{\left(1 - x_1^{CB}\right)\left(1 - x_2^{CB}\right)} + \frac{x^T}{1 - x^T} \right)^{-1} < L^{CB(M)}. \tag{16}$$

As simple New Keynesian economics states, if we consider that minimizing inflation variability leads to maximizing social welfare, then this proposition suggests that "R" is socially more desirable than "M." This proposition is intuitive. In "R," CB can learn T's information about the shock u, so CB can improve its forecast accuracy for u and set a better interest rate to stabilize inflation than in "M," where CB has to rely only on its own information. T's information, even if its precision is low, is always useful for CB if CB extracts the signal appropriately. It is also worth pointing out that equation (16) does not depend on the transparency parameter τ. The degree of transparency does not make any difference to CB's loss.

To find a way to eliminate the undesirable equilibrium "M," we next examine the condition of multiple equilibria in more detail. A condition on the fixed costs to acquire private information c is obvious. As the fixed costs become lower, profits from saving the fixed costs become smaller. Therefore, the equilibrium "M" is more likely to be eliminated. Regarding transparency and forecast precision parameters, the following lemma provides a way to eliminate "M."

LEMMA 3.

$$dF\left(x_1^{CB}, x_2^{CB}, x^T, x_i^T, \tau\right)/dx_2^{CB} \geq 0, dF\left(x_1^{CB}, x_2^{CB}, x^T, x_i^T, \tau\right)/dx^T > 0,$$
$$dF\left(x_1^{CB}, x_2^{CB}, x^T, x_i^T, \tau\right)/dx_i^T > 0, dF\left(x_1^{CB}, x_2^{CB}, x^T, x_i^T, \tau\right)/d\tau \leq 0.$$

Many valuable policy implications can be obtained from this lemma. Firstly, too-high transparency is bad because transparency that is too high increases the parameter space over which the socially bad equilibrium "M" is possible. The fact that $dF/d\tau$ is non-positive suggests that as CB's transparency becomes higher, "M" becomes more likely to arise. The reason why $dF/d\tau$ is non-positive is simple. As transparency increases, T can infer more accurately CB's valuation. However, if traders reveal their information truthfully, it makes CB's action uncertain because truthfully revealed information by other traders is uncertain. Therefore, high transparency increases an incentive to mimic CB's forecast, which yields the "dog-chasing-its-tail" equilibrium. The cost of transparency depends on the fixed costs to acquire private information c. As c becomes lower, the bad equilibrium "M" is more likely to be eliminated, so high transparency becomes less detrimental.

Secondly, CB should conduct continuous monitoring or emphasize that CB's forecasts are conditional. The fact that dF/dx_2^{CB} is non-negative suggests that as CB's forecast after disclosure becomes more accurate, "M" becomes less likely to arise. The reason why dF/dx_2^{CB} is non-negative is as follows. If x_2^{CB} is high, CB weighs its own information after disclosure more than that before disclosure, so CB's disclosed information does not provide T with a good sign for future interest rate decisions. Therefore, mimicking CB becomes less valuable to T than the case of truthfully revealing. By conducting continuous monitoring or emphasizing that CB's forecasts are conditional, CB can increase its forecast precision after the disclosure and make itself less committed to future policy, which can reduce the incentive for T to choose "M."

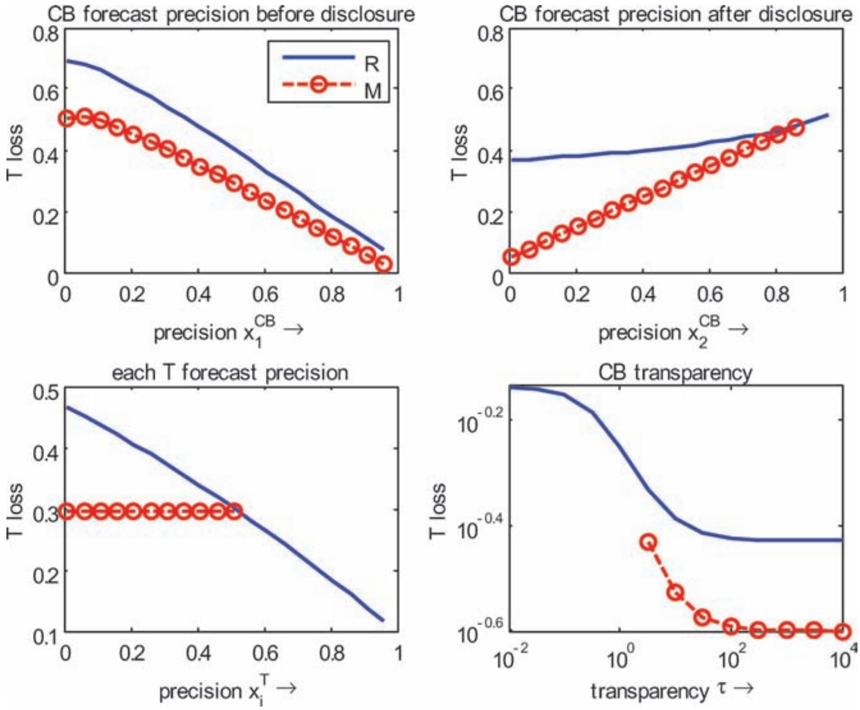
Thirdly, we find that both dF/dx^T and dF/dx_i^T are positive. This suggests that as T's forecast precision is lower, "M" becomes more likely to arise. This is because if T's forecast precision is low, then T's payoff by truthful revelation becomes small, which lowers the incentive for T to deviate from "M." Since policymakers cannot adjust the parameters x^T and x_i^T, this finding does not necessarily give them a direct policy implication. However, it gives them a warning of "M" when T's ability is low. In this respect, it is important for CB to evaluate T's ability.

To check these results visually, we implement numerical evaluations by changing some parameters. As a baseline, we assume the values of $x_1^{CB} = 0.5$, $x_2^{CB} = 0.5$, $x_i^T = 0.2$, and $\tau = 10$. The forecast precision of each trader is worse than that of CB. The aggregate forecast accuracy of traders, x^T, is chosen to be 0.8 so that traders' aggregate information has the same accuracy as CB's information. This corresponds to the conservative benchmark in Svensson (2006). Fixed costs, c, are chosen to be 0.05 so that our numerical calculation clearly demonstrates this model's characteristics. By changing one parameter while keeping the other parameters fixed, we examine how T's loss changes depending on forecast precision and transparency.

Figure 1 shows the results. The solid line shows T's loss in "R," while the dotted line with circles shows T's loss in "M." We find that the dotted line disappears in some regions. This suggests that, as proposition 1 and lemma 3 state, the bad equilibrium "M" is eliminated when x_2^{CB} is high, x_i^T (so as x^T) is high, or τ is low.

Finally, we consider the possibility that there exists an optimal degree of transparency. A candidate for this is the highest τ^* that leads to the unique equilibrium. In our simulation, the bottom-right panel of figure 1 suggests that τ^* is approximately one. If the transparency parameter becomes lower than τ^*, the undesirable equilibrium "M" is eliminated but T suffers more losses, as the bottom-right panel of figure 1 demonstrates. In this sense, as in Hellwig (2005), it is good to be transparent—for traders as well as a central bank—so as to promote good coordination. On the other hand, if the transparency parameter exceeds τ^*, it produces the socially bad equilibrium "M" as in Morris and Shin (2002, 2005). Therefore, too much transparency is risky. Of course, higher transparency than τ^* does not necessarily lead to the selection of "M" because, as long as all other T believes that the equilibrium "R" is achieved, the equilibrium "R" becomes stable. However, T prefers "M" to "R," as the bottom-right panel suggests, so T may be inclined to coordinate to achieve "M."

Figure 1. Trader's Loss



4. Concluding Remarks

This paper constructed a very simple model for communication between a central bank and money-market traders in the financial market. We assume that a central bank aims to stabilize inflation, while traders aim to forecast the interest rate set by the central bank, and that traders have to pay a fixed cost when gathering information. With these assumptions, we demonstrate that there may be multiple equilibria. In one equilibrium, "R," traders truthfully reveal their own information, and by learning this, the central bank can make a better forecast. Another equilibrium, "M," corresponds to a "dog-chasing-its-tail" equilibrium described by Blinder (1998): traders mimic the central bank's forecast, so the central bank simply observes its own forecast from traders. The latter equilibrium is worse for the central bank and ultimately for consumers because inflation variability increases.

Policy implications are, first, that too-high transparency of central banks is bad because it yields the socially bad equilibrium "M." Pleasing the markets may be good in the short run but detrimental in the long run because it prevents the markets from functioning, which induces instability in the economy. Second, the cost of transparency depends positively on the fixed costs to acquire private information c and negatively on traders' forecast precision x^T and x_j^T and a central bank's forecast precision after disclosure x_2^{CB}. In this respect, for central banks, it is important to evaluate traders' ability to judge whether the equilibrium "M" is likely to arise. Furthermore, central banks should conduct continuous monitoring or emphasize that their forecasts are conditional because it increases x_2^{CB} and eliminates "M." Third, we consider the possibility that there exists an optimal degree of transparency. The highest τ^* that eliminates "M" may be interpreted as the optimal degree of transparency.

There are remaining tasks, however. In particular, it is important to extend this model into a dynamic model with consideration of the inflation target and inflation expectations. In this model, we assume that traders in the financial market do not influence inflation except for their expectation formation, but it is natural to think that traders' expected interest rates or traders' reactions to a central bank's action directly influence inflation by affecting inflation expectations and long-term interest rates. Our paper neglects this aspect, although many people emphasize its importance (e.g., Faust and Svensson 2001, Woodford 2005, Aoki and Kimura 2008, and Demertzis and Viegi 2008). This may well reestablish the importance of greater transparency.

Appendix. Model without Fixed Costs

In this paper, we assume that T has to pay a fixed cost when gathering information. This is clearly one of the key assumptions used to demonstrate the existence of multiple equilibria—in particular, "M." This appendix considers another way of obtaining similar multiple equilibria without introducing fixed costs.

We make four modifications in our model. Firstly, we do not assume a fixed cost in gathering information. Secondly, we consider a finite number of money-market traders given by N. Thirdly and most importantly, we assume that CB cannot observe the deviation of each strategy of T (a_1^j, a_2^j) from a certain aggregate equilibrium strategy (a_1, a_2). At step 3, CB observes i^T, but CB considers that its change comes not from the deviation of each strategy of T (a_1^j, a_2^j) but from a change in the shock traders receive, u_j^T. Therefore, each trader's strategy does not influence CB's strategy, and the trader optimizes its strategy with CB's strategy fixed. Fourthly, for simplicity we neglect aggregate uncertainty e^T. In other respects, our model setup is the same as before.

With this setup, we can obtain the following two propositions.

PROPOSITION A1. *There always exist multiple equilibria. In equilibrium "M" (or "dog-chasing-its-tail"), T mimics CB's forecast. In equilibrium "R," T truthfully reveals its own forecast.*

Proof. We can show that there are two equilibria, where the coefficients a and b are written as

$$\begin{aligned}
a_1 &= -\left\{x_1^{CB} - \frac{1}{x^T}\left(\frac{1}{\tau}\frac{1-x_1^{CB}}{x_1^{CB}} + 1\right)\right\}^{-1}\left(\frac{1-x_j^T}{x_j^T}\right),\\
a_2 &= -1 + a_1\left(\frac{1}{\tau}\frac{1-x_1^{CB}}{x_1^{CB}} + 1\right),\\
b_1 &= \frac{1}{a_2}\frac{\left(1-x_1^{CB}\right)\left(1-x_2^{CB}\right)}{\left(1-x_1^{CB}\right)\left(1-x_2^{CB}\right) + \frac{1-x_j^T}{Nx_j^T}}, \quad b_2 = x_2^{CB}(1-a_2 b_1),\\
b_3 &= x_1^{CB}\left(1 - a_2 b_1 - x_2^{CB} + x_2^{CB} a_2 b_1 + a_1 b_1\right), \quad b_4 = -a_1 b_1, \quad (17)
\end{aligned}$$

or

$$\begin{aligned}
&a_1 = -b_4/b_1, \quad a_2 = 0,\\
&b_1 = -N,\ \ b_2 = x_2^{CB},\ \ b_3 = x_1^{CB}\left(1 - x_2^{CB} + a_1 b_1\right),\ \ b_4 = \text{arbitrary.} \quad (18)
\end{aligned}$$

In equation (18), $a_2 = 0$, which suggests that T mimics CB's forecast. A parameter b_4 is arbitrary, but we can easily show that equation (7) is transformed into

$$E(u|I^{CB}) = b_1 i^T + b_2 u_2^{CB} + b_3 u_1^{CB} + b_4 v$$
$$= x_2^{CB} u_2^{CB} + x_1^{CB}\big(1 - x_2^{CB}\big) u_1^{CB}. \tag{19}$$

On the other hand, in equation (17), a_2 is non-zero, which suggests that T truthfully reveals its own forecast.

PROPOSITION A2. *For CB, "R" is better than "M."*

Proof. CB's loss, $L^{CB} = \pi^2$, is given by

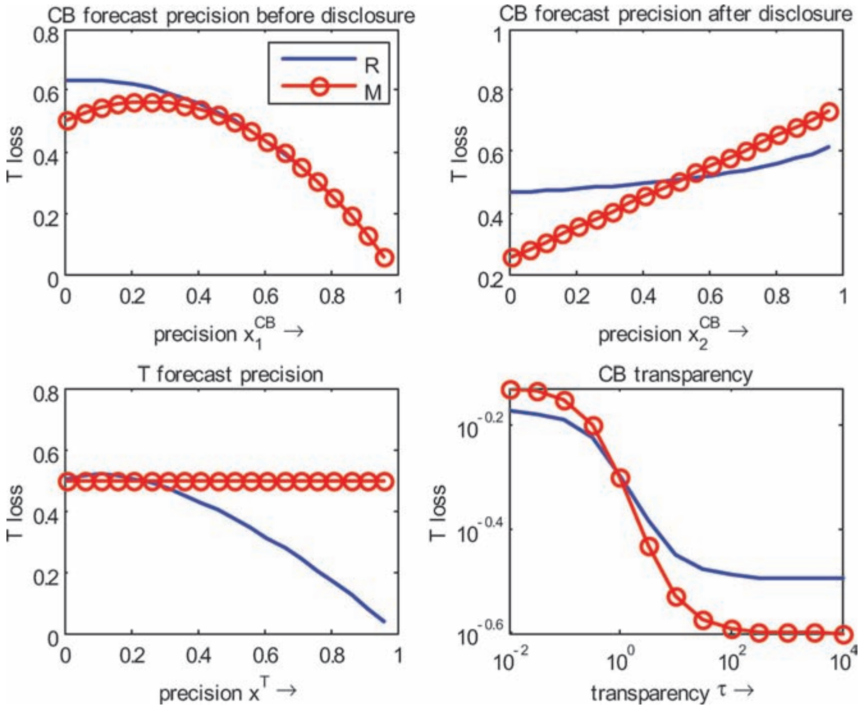
$$\text{(M)}\ L^{CB(M)} = \big(1 - x_1^{CB}\big)\big(1 - x_2^{CB}\big),$$
$$\text{(R)}\ L^{CB(R)} = \left(\frac{1}{\big(1 - x_1^{CB}\big)\big(1 - x_2^{CB}\big)} + \frac{N x_j^T}{1 - x_j^T}\right)^{-1} < L^{CB(M)}. \tag{20}$$

These multiple equilibria are highly similar to those in our main model. A notable difference is that in this modified model, multiple equilibria always exist irrespective of the forecast precision parameters α_1^{CB}, α^T, and α_2^{CB} and the transparency parameter τ.

As this model demonstrates, fixed costs are not necessarily needed to yield multiple equilibria—in particular, "M." For T, deviating from "M" is not worthwhile because CB can respond to the deviation of a single trader's strategy by choosing $b_1 = -N$, which lowers T's profit by deviating. However, as we noted at the beginning of this appendix, this result crucially depends on the assumption that CB observes i^T but CB considers that its change comes not from the deviation of each T's strategy (a_1^j, a_2^j) but from a change in the shock traders receive, u_j^T.

Finally, in order to evaluate T's loss, we assume the values of $x_1^{CB} = 0.5$, $x_2^{CB} = 0.5$, $x_j^T = 0.2$, $\tau = 10$, and $N = 16$. The number of traders, N, is set so that T's pooled forecast precision is the same as CB's forecast precision. Figure 2 demonstrates the results. Although multiple equilibria always exist, these panels closely resemble figure 1. Comparing the losses in "M" and "R" implies that high transparency increases the attractiveness of "M" over "R" for T. On the other hand, CB's high forecast precision after disclosure decreases the attractiveness of "M" for T.

Figure 2. Trader's Loss



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