The Expected Interest Rate Path: Alignment of Expectations vs. Creative Opacity*

Pierre Gosselin,[a] Aileen Lotz,[b] and Charles Wyplosz[b]
[a]Institute Fourier, University of Grenoble
[b]The Graduate Institute, Geneva

We examine the effects of the release by a central bank of its expected future interest rate in a simple two-period model with heterogeneous information between the central bank and the private sector. The model is designed to rule out common-knowledge and time-inconsistency effects. Transparency—when the central bank publishes its interest rate path—fully aligns central bank and private-sector expectations about the future inflation rate. The private sector fully trusts the central bank to eliminate future inflation and sets the long-term interest rate accordingly, leaving only the unavoidable central bank forecast error as a source of inflation volatility. Under opacity—when the central bank does not publish its interest rate forecast—current-period inflation differs from its target not just because of the unavoidable central bank expectation error but also because central bank and private-sector expectations about future inflation and interest rates are no longer aligned. Opacity may be creative and raise welfare if the private sector's interpretation of the current interest rate leads it to form a view of expected inflation and to set the long-term rate in a way that systematically offsets the effect of the central bank forecast error on inflation volatility. Conditions that favor the case for transparency are a high degree of precision of central bank information relative to private-sector information, a high precision of early information, and a high elasticity of current to expected inflation.

JEL Codes: D78, D82, E52, E58.

*We acknowledge with thanks helpful comments from an anonymous referee, Alex Cukierman, Martin Ellison, Hans Genberg, Petra Geraats, Charles Goodhart, Craig Hakkio, Glenn Rudebusch, Laura Veldkamp, Anders Vredin,

1. Introduction

A number of central banks—the Reserve Bank of New Zealand, the Bank of Norway, the Central Bank of Iceland, and the Swedish Riksbank—now announce their expected interest rate paths, in addition to their inflation and output-gap forecasts. One reason for this practice is purely logical. Inflation-targeting central banks publish the expected inflation rate and the output gap, typically over a two- or three-year horizon, but what assumptions underlie their forecasts? Obviously, they make a large number of assumptions about the likely evolution of exogenous variables. One of these is the policy interest rate. Most banks used to assume a constant policy interest rate. If, however, the resulting expected rate of inflation exceeds the inflation target, the central bank is bound to raise the policy rate, which implies that the inflation forecast does not really reflect what the central bank expects. This is why many central banks now report that their inflation-forecasting procedure relies on the interest rate implicit in the yield curve set by the market. As long as the central bank agrees with the market forecasts, this might seem to be an acceptable procedure. But what if the market forecasts do not lead, in the central bank's view, to the desirable outcome? Then the inflation forecasts are not what the central bank expects to see and, therefore, the market interest forecasts must differ from those of the central bank. As noted by Woodford (2006), consistency requires that the central bank report the expected path of the policy rate along with its inflation and output-gap forecasts.

Why then do most central banks conceal their conditional inflation forecasts by not revealing their expected interest rate paths? Would it not be preferable for central banks to reveal their own expectations of what they anticipate to do? Most central banks reject this idea. Goodhart (2006) offers a number of reasons of why they do so:

Carl Walsh, and John Williams, as well as from participants in seminars at the University of California, Berkeley; the Federal Reserve Bank of San Francisco; the Bank of Korea; the Bank of Norway; the Riksbank; and the Third Banca d'Italia-CEPR Conference on Money, Banking and Finance. All errors are our own.

> If, as I suggest, the central bank has very little extra (private, unpublished) information beyond that in the market, [releasing the expected interest rate path forces the bank to choose between] the Scilla of the market attaching excess credibility to the central bank's forecast (the argument advanced by Stephen Morris and Hyun Song Shin), or the Charybdis of losing credibility from erroneous forecasts.

The first concern is that the central bank could become unwillingly committed to earlier announcements even though the state of the economy has changed in ways that were then unpredictable. The risk is that either the central bank validates the pre-announced path, and enacts suboptimal policies, or it chooses a previously unexpected path and loses credibility since it does not do what it earlier said it would be doing. This argument is a reminder of the familiar debate on time inconsistency. The debate has shown that full discretion is not desirable. Blinder et al. (2001) and Woodford (2005) argue instead in favor of a strategy that is clearly explained and shown to the public to guide policy decisions.

The second concern is related to the result by Morris and Shin (2002) that the public tends to attribute too much weight to central bank announcements—not because central banks are better informed, but because these announcements are common knowledge. This argument is far from convincing. It is based on the doubtful assumption that the central bank is poorly informed relative to the private sector (Svensson 2005a). It also ignores the fact that central banks must reveal at least the current interest rate (Gosselin, Lotz, and Wyplosz 2008).

The third, related, concern is that revealing future interest rates might create a potential credibility problem. The central bank's announcement is bound to shape the market-set yield curve, but what if the implied short-term rates do not accord with those announced by the central bank? Since it is the long end of the yield curve that affects the economy, and therefore acts as a key transmission channel of monetary policy, it could force the central bank to take more abrupt actions to move the yield curve to match its own interest rate forecasts. Would this note be countereffective?

Finally, central bank decisions are normally made by committees—the Reserve Bank of New Zealand is an exception among inflation-targeting central banks—which, it is asserted, are unlikely to be able to agree on future interest rates. The Bank of Norway and the Riksbank show that this is not really the case. Quite to the contrary, these central banks not only explain that committees can think about the expected interest rate path, but they also report that doing so improves the quality of analyses carried out by both the decision makers and the staff.[1]

We deal with some, not all, of these questions. Because they have been extensively studied, we deliberately ignore the time-consistency issue and the Morris-Shin effect. Instead, we focus on the information role of interest rate forecasts with two aims. First, we examine how the publication of the expected interest rate path affects private-sector expectations in a simple model characterized by information heterogeneity—the central bank and the private sector receive different information about a random shock. Second, we ask whether revealing the forecasted policy rates is desirable.

In our model, full central bank transparency is not necessarily desirable because an imperfectly informed central bank policy inevitably makes forecast errors; this is indeed one argument put forward against the publication of the interest rate path. The private sector recognizes that the central bank's forecast errors result in misguided policy choices, but it fully trusts the central bank to do the best that it can given its information set. With no further information about this information set, the private sector does not fully understand the policy choice about the current interest rate and therefore draws wrong conclusions about this choice. When it publishes its interest rate forecast, the central bank reveals its information set, which helps the private sector to more accurately interpret the current interest rate decision; yet, this is not always optimal. In a typical second-best fashion, it may be that the private sector's erroneous inference of the central bank's erroneous policy choice delivers a welfare-superior outcome. For the publication of the expected interest rate path to be desirable, the central bank

[1]This information was obtained via private communication from Anders Vredin.

information must be precise relative to that of the private sector and early signals must be precise relative to subsequent updates.[2]

Two other results are worth mentioning at the outset. First, because they receive different signals, the central bank and the private sector do not generally agree on expected future inflation. In our model, the publication of the interest rate path forecast fully aligns expectations, not because the information sets become identical but because expectations coincide. Second, the publication of the interest rate path forecast leads to a process of information swapping between the central bank and the private sector: we call this a mirror effect. The central bank initially provides information about its signals and subsequently recovers information about the private-sector signals.

The literature on the revelation of expected future policy interest rates is limited so far. Archer (2005) and Qvigstad (2005) present, respectively, the approach followed by the Reserve Bank of New Zealand and the Bank of Norway. Svensson (2005b) presents a detailed discussion of the shortcomings of central bank forecasts based on the constant interest rate assumption or on market rates to build up the case for using and revealing the policy interest rate path. Faust and Leeper (2005) emphasize the distinction between conditional and unconditional forecasts. They assume that the central bank holds an information advantage over the private sector, which in their model implies that sharing that information is welfare enhancing. They show that conditional forecasts—i.e., not revealing the policy interest rate path—provide little information on the more valuable unconditional forecasts, for which they find some supporting empirical evidence.

Similarly, Rudebusch and Williams (2006) assume an information asymmetry between the central bank and the private sector regarding both policy preferences and targets.[3] The private sector

[2]This second-best result is related to the demonstration by Hellwig (2005) that the reason why nontransparency may be desirable in Morris and Shin (2002) is the existence of a market failure due to the combination of asymmetric information and incomplete markets.

[3]Rudebusch and Williams (2006) also offer an excellent overview of the policy debate about how central banks signal their intentions regarding future policy actions.

learns about these factors by running regressions on past information, which may include the expected interest rate path. The paper also allows for a "transmission noise" that distorts its communication. Through simulations, they find that revealing the expected path improves the estimation process and welfare, with a gain that declines as the transmission noise increases. Additionally, they explore the case when the accuracy of the central bank signals is not known by the public. They find that accuracy underestimation limits the gains from releasing the expected interest rate path, while overestimation may be counterproductive. This result is not of the Morris-Shin variety, however, because what is at stake is not the precision of information but the size of the transmission noise, a very different phenomenon.

Walsh (2007) considers a model where the central bank and individual firms receive different signals about aggregate demand and firm-level cost shocks. As a consequence, as in Morris and Shin (2002), the publication by the central bank of its output-gap forecasts—which is equivalent in his model to revealing expected inflation—has a large effect on individual firm forecasts, which can be welfare reducing if the central bank is poorly informed. Walsh examines the possibility that the central bank information is not received by all firms. Partial transparency may offset the common-knowledge effect. The optimal degree of transparency—the proportion of firms that receive the central bank's information—depends on the relative accuracy of the central bank's information about demand and supply shocks.

Our contribution differs from Faust and Leeper (2005) and Rudebusch and Williams (2006). They assume the existence of an information *asymmetry*, which makes transparency always desirable as long as the central bank is credible. Instead, we assume that the central bank is credible with known preferences—which fully accord with social preferences—and we focus on information *heterogeneity* between the central bank and the private sector. Walsh (2007) too deals with information heterogeneity but, as we consider a single representative private agent, we eliminate the common-knowledge effect that is at the center of his analysis.

The next section presents the model, a simple two-period version of the standard New Keynesian log-linear model. Section 3 looks at the case when the central bank optimally chooses the interest rate and announces its expected future interest rate. In section 4, the central bank follows the same rule as in section 3 but does not reveal its expected future interest rate. Section 5 compares the welfare outcomes of the two policy regimes, and the last section concludes with a discussion of arguments frequently presented to reject the release of interest rate expectations by central banks.

2. The Model

2.1 Macroeconomic Structure

We adopt the now-standard New Keynesian log-linear model, as in Woodford (2003). It includes a Phillips curve:

$$\pi_t = \beta E_t^P \pi_{t+1} + \kappa_1 y_t + \varepsilon_t, \tag{1}$$

where y_t is the output gap and ε_t is a random disturbance, which is assumed to be uniformly distributed over the real line, therefore with an improper distribution and a zero unconditional mean. In what follows, without loss of generality, we assume a zero rate of time preference so that $\beta = 1$. The output gap is given by the forward-looking IS curve:

$$y_t = E_t^P y_{t+1} - \kappa_2 \big(r_t - E_t^P \pi_{t+1} - r^*\big), \tag{2}$$

where r_t is the nominal interest rate. We do not allow for a demand disturbance because allowing for two sources of uncertainty would greatly complicate the model.[4] We assume that the natural real interest rate $r^* = 0$. Note that all expectations E^P are those of the private sector, which sets prices and decides on output after the central bank has decided on the contemporaneous interest rate.

We limit our horizon to two periods by assuming that the economy is in steady state at $t = 0$ and $t \geq 3$, i.e., when inflation, output gap, and the shocks are nil. This simplifying assumption is meant to describe a situation where past disturbances have been absorbed so that today's central bank action is looked upon as dealing with the current situation ($t = 1$) given expectations about the near future

[4] A generalization to both demand and supply disturbances, which could preclude obtaining closed-form solutions, is left for future work. Walsh (2007) examines the different roles of these disturbances.

$(t = 2)$—say two to three years ahead—while too little is known about the very long run $(t \geq 3)$ to be taken into consideration. Consequently, (1) and (2) imply

$$\pi_1 = E_1^P \pi_2 - \kappa\bigl(r_1 - E_1^P \pi_2 + E_1^P r_2 - E_1^P \pi_3\bigr) + \kappa_1 E_1^P y_3 + \varepsilon_1,$$

where $\kappa = \kappa_1 \kappa_2$. Note that the channel of monetary policy is the real long-term interest rate, the second term in the above expression. This long-term rate is decided partly by the central bank—it chooses r_1—and partly by the private sector, which sets the longer end of the yield curve $E_1^P r_2$ and the relevant expected inflation rates $E_1^P \pi_2$ and $E_1^P \pi_3$. This implies that, when it sets the interest rate r_1, the central bank must take into account the effect of its decision on market expectations. Put differently, the central bank must forecast how private-sector forecasts will react to the choice of r_1.

Since the economy is known to return to steady state in period 3, $E_1^P \pi_3 = 0$ and $E_1^P y_3 = 0$ and the previous equation simplifies to

$$\pi_1 = (1 + \kappa) E_1^P \pi_2 - \kappa\bigl(r_1 + E_1^P r_2\bigr) + \varepsilon_1, \tag{3}$$

where $r_1 + E_1^P r_2$ is the long-run (two-period) nominal interest rate. Similarly,

$$\pi_2 = -\kappa r_2 + \varepsilon_2, \tag{4}$$

where we also assume that the central bank sets $r_t = r^*$ for $t \geq 3$, which is indeed optimal, as will soon be clear.

The loss function usually assumes that society is concerned with stabilizing both inflation and the output gap around some target levels, which allows for a well-known inflation-output trade-off. Much of the literature on central bank transparency additionally focuses on the idea that the public at large may not know how the central bank weighs these two objectives. This assumption creates an information asymmetry, which makes transparency generally desirable, as shown in Rudebusch and Williams (2006). Here, instead, we ignore this issue by assuming that the weight on the output gap is zero and that the target inflation rate is also nil. Since the rate of time preference is zero, the loss function is, therefore, evaluated as the unconditional expectation:

$$L = E\bigl(\pi_1^2 + \pi_2^2\bigr) \tag{5}$$

and this is known to everyone.

2.2 Information Structure

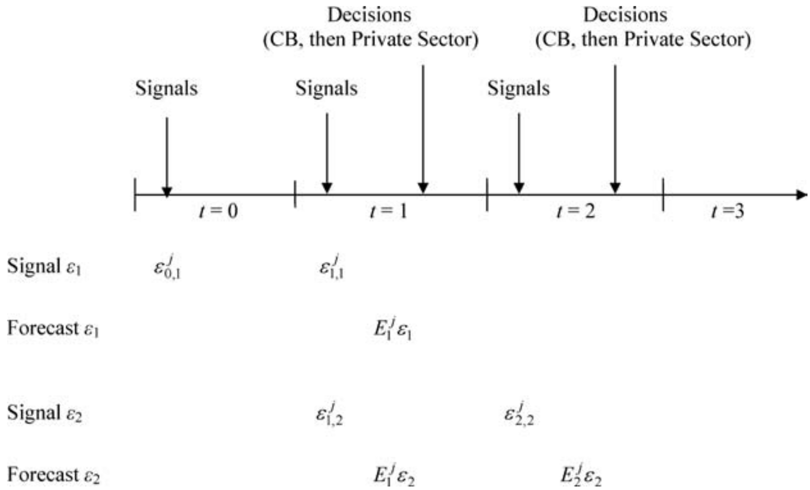
The information structure is crucial. Information asymmetry requires that the central bank and the private sector receive different signals about the shock ε_t. In addition, in order to meaningfully discuss the publication of interest rate forecasts, we allow for the central bank to discover new information between the release of its forecast and the decision on the corresponding interest rate. To that effect, we assume that two signals are received for each shock ε_t, both of which are centered around the shock: (i) an early signal $\varepsilon^j_{t-1,t}$ obtained in the previous period, which leads to the forecast $E^j_{t-1}\varepsilon_t$, and (ii) a contemporaneous signal $\varepsilon^j_{t,t}$, where $j = CB$, P denotes the recipient of the signals—the central bank and the private sector, respectively. Both of them then combine the early and updated signals to form new forecasts $E^{CB}_t\varepsilon_t$ and $E^{P'}_t\varepsilon_t$.[5] Note that the private-sector forecast based on its own signals is denoted with a prime to distinguish it from the forecasts made subsequently, after the central bank has decided on the interest rate, which is instantly revealed. Thus the operator E^P_1 in (3) combines $E^{P'}_1$ with the information content of r_1.

Figure 1 presents the information structure and the timing of decisions. At the beginning of period 0, the central bank and the private sector receive an early signal $\varepsilon^j_{0,1}$ on the shock ε_1. These signals have known variances $(k\alpha)^{-1}$ and $(k\beta)^{-1}$ for the central bank and the private sector, respectively. Equivalently, the signal precisions are $k\alpha$ and $k\beta$. At the beginning of period 1, updated signals on $\varepsilon^{CB}_{1,1}$ and $\varepsilon^P_{1,1}$—with variances $[(1-k)\alpha]^{-1}$ and $[(1-k)\beta]^{-1}$, respectively—are received by the central bank and the private sector. Using Bayes's rule to exploit both signals, the central bank and the private sector infer expectations $E^{CB}_1\varepsilon_1$ and $E^{P'}_1\varepsilon_1$, respectively, with variances α^{-1} and β^{-1} or, equivalently, precisions α and β. The parameter k measures the relative precision of early signals vis à vis the updated signals, and we assume that $0 \leq k \leq 1$.

Much the same occurs concerning the period 2 disturbance ε_2, with a slight but importance difference. At the beginning of period 1,

[5]The assumption that ε_t is uniformally dsitributed implies that Bayes's rule is only applied to the signals. Note that $cor(\varepsilon_t, \varepsilon^j_t) = 1$.

Figure 1. Timing of Information and Decisions



the central bank and the private sector receive, respectively, the early signals $\varepsilon^{CB}_{1,2}$ and $\varepsilon^P_{1,2}$ with variances $(k\alpha)^{-1}$ and $(k\beta)^{-1}$. The central bank then forms $E^{CB}_1\varepsilon_2 = \varepsilon^{CB}_{1,2}$ and sets r_1 to minimize $E^{CB}_1 L$. The private sector waits until r_1 is set and announced to form $E^P_{1,2}\varepsilon_2$, using both its early signal $\varepsilon^P_{1,2}$ and whatever information it can extract from r_1. Thus, as previously noted, $E^{CB}_1\varepsilon_2$ and $E^P_1\varepsilon_2$ are formed at different times during period 1: $E^{CB}_1\varepsilon_2$ before r_1 is known and $E^P_1\varepsilon_2$ afterwards. The reason is that r_1 conveys new information to the private sector, not to the central bank.

At the beginning of period 2, the central bank and the private sector receive contemporaneous signals $\varepsilon^{CB}_{2,2}$ and $\varepsilon^P_{2,2}$, with variances $[(1-k)\alpha]^{-1}$ and $[(1-k)\beta]^{-1}$, respectively. We further assume that, at the beginning of period 2, the realized values of π_1 and ε_1 become known to both the central bank and the private sector. The central bank uses all information available—the early and contemporaneous signals $\varepsilon^{CB}_{1,2}$ and $\varepsilon^{CB}_{2,2}$ as well as π_1 and ε_1—to form its forecast $E^{CB}_2\varepsilon_2$ and sets r_2 to minimize $E^{CB}_2 L$. After the central bank decision, the private sector observes r_2, forms its expectations, and decides on output and prices.

The focus of the paper is whether, in addition to choosing and announcing r_t, the central bank should also reveal its expectation of the interest rates in the following periods r_{t+i}. This issue is made simpler once we recognize that $r_t = 0$ for all $t \geq 3$, so that we will only need to consider the choice of r_1 and r_2 and whether the central bank reveals $E_1^{CB} r_2$.

2.3 Comments

The model combines some highly stylized features with a rather complicated information structure. The objective is to work with the simplest possible model that can meaningfully explore the role of interest rate forecasts. Two periods allow us to distinguish between the current and the future interest rate. Two signals—early and updated—make it possible for the actually chosen interest rate to differ from its forecast. Information heterogeneity provides a channel for central bank release of information to affect private-sector decisions, raising a few issues along the way.

We intentionally shut down two prominent channels that provide arguments against full central bank transparency: creative ambiguity and the common-knowledge effect.[6] Creative ambiguity emerges in the presence of time inconsistency due to uncertainty about the central bank preferences, presumed to differ from those of the private sector. It is desirable in a model where only unanticipated money matters so that central banks need to preserve some secrecy margin, a dubious assumption rejected by the New Keynesian Phillips curve. Here, instead, we assume that the central bank and the private sector only care about inflation, a special—simple—case of identical preferences further discussed below. The beauty-contest effect arises when the private sector includes a large number of agents who each receive a different signal and pay excessive attention to central bank signals simply because these signals are seen, and are known to be seen, by all. We previously voiced doubts about the practical relevance of this effect. Here we assume that there is a single representative private-sector agent.

We also rule out information asymmetry between the central bank and the private sector and focus instead on information heterogeneity. Information asymmetry generally provides support for

[6]The seminal contribution for creative ambiguity is Cukierman and Meltzer (1986); the common-knowledge effect is due to Morris and Shin (2002).

transparency. However, except for its own preferences, it is hard to imagine what information advantage is enjoyed by central banks.[7] Information heterogeneity arises when central banks and the private sector have different (non-nested) information about the economy or the "right model," a highly plausible assumption.

In our model, information heterogeneity occurs because the central bank does not observe the long-term rate before it sets the interest rate. This may seem unrealistic. Central banks, which move at discrete times, can and do observe the continuously updated yield curve and other financial variables whenever they make decisions. But if we allow the private sector to set the long-term interest rates before the central bank makes its decision, the result is information asymmetry, not heterogeneity. Indeed, the central bank would know both the private-sector signals and its own signals. As we explain below, this would eliminate the welfare difference between transparency and opacity. We could allow the private sector to move first and yet preserve information heterogeneity by allowing for more than one source of uncertainty. In this case, observing the yield curve would provide the central bank with information on the combination of shocks, not on the individual shocks.[8] However, adding one more shock would make our model intractable analytically.

More generally, in this kind of linear model, the fact that the central bank observes the variables (interest rate, asset prices, etc.) set by the private sector does not lead to information asymmetry as long as the number of these variables is smaller than the number of private-sector signals. Allowing for a single private-sector signal that is not observed by the central bank is a parable meant to capture the idea that the central bank cannot uncover all private signals. This is the simplest possible framework that gives rise to information heterogeneity.

Finally, our loss function (5) implies that the central bank pursues a strict inflation-targeting strategy. In practice, however, the commonly adopted strategy is flexible inflation targeting. This issue is, again, related to the general issue of the number of shocks and signals. Meaningfully adding the output gap to the loss function would

[7] We ignore confidential central bank information about the situation of banks during financial crises, a different phenomenon from the one at hand.

[8] This is the modeling strategy adopted by Walsh (2007).

require allowing for demand shocks. Two variables and their signals would make the model considerably more complicated, most likely analytically intractable. We believe that our generic results would be qualitatively preserved.

3. The Central Bank Reveals Its Interest Rate Forecast

We first look at the case where the central bank reveals $E_1^{CB} r_2$, which we refer to as the transparency case. In period 2, the central bank sets the interest rate in order to minimize $E_2^{CB}(\pi_2)^2$ conditional on the information available at the beginning of this period, i.e., after it has received the signal $\varepsilon_{2,2}^{CB}$.[9] The central bank seeks to offset the perceived shock and sets

$$r_2 = \frac{1}{\kappa} E_2^{CB} \varepsilon_2. \tag{6}$$

The simplicity of this choice is a consequence of our assumption that the economy will return to the steady state in period $t = 3$. It can be viewed either as a rule or as discretionary action given the new information received at the beginning of the period.

Moving backward to period 1, the central bank publishes $E_1^{CB} r_2 = \frac{1}{\kappa} E_1^{CB} \varepsilon_2 = \frac{1}{\kappa} \varepsilon_{1,2}^{CB}$. This shows that publishing the interest rate is equivalent to fully revealing the central bank signal $\varepsilon_{1,2}^{CB}$.[10] As a consequence, in period 1 the private sector receives two signals about ε_2: its own signal $\varepsilon_{1,2}^{P}$ with precision $k\beta$ and, as just noted, the central bank signal $\varepsilon_{1,2}^{CB}$ with precision $k\alpha$. Denoting the relative precision of the central bank and private-sector signals as $z = \frac{\alpha}{\beta}$, the private sector uses Bayes's rule in period 1 to optimally forecast ε_2:

$$E_1^P \varepsilon_2 = \gamma_1^{tr} \varepsilon_{1,2}^P + \left(1 - \gamma_1^{tr}\right) \varepsilon_{1,2}^{CB} = \frac{1}{1+z} \varepsilon_{1,2}^P + \frac{z}{1+z} \varepsilon_{1,2}^{CB}. \tag{7}$$

---

[9] Note that we do not allow for the private sector to use newly received information $\varepsilon_{2,2}^P$, which arrives too late to be of any use.

[10] This is so because the model allows for one signal and one policy instrument. If there were more signals than instruments, publishing the expected future value of one instrument would not be fully revealing.

In order to set the long-term interest rate, the private sector also needs to forecast the future short-term interest rate given by (6) and therefore the central bank's own forecast of the future shock. Conjecture that, similarly to (7), the optimal forecast is

$$E_1^P E_2^{CB} \varepsilon_2 = \gamma_2^{tr} \varepsilon_{1,2}^P + \left(1 - \gamma_2^{tr}\right) \varepsilon_{1,2}^{CB} \tag{8}$$

with unknown coefficient γ_2^{tr} to be determined.

When period 2 starts, π_1 and ε_1 become known. As a consequence, (3) and (6) show that $\pi_1 + \kappa r_1 - \varepsilon_1 = (1+\kappa)(E_1^P \varepsilon_2 - E_1^P E_2^{CB} \varepsilon_2) - E_1^P E_2^{CB} \varepsilon_2$ is known to both the central bank and the private sector. Using (7) and (8) we have

$$\pi_1 + \kappa r_1 - \varepsilon_1 = \left[(1+\kappa)\left(\gamma_1^{tr} - \gamma_2^{tr}\right) - \gamma_2^{tr}\right]\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right) + \gamma_2^{tr} \varepsilon_{1,2}^{CB}.$$

This implies that, at the beginning of period 2, when π_1 and ε_1 become known, the central bank can recover the private signal $\varepsilon_{1,2}^P$. We have a delayed mirror effect: by revealing the expected future interest rate, the central bank gives out its period 1 information $\varepsilon_{1,2}^{CB}$ and gets in return, in period 2, the private information $\varepsilon_{1,2}^P$. Put differently, by observing how its own information was previously interpreted, the central bank now recovers the signal previously received by the private sector. Importantly, the mirror image is not identical to the original; it provides the central bank with useful information when it decides on the interest rate r_2. Indeed, it can use three signals about ε_2: $\varepsilon_{1,2}^{CB}$ received in period 1 with precision $k\alpha$, $\varepsilon_{2,2}^{CB}$ received in period 2 with precision $(1-k)\alpha$, and now $\varepsilon_{1,2}^P$ with precision $k\beta$. Applying Bayes's rule we have

$$E_2^{CB} \varepsilon_2 = \frac{z\left[k\varepsilon_{1,2}^{CB} + (1-k)\varepsilon_{2,2}^{CB}\right] + k\varepsilon_{1,2}^P}{z+k}.$$

Noting that $E_1^P \varepsilon_{2,2}^{CB} = E_1^P \varepsilon_2$, it follows that $\gamma_1^{tr} = \gamma_2^{tr}$ and therefore[11]

$$E_1^P E_2^{CB} \varepsilon_2 = \frac{1}{1+z} \varepsilon_{1,2}^P + \frac{z}{1+z} \varepsilon_{1,2}^{CB} = E_1^P \varepsilon_2.$$

The private sector's own forecast of the future shock is perfectly aligned with its perception of the future central bank estimate of this shock, which it knows will lead to the choice of the future interest rate. As they swap signals, both the central bank and the private sector learn from each other. As a consequence, the private sector knows that its own forecast will be taken into account by the central bank when it applies Bayes's rule before deciding on r_2.

PROPOSITION 1. *When the central bank reveals its expected future interest rate, the private sector and the central bank exchange information about their signals received in period 1 about the period 2 shock:*

- *In period 1, the central bank fully reveals its early signal about the period 2 shock, which is then used by the private sector to improve its own forecast.*
- *In period 2, the central bank can identify the corresponding early signal previously received by the private sector.*
- *As a result, central bank and private-sector expectations are fully aligned and, in period 1, both expect future inflation to be zero.*

The last statement in the proposition is readily established. In period 2, the interest rate r_2 is set by the central bank according to

[11] Proof:

$$\begin{aligned} E_1^P E_2^{CB} \varepsilon_2 &= \frac{z E_1^P \left[k \varepsilon_{1,2}^{CB} + (1-k) \varepsilon_{2,2}^{CB}\right] + k \varepsilon_{1,2}^P}{z+k} \\ &= \frac{z \left[k \varepsilon_{1,2}^{CB} + (1-k) \left(\frac{\varepsilon_{1,2}^P}{1+z} + \frac{z \varepsilon_{1,2}^{CB}}{1+z} \right)\right] + k \varepsilon_{1,2}^P}{z+k} \\ &= \frac{\varepsilon_{1,2}^P}{1+z} + \frac{z \varepsilon_{1,2}^{CB}}{1+z}. \end{aligned}$$

(6), which fully reveals $E_2^{CB}\varepsilon_2$, the central bank updated information about the shock ε_2. Using (4), it follows that

$$\pi_2 = \varepsilon_2 - E_2^{CB}\varepsilon_2. \tag{9}$$

As a consequence, $E_1^P\pi_2 = E_1^P\varepsilon_2 - E_1^P E_2^{CB}\varepsilon_2 = 0 = E_1^{CB}\pi_2$. The publication in period 1 by the central bank of its inflation forecast $E_1^{CB}\pi_2$ is uninformative: it simply restates that the central bank aims at bringing inflation to its target level. This is similar to forecasts of inflation-targeting central banks, which are invariably on target at the chosen horizon, typically two to three years ahead.[12]

We can characterize the optimal monetary policy. In period 2, it is described by (6). In period 1, the central bank sets the interest rates to minimize $E_1^{CB}(\pi_1^2 + \pi_2^2)$ conditional on available information. Since (9) shows that r_1 does not affect π_2, in period 1 the central bank can simply minimize $E_1^{CB}\pi_1^2$. Since $E_1^P\pi_2 = 0$, from (3) we see that the central bank chooses the short-term interest rate r_1 such that, in expectation, the long-term interest rate—which is what matters for aggregate demand—fully offsets the current shock:

$$r_1 + E_1^{CB}E_1^P r_2 = \frac{1}{\kappa}E_1^{CB}\varepsilon_1.$$

Since $E_1^P\varepsilon_2 = E_1^P E_2^{CB}\varepsilon_2$, $E_1^P r_2 = E_1^{CB} r_2$ and, using (6), we find the optimal policy decision in period 1:

$$r_1 = \frac{1}{\kappa}\left(E_1^{CB}\varepsilon_1 - E_1^{CB}\varepsilon_2\right). \tag{10}$$

In period 1, having observed r_1, the private sector uses (6) to set the long-term interest rate:

$$r_1 + E_1^P r_2 = r_1 + \frac{1}{\kappa}E_1^P E_1^{CB}\varepsilon_2 = r_1 + \frac{1}{\kappa}E_1^{CB}\varepsilon_2,$$

[12] On the other hand, evidence so far by Archer (2005) and Ferrero and Secchi (2007) suggests that market expectations only partially adjust following the publication of the interest rate path. We find that expectations are fully aligned because we assume that there is only one source of uncertainty. Allowing for more shocks would mean that the central bank revelation of its expected interest rate path would not fully reveal all its information, as noted in section 2.3 above. We are grateful to the anonymous referee for attracting our attention to this point.

which is the same as the central bank's own forecast. Thus the yield curve exactly matches the interest rate path published by the central bank.

Collecting the previous results, we obtain

$$\pi_1 = \left(\varepsilon_1 - E_1^{CB}\varepsilon_1\right) + \frac{1}{1+z}\left(\varepsilon_{1,2}^{CB} - \varepsilon_{1,2}^{P}\right).$$

Period 1 inflation depends on two forecasting errors: the period 1 central bank forecasting error and the discrepancy between the central bank and the private-sector signals regarding period 2 shock.[13] Note that the impact of this last discrepancy is less than one for one ($\frac{1}{1+z} < 1$) because the revelation of $\varepsilon_{1,2}^{CB}$ by the central bank leads the private sector to discount its own signal $\varepsilon_{1,2}^{P}$ and to bring its forecast $E_1^P\varepsilon_2$ in the direction of $\varepsilon_{1,2}^{CB}$. Note also that $E_1^{CB}\pi_1 = 0$: the central bank always forecasts inflation rate to be on target because its objective does not call for any trade-off with other objectives. That forecast is also uninformative.

The private sector is well aware that the central bank's interest rate forecast is bound to be inaccurate. Indeed, in general, there is no reason for $E_1^P E_2^{CB}\varepsilon_2$ to be equal to ε_2, but the eventual realization of this difference is irrelevant. The private sector fully understands that the future interest rate will usually differ from what was announced, since the central bank will then respond to newly received information $\varepsilon_{2,2}^{CB}$; see (6). This eventual discrepancy is fully anticipated by the private sector because the central bank strategy—its loss function—is public knowledge, so credibility is not an issue here. The difference between the pre-announced rate $E_1^{CB}r_2$ and the actually chosen rate r_2 is purely random and therefore uninformative. Importantly, this result holds independently of the degree of precision of the signals received by the central bank and the private sector. What matters is that signal precision be known.[14]

[13] More precisely, inflation is the result of three forecasting errors since $\pi_1 = (\varepsilon_1 - E_1^{CB}\varepsilon_1) + \frac{1}{1+z}[(\varepsilon_{1,2}^{CB} - \varepsilon_2) - (\varepsilon_{1,2}^{CB} - \varepsilon_2)]$, which includes the central bank and private-sector early forecast errors about ε_2.

[14] The case when the signal precisions are not known is left for further research. For a study of this case in a different setting, see Gosselin, Lotz, and Wyplosz (2008).

Finally, for future reference, in this case of transparency the unconditional loss function is

$$L^{tr} = E(\pi_1)^2 + E(\pi_2)^2 = \frac{1}{\beta}\left[\frac{1}{z} + \frac{1}{k}\left(\frac{1}{1+z}\right)^2\left(\frac{1}{z}+1\right) + \frac{1}{z+k}\right].$$

4. The Central Bank Does Not Reveal Its Interest Rate Forecast

We consider now the case when the central bank does not announce its expectation of the future interest rate. We call this the opacity case. The optimal interest rate in period 2 remains given by (6). The resulting inflation rate is also the same as in (9), although the information available to the central bank is different from that in the previous case, as will be emphasized below.

In period 1, the central bank still reveals the current interest rate, which is set on the basis of its available information, i.e., $E_1^{CB}\varepsilon_1$ and $E_1^{CB}\varepsilon_2$. We restrict our attention to the following policy linear rule, which optimally uses all available information:[15]

$$r_1 = \mu E_1^{CB}\varepsilon_1 + \nu E_1^{CB}\varepsilon_2, \tag{11}$$

where μ and ν are unknown parameters to be determined by the optimal policy.

Having observed r_1, the private sector sets the inflation rate according to (3). To that effect, it needs to forecast future inflation, which by (9) depends on $E_2^{CB}\varepsilon_2$, the central bank's forecast. In forming this forecast, the central bank uses its signals $\varepsilon_{1,2}^{CB}$ and $\varepsilon_{2,2}^{CB}$ as well as period 1 inflation, which has now become known. In contrast to the previous case, $\varepsilon_{1,2}^{CB}$ is now unknown to the private sector. As a consequence, $E_1^P\varepsilon_2$ no longer coincides with $E_1^P E_2^{CB}\varepsilon_2$. In order to form its forecast $E_1^P E_2^{CB}\varepsilon_2$, following Bayes's rule, the

[15]There is no reason to presume that a linear rule is optimal. This restrictive assumption, required to carry through the calculations that follow, can be seen as a linear approximation of the optimal policy. This introduces some asymmetry between the transparency and opacity cases: in the former, the rule is optimal; in the latter, it may not be. Unfortunately, we are not able to derive the optimal policy choice under opacity.

private sector uses its three available signals $E_1^P\varepsilon_1$, $\varepsilon_{1,2}^P$, and r_1.[16] It can use $\varepsilon_{1,2}^P$ directly. In addition, the interest rate rule (11) implies that $E_1^{CB}\varepsilon_2 = (r_1 - \mu E_1^{CB}\varepsilon_1)/\nu$, so r_1 can be used to make inference about $E_2^{CB}\varepsilon_2$. The optimal forecast is necessarily of the form

$$\begin{aligned} E_1^P E_2^{CB}\varepsilon_2 &= \gamma_2^{op}\varepsilon_{1,2}^P + \left(1-\gamma_2^{op}\right)\left(\frac{r_1 - \mu E_1^{P'}\varepsilon_1}{\nu}\right) \\ &= \gamma_2^{op}\varepsilon_{1,2}^P + \left(1-\gamma_2^{op}\right)\left[\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^{P'}\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right] \quad (12) \end{aligned}$$

with γ_2^{op} to be determined. The same reasoning can be applied to $E_1^P\varepsilon_2$ to obtain

$$E_1^P\varepsilon_2 = \gamma_1^{op}\varepsilon_{1,2}^P + \left(1-\gamma_1^{op}\right)\left[\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^{P'}\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right], \quad (13)$$

where $\gamma_1^{op} = \frac{k(1+z)+\left(\frac{\nu}{\mu}\right)^2}{(1+z)\left[k+\left(\frac{\nu}{\mu}\right)^2\right]}$.

As in the transparency case, the unknown weighting coefficient γ_2^{op} can be found by identification. In this case, there is no simple analytical solution. The appendix shows that $\gamma_1^{op} - \gamma_2^{op}$ is the solution to a third-order equation that satisfies the following relation:

$$\gamma_1^{op} - \gamma_2^{op} = \frac{z\theta k\left[\theta k - \left(\frac{\nu}{\mu}\right)^2\right]}{(1+z)\left[k + \left(\frac{\nu}{\mu}\right)^2\right]\left[\left(\frac{\mu}{\nu}\right)^2\theta^2 zk + z + k\right]}, \quad (14)$$

where θ is defined as

$$\theta = 1 + \frac{1}{(1+\kappa)\left(\gamma_1 - \gamma_2^{op}\right) - \gamma_2^{op}}.$$

In comparison with the case where the central bank publishes its expected future interest rate, (14) implies that, in general, $\gamma_1^{op} \neq \gamma_2^{op}$ so that $E_1^P\varepsilon_2 \neq E_1^P E_2^{CB}\varepsilon_2$. From (4) and (6), it follows that

$$E_1^P\pi_2 = E_1^P\varepsilon_2 - E_1^P E_2^{CB}\varepsilon_2 \neq 0. \quad (15)$$

[16]More precisely, $E_1^P\varepsilon_1$ is not a signal but the expectation formed on the basis of signals $\varepsilon_{0,1}$ and $\varepsilon_{1,1}$.

Well aware that its own period 1 forecast of the disturbance ε_2 differs from that of the central bank, the private sector is no longer sure that the central bank can achieve its aim. This is the key difference between transparency and opacity. Private-sector doubt is reflected in the discrepancy between central bank and private-sector expectations, which is captured by $\gamma_1^{op} - \gamma_2^{op}$.

The appendix shows that the optimum interest rate rule in period 1 requires $\mu = -\nu = \kappa^{-1}$. The monetary policy rule is formally identical to (10) in the transparency case. As before, the reason is that, in order to minimize the volatility of π_1, the central bank seeks to set the nominal long-term interest rate to offset the first-period shock, which it expects to be $E_1^{CB}\varepsilon_1$; to do so, it must take into account its future interest rate, which it expects to choose so as to offset the future shock, which is expected to be $E_2^{CB}\varepsilon_2$. Thus, even if the central bank is transparent, it must still form a view of its future action.[17] The above results can be summarized as follows.

PROPOSITION 2. *Private-sector and central bank expectations are no longer aligned under opacity. While the interest rate rule is the same as when the central bank announces its expected future interest rate, the yield curve no longer matches the central bank forecast of the interest rate path.*

The resulting inflation rate in period 1 is

$$\pi_1 = \frac{1}{\theta - 1}\left[\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right) - \theta\left(E_1^{P'}\varepsilon_1 - \varepsilon_1\right) + \left(E_1^{CB}\varepsilon_1 - \varepsilon_1\right)\right], \quad (16)$$

which combines the forecast errors of both the private sector and the central bank. It follows that

$$E_1^P\pi_2 = \left(\gamma_1^{op} - \gamma_2^{op}\right)\left[\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right) - \left(E_1^{P'}\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right], \quad (17)$$

which shows the role of the doubt factor $\gamma_1^{op} - \gamma_2^{op}$: the private sector will not expect the central bank to eliminate inflation in period 2 unless $\gamma_1^{op} - \gamma_2^{op} = 0$.

[17]Note that even though the interest rate rules are formally the same under both transparency regimes, this does not imply the same interest and inflation rates. Indeed, the information sets of the central bank and of the private sector change with the transparency regime.

Using the expression for $E(\pi_2)^2$ provided in the appendix, we find the loss function under central bank opacity:

$$L^{op} = E(\pi_1)^2 + E(\pi_2)^2$$
$$= \frac{1}{\beta}\left[\left(\frac{1}{\theta-1}\right)^2\left(\frac{1}{z}+\frac{1}{k}+\frac{1}{kz}\right)+\left(\frac{\theta}{\theta-1}\right)^2+\frac{1+\theta^2 k}{\theta^2 zk+z+k}\right].$$

We mentioned in section 2.3 that there would be no welfare difference between transparency and opacity if the central bank could observe in period 1 the yield curve before making its decision. Indeed, in this case, observing $E_1^P r_1$ and $E_1^P r_2$ fully reveals the two private-sector signals $\varepsilon_{1,1}^P$ and $\varepsilon_{1,2}^P$. Similarly, observing $E_2^P r_2$ fully reveals $\varepsilon_{2,2}^P$. It follows that, in period 2, independently of the transparency regime, the central bank knows everything that the private sector knows, there is no mirror effect, and L_2 is the same. In period 1, independently of the regime, we have $E_1^P \pi_2 = E_1^P \varepsilon_2 - E_1^P E_2^{CB} \varepsilon_2 = E_1^P \varepsilon_2 - E_1^P \varepsilon_2 = 0$, i.e., inflation expectations are always aligned. It follows that $\pi_1 = -\kappa(r_1 + E_1^P r_2) + \varepsilon_1$ even though $E_1^P r_2$ is not the same under transparency and opacity. The central bank optimal decision then is $r_1 = -E_1^P r_2 + \frac{1}{\kappa} E_1^{CB} \varepsilon_1$, which implies $\pi_1 = \varepsilon_1 - E_1^{CB} \varepsilon_1$.[18] This shows that period 1 inflation, and therefore welfare, does not depend on the transparency regime.

5. Welfare Analysis

We now compare welfare when the central bank reveals its expected interest rate—labeled transparency—and when it does not—labeled opacity. To do so we study the difference of welfare losses under the two regimes: $\Delta L = L^{op} - L^{tr}$. In spite of the model's extreme simplicity, we cannot derive an explicit condition that determines the sign of ΔL. Consequently, we proceed in three steps. In section 5.1, we derive a sufficient condition for period 1 loss difference ΔL_1 to be positive; since $\Delta L_2 > 0$, this is also a sufficient condition for

[18] Proof: Note that $r_2 = \frac{1}{\kappa} E_2^{CB} \varepsilon_2$ so $E_1^P r_2 = \frac{1}{\kappa} E_1^P E_2^{CB} \varepsilon_2 = \frac{1}{\kappa} E_1^P \varepsilon_2$. This, in turn, implies that $E_1^{CB} E_1^P r_2 = \frac{1}{\kappa} E_1^{CB} E_1^P \varepsilon_2 = \frac{1}{\kappa} E_1^P \varepsilon_2$. Optimal monetary policy in period 1 is $r_1 = -E_1^{CB} E_1^P r_2 + \frac{1}{\kappa} E_1^{CB} \varepsilon_1$. With the previous result, this means $r_1 = -\frac{1}{\kappa} E_1^P \varepsilon_2 + \frac{1}{\kappa} E_1^{CB} \varepsilon_1$.

transparency to dominate opacity. Then, in section 5.2, we provide a necessary condition for $\Delta L_1 < 0$. Finally, we present in section 5.3 the results from the formal analysis of ΔL that is described in the appendix.

5.1 Preliminary Observation

We first compare the welfare losses separately period by period. Starting with period 2, we have

$$\beta \Delta L_2 = L_2^{op} - L_2^{tr} = \frac{1}{\beta} \frac{\theta^2 k^2}{(\theta^2 zk + z + k)(z + k)} > 0. \tag{18}$$

PROPOSITION 3. *Transparency is always welfare increasing in period 2.*

The reason is that the central bank is better informed when it can recover the private-sector signal $\varepsilon_{1,2}^P$; see (9).

Thus, a sufficient condition for transparency to be welfare improving is that the period 1 welfare difference $\Delta L_1 = L_1^{op} - L_1^{tr} \geq 0$. In the appendix we show that

$$\beta \Delta L_1(\theta) = \left(\frac{1}{\theta - 1}\right)^2 \frac{1 + k + z}{kz} + \left(\frac{\theta}{\theta - 1}\right)^2 - \frac{1 + k(1 + z)}{kz(1 + z)}, \tag{19}$$

and we study this expression as a function of θ. This analysis yields the following sufficient condition for transparency to be welfare improving.

PROPOSITION 4. *A sufficient condition for the release by the central bank of its expected future interest rate to be welfare improving is that* $z > \frac{1+k}{\sqrt{k}}$.

The more precise is the central bank signal α relative to the private-sector signal β—the higher is z—the more likely it is that transparency pays off. Conversely, if central bank information is of poor quality—i.e., when $z < \frac{1+k}{\sqrt{k}}$—the situation becomes ambiguous.[19]

[19] Over the relevant range of $\not{k}$, from zero to one, the function $(1 + k)/\sqrt{k}$ is decreasing from ∞ when $k = 0$ to 2 when $k = 1$.

The intuition is as follows. Transparency allows for the exchange of early signals between the central bank and the private sector: in period 1, the central bank reveals $\varepsilon_{1,2}^{CB}$; in period 2, it discovers $\varepsilon_{1,2}^{P}$. The ambiguous period 1 welfare effect of transparency, therefore, depends on the precision of the central bank early signal $\varepsilon_{1,2}^{CB}$, i.e., on k and z. Thus k and z act as complementary factors favoring transparency. A higher k means that transparency is achieved with a lower z, and conversely.

5.2 Why May Opacity Raise Welfare?

We now ask why opacity could ever raise welfare. It might seem that more information is always better than less. This is not necessarily true here since we have two agents—the central bank and the private sector—who strategically interact under heterogeneous information.[20]

The appendix provides a formal explanation of why less information may be welfare increasing. Here we use a very simple example to provide an intuitive interpretation. Consider the case where it will turn out that the two shocks ε_1 and ε_2 are nil, but the signals received by the central bank lead it to mistakenly infer an inflationary shock in period 1 ($E_1^{CB}\varepsilon_1 > 0$) and, correctly, no shock in period 2 ($E_1^{CB}\varepsilon_2 = 0$). Assume also that the private-sector signals turn out to be accurate, so $E_1^{P}\varepsilon_1 = E_1^{P}\varepsilon_2 = 0$. From (10) we know that, expecting an inflationary shock, the central bank raises the interest rate ($r_1 > 0$). This will turn out to be a policy mistake—optimal policy would call for $r_1 = 0$. When it observes the positive interest rate, knowing that it is set according to (10), the private sector can infer either that $E_1^{CB}\varepsilon_1 > 0$ or that $E_1^{CB}\varepsilon 2 < 0$, or a suitable combination of both. Under transparency, the central bank reveals $E_1^{CB}\varepsilon_2 = 0$, so the private sector understands that the central bank has raised the period 1 interest rate because of an inflationary signal. The private sector correctly expects the central bank to bring inflation to target in period 2 ($E_1^{P}\pi_2 = 0$) by keeping $r_2 = 0$. Then (3)

[20]In period 1, the central bank acts as a Stackelberg leader in setting r_1 and then the private sector reacts, setting π_1 and the long-term interest rate. Then, in period 2, the central bank reacts and sets r_2.

shows that $\pi_1 < 0$ because the central bank policy is too restrictive in period 1.

When the central bank does not publish its interest rate forecast, the private sector no longer knows for sure why the interest rate has been raised. For the sake of reasoning, let us consider two possible extreme assumptions about private-sector inference in this situation. If the private sector correctly guesses that $E_1^{CB}\varepsilon_1 > 0$ and $E_1^{CB}\varepsilon_2 = 0$, the situation is the same as under transparency. If instead the private sector incorrectly infers from the interest rate increase that the central bank expects a deflationary shock in period 2 ($E_1^{CB}\varepsilon_1 = 0$ and $E_1^{CB}\varepsilon_2 < 0$), it will conclude that the central bank plans to lower r_2 and has raised r_1 to keep the long-term rate unchanged in shockless period 1. Its expectation of a lower interest rate ($E_1^P r_2 < 0$) leads the private sector to raise its inflation forecast ($E_1^P \pi_2 > 0$). Both terms tend to offset in (3) the effect on π_1 of the contractionary policy actually carried out by the central bank.

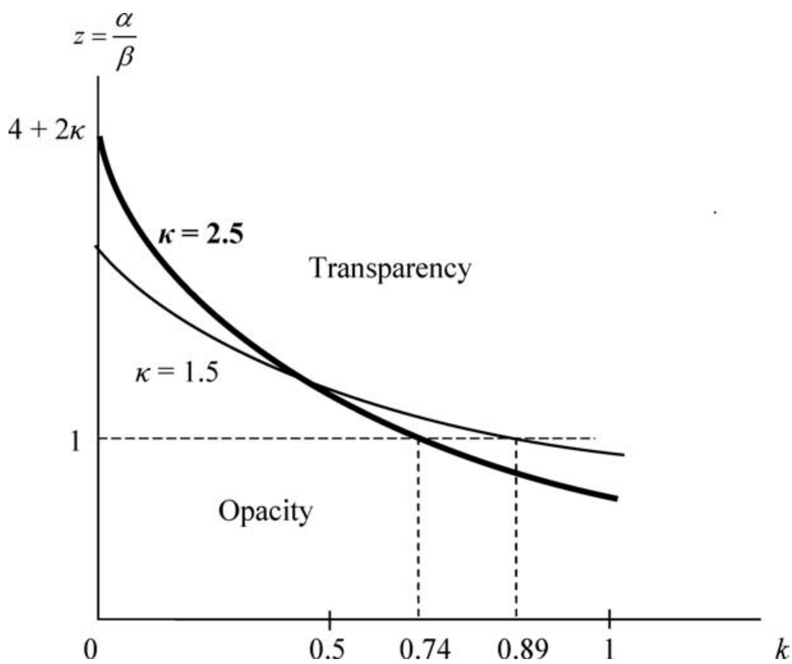
This example illustrates how, under opacity, the private sector's misinterpretation of the central bank action mitigates the effect of a policy mistake and possibly raises welfare. This is a special case of the more general result, developed in the appendix, that opacity is desirable when it leads the private sector to systematically draw inference from the central bank action in a way that offsets policy mistakes due to imperfect signals. This leads to the following proposition.

PROPOSITION 5. *(Creative Opacity) A necessary condition for opacity to welfare dominate transparency is that the private sector's own forecasts systematically offset the impact on inflation volatility of the central bank forecast errors.*

5.3 Welfare Ranking

We have derived a *sufficient* condition for transparency to be desirable and a *necessary* condition for opacity to dominate. We now study how the necessary and sufficient sign condition for ΔL relates to the three model parameters z, κ, and k, with $z \geq 0$, $k \in [0, 1]$, and $\kappa \geq 0$. Figure 2 summarizes the results established in the appendix. It displays two curves that correspond to two values of κ. The area below each curve corresponds to $\Delta L = L^{op} - L^{tr} < 0$, i.e., to the

Figure 2. Welfare Outcomes



case where welfare is higher when the central bank does not reveal its interest rate path forecast.

The following proposition summarizes the results of this analysis.

PROPOSITION 6. *When the central bank follows the optimal linear interest rate rule (11), ceteris paribus, transparency dominates when z is large and when k is large. The role of κ is ambiguous: when k is small, an increase in κ favors opacity, while it favors transparency when k is large.*

We interpret these results below.

The Role of Relative Signal Precision. We first look at the role of $z = \alpha/\beta$, the ratio of central bank signal precision α to private-sector signal precision β. The higher is z, the more likely it is that transparency is desirable. The reason is clear: the publication of the interest rate path provides the private sector with a central bank signal that is more useful the more precise it is relative to its own signals.[21] As z becomes smaller, the benefit from information disclosure declines because the private sector increasingly doubts any signal from the central bank, for good reason. Opacity increases welfare when, having observed the current interest rate r_1, the private sector sets prices and the long-term interest rate so as to systematically offset the effects of potential large central bank forecast errors. Note that this is not the same result as in Morris and Shin (2002), which deals with a common-knowledge effect that is not considered here.

The Role of Early Information Precision. The parameter k, which ranges from zero to one, represents that precision of early signals relative to updated signals. By releasing in period 1 its forecast of the interest rate that it expects to set in period 2, the central bank reveals its early signal $\varepsilon_{1,2}^{CB}$ of the shock ε_2 expected in period 2. Then, in period 2, the central bank can decipher the early signal $\varepsilon_{1,2}^{P}$ received by the private sector in period 1 concerning the same shock ε_2. Since transparency makes this exchange of early signals possible, it is more desirable the more precise are these signals. Indeed, when $k = 0$, these signals become nearly useless.

Yet, figure 2 shows that there always exists a high enough z to make transparency desirable.[22] The reason is that, even when when $k = 0$, in period 1 the private sector still needs to set inflation and the long end of the yield curve. Under opacity, it must rely on its infinitely imprecise early signal $\varepsilon_{1,2}^{P}$ as well as on the interest rate r_1 announced by the central bank, on the basis of its own infinitely imprecise signal $\varepsilon_{1,2}^{CB}$. When the central bank is generally better informed than the private sector—when z is large—it therefore helps the private sector to know $\varepsilon_{1,2}^{CB}$. Put differently, with $k = 0$, the private sector "buys" whatever information it gets from the central bank when z is large. Under opacity, it will not assume that the central bank is misled and will still conclude that $E_1^P \pi_2 \simeq 0$. As expectations are aligned, there is no room for creative

[21]Under opacity, as $z \to \infty$, $\gamma_1^{op} - \gamma_2^{op} \to 0$, so $E_1^P \pi_2 = 0$: opacity converges to transparency. From (18) we know that $\lim_{z\to\infty} \Delta L_2 = 0$ and, from (19), $\lim_{z\to\infty} \beta \Delta L_1 = \frac{1/k+\theta^2}{(\theta-1)^2} > 0$, so transparency dominates when z is large enough.

[22]The intercept of the curve on the vertical axis is $4 + 2\kappa$.

opacity to trigger the kind of welfare-improving correction described in section 4.[23]

As k increases, more attention is paid by both the central bank and the private sector to their own early signals, not just to the other agents' early signals. Under opacity, this heightened attention increases the expectation discrepancy, which is a source of welfare loss. At the same time, because it interprets the current interest rate as conveying information on the central bank's early signal when it sets the long-term rate, the private sector may offset the central bank forecast error, which improves welfare. The expectation discrepancy, which rises with k, directly hurts welfare but may be exploited to raise it indirectly.

Put differently, when it is welfare improving under opacity, the private-sector correction of the mistaken central bank policy decision is more effective the lower is the precision k of early signals. This is because early information swapping under transparency is less effective when k is low, which makes the private-sector correction relatively more helpful.

The Role of the Elasticity of Current to Expected Inflation. Parameter κ represents the channel through which private forecasts of inflation and the long-term interest rate affect current inflation; see (3). As κ increases, the curve that marks the frontier between transparency and opacity in figure 2 shifts up on the left where k is small (the intercept with the horizontal axis is $4 + 2\kappa$) and down on the right where k is large.

To understand why, we need to consider two different effects. First, remember that, due to the non-alignment of central bank and private-sector expectations, opacity tends to increase the volatility of private-sector forecasts and therefore inflation volatility. This effect increases when κ rises, which tends to make transparency more desirable, i.e., to shift the curve down. Second, we have previously noted that the private-sector correction of the central bank error is more likely to stabilize inflation, and therefore to be welfare increasing, the lower is k; we also noted that this effect is reinforced when κ rises. This explains why an increase in κ favors opacity for low values of k. When, instead, k is large, the exchange of noisy early signals under

[23]The appendix shows that the non-alignment of inflation expectations is proportional to the doubt factor $\gamma_1^{op} - \gamma_2^{op}$, which becomes nil as $k \rightarrow 0$.

transparency stabilizes expectations and, through κ, the inflation rate.

Lessons from Calibrated Models. The model that we use to derive this result has been calibrated in the literature. In this section, we look at the welfare implications of the parameter values suggested by Galí and Gertler (2007). In their quarterly model, they set $\kappa = 0.167$. In our model, a period is better thought of as lasting two or three years, which approximately implies that κ should range between 1.33 and 2. Svensson (2005a) argues that z cannot be lower than unity and is probably larger. Estimates of z from Clark and McCrackin (2006b) range from 0.83 to 1.55. Taking $z = 1$ as a reference, we ask what the minimum value of k must be in order for transparency to welfare dominate opacity. As shown in figure 2, the critical values of k are 0.74 for $\kappa = 2.5$ and 0.89 for $\kappa = 1.5$. Estimates of k by Clark and McCrackin (2006a) range from 0.43 to 1.

On this basis, the conclusion is that the desirability of publishing the interest rate path is a close call, with most parameter values falling in the no-transparency zone. Of course, these parameter values are to be taken with considerable precaution and, even more importantly, the model is far too simple to be taken at face value. Our purpose is emphatically not to reach normative conclusion but to explore what mechanisms come into play when a central bank publishes its interest rate forecast.

6. Conclusions

The general presumption in the (so far limited) academic literature is that transparency is welfare superior. With few exceptions, most central banks take the opposite view. This paper is a first step to breach the gap. For opacity to welfare dominate transparency, we must identify a market failure. This paper is based on the view that there exists an important degree of information heterogeneity between central banks and the private sector. The results imply that neither side can ever fully recover the information of the other side by simply observing its actions—the private sector observes the interest rate set by the central bank and the central bank observes financial prices. This makes it possible for opacity to welfare dominate transparency. In the simple setup adopted here, opacity is desirable when the private sector misinterprets the central bank decisions and sets its forecasts in a way that offsets the effect of central bank forecasting errors. This double coincidence of forecast errors makes the case for opacity quite weak.

In contrast, the case made by central banks against transparency relies on private-sector confusion between forecasts and commitments. We show that, when it is assumed that both the central bank and the private sector act optimally on the basis of optimal signal extraction and in the absence of a time-inconsistency problem, this is a non-issue. This is so because the private sector has no reason to doubt that the interest rate path announced by the central bank is optimal given its information set. As the information set changes, so must the optimal path. Put differently, the standard case against transparency relies either on suboptimal private behavior—the private sector does not form its expectations on the basis of available information—or on the dubious assumption that central banks act strategically in a way that gives rise to time inconsistency. Narrowing down the policy debate is, we hope, a relevant contribution.

Another insight is that the case for transparency is enhanced when early signals are precise relative to contemporaneous signals. Put differently, the interest rate path becomes less useful when the outlook becomes more uncertain. This resembles the Morris-Shin result, but the mechanism is completely different: it pits early against contemporaneous information precision. This aspect does not seem to have been noted in the literature so far. It suggests that the benefits from transparency can change over time, depending on the prevailing situation. For instance, revealing the expected interest rate path may be undesirable when longer-run uncertainty rises.

Transparency does not just allow a central bank to better (i.e., more credibly) share its information with the private sector; it also gives rise to the mirror effect whereby the central bank also obtains some information back. In our simple model, this means that inflation expectations of the private sector and the central bank are perfectly aligned. The realistic version of this result, which would follow from allowing for more signals, is that transparency lowers the volatility of expected future inflation and therefore the volatility of current inflation. This is a testable proposition, which could narrow down the policy debate when enough observations from the current experiments become available.

A last, fairly obvious result is that transparency is more desirable the better informed is the central bank and the more elastic is output to the long-term interest rate, i.e., the more effective is monetary policy. In other words, the case for transparency is stronger when the central bank is well informed and powerful.

Obviously, we do not address all the arguments against the publication of the interest rate path. Consider, for example, the articulate presentation of the case against transparency by Goodhart (2005):

> If an MPC's non-constant forecast was to be published, there is a widespread view, in most central banks, that it would be taken by the public as more of a commitment, and less of a rather uncertain forecast than should be the case (though that could be mitigated by producing a fan chart of possible interest rate paths, rather than a point estimate: no doubt, though, measuring rulers and magnifying glasses would be used to extract the central tendency). Once there was a published central tendency, then this might easily influence the private sector's own forecasts more than its own inherent uncertainty warranted, along lines analyzed by Morris and Shin (1998, 2002, 2004). Likewise when new, and unpredicted, events occurred, and made the MPC want to adjust the prior forecast path for interest rates, this might give rise to criticisms, ranging from claims that the MPC had made forecasting errors to accusations that they had reneged on a (partial) commitment.

Part of the argument directly refers to Morris and Shin's common-knowledge effect. We do not address this issue here because it has been shown to rest on highly unlikely assumptions. Indeed, it assumes that the central bank is relatively poorly informed (z is low) and that the central bank does not even reveal the current interest rate.[24] Another part of the argument is that releasing the expected interest rate might lock the central bank into setting its interest rate in the future at forecasted level, even though it is no longer desirable given newly available information. This is the classic rules-versus-discretion argument in the presence of time inconsistency, as discussed in Woodford (2005). In our model, time inconsistency is

[24]See Hellwig (2005), Svensson (2005a), and Gosselin, Lotz, and Wyplosz (2008).

eliminated because we do not allow for its two constituent ingredients, the presence of an inflation bias and unknown central bank preferences—two assumptions that we consider unrealistic.[25]

Of course, we too make a large number of assumptions. Some of them, discussed in section 2.3, are simplifications that can be generalized to be more realistic without, we believe, affecting the policy conclusions. We assume that all signal precisions are known. In Gosselin, Lotz, and Wyplosz (2008), in a different setup that focuses on the common-knowledge effect, we show that uncertainty about signal precision carries subtle changes, most of which tend to favor opacity.

Our assumption that the economy starts from and ends at the steady state is not innocuous. In particular, it implies that inflationary expectations are perfectly anchored. Along with a loss function that focuses only on inflation, it implies that the central bank does not aim at a gradual path guiding inflation to its target. Preliminary investigation of an extension of our model to an arbitrary number of periods suggests the following tentative observations. The sharp distinction between periods 1 and 2 would disappear. With it, the result that transparency is always welfare superior in period 2 would be lost. This would work against transparency. On the other hand, in each period the central bank would benefit from the mirror effect as the result of previous publication of the expected interest rate path. This is likely to strengthen the welfare case for transparency.

Appendix

Proof of Equation (14)

Using (11), note that $E_1^{CB}\varepsilon_1 = (r_1 - \nu\varepsilon_{1,2}^{CB})/\mu$ is a signal about ε_1. In period 1, the private sector observes $\frac{r_1 - \nu\varepsilon_{1,2}^{P}}{\mu} = E_1^{CB}\varepsilon_1 + \frac{\nu}{\mu}(\varepsilon_{1,2}^{CB} - \varepsilon_{1,2}^{P})$, which is therefore also a signal about ε_1 available for the private sector with variance $\frac{1}{\alpha} + (\frac{\nu}{\mu})^2(\frac{1}{k\alpha} + \frac{1}{k\beta})$. Similarly, in period 1,

[25] The experience of the Bank of Norway is particularly interesting in this respect. Realizing that credibility is necessary to avoid misinterpretations of the difference between the forecasted and actual interest rate, the Bank of Norway is actively engaged in describing its preferences.

the private sector observes $\frac{r_1-\mu\varepsilon_1^P}{\nu} = \varepsilon_{1,2}^{CB} + \frac{\mu}{\nu}(E_1^{CB}\varepsilon_1 - E_1^{P'}\varepsilon_1)$, which is a signal about ε_2 with variance $\frac{1}{\beta}\left[\frac{1}{kz} + \left(\frac{\mu}{\nu}\right)^2\left(1+\frac{1}{z}\right)\right]$. Using these signals, we can apply Bayes's theorem to obtain

$$E_1^P\varepsilon_1 = \frac{\left[k+(1+z)\left(\frac{\nu}{\mu}\right)^2\right]E_1^{P'}\varepsilon_1 + kz\left[E_1^{CB}\varepsilon_1 - \frac{\nu}{\mu}\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right)\right]}{k(1+z)+(1+z)\left(\frac{\nu}{\mu}\right)^2}$$

$$E_1^P\varepsilon_1^{CB} = \frac{\left(\frac{\nu}{\mu}\right)^2 E_1^{P'}\varepsilon_1 + k\left[E_1^{CB}\varepsilon_1 - \frac{\nu}{\mu}\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right)\right]}{k+\left(\frac{\nu}{\mu}\right)^2}$$

$$E_1^P\varepsilon_2 = \frac{\left[k(1+z)+\left(\frac{\nu}{\mu}\right)^2\right]\varepsilon_{1,2}^P + z\left(\frac{\nu}{\mu}\right)^2\left[\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^{P'}\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right]}{(1+z)\left(k+\left(\frac{\nu}{\mu}\right)^2\right)}$$

$$= \gamma_1^{op}\varepsilon_{1,2}^P + \left(1-\gamma_1^{op}\right)\left[\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^{P'}\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right],$$

which defines $\gamma_1^{op} = \frac{k(1+z)+\left(\frac{\nu}{\mu}\right)^2}{(1+z)\left(k+\left(\frac{\nu}{\mu}\right)^2\right)}$.

$$E_1^P\varepsilon_2^{CB} = \frac{k\varepsilon_{1,2}^P + \left(\frac{\nu}{\mu}\right)^2\left[\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^{P'}\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right]}{k+\left(\frac{\nu}{\mu}\right)^2}.$$

It follows that

$$E_1^P\varepsilon_1 - E_1^P\varepsilon_1^{CB} = \frac{E_1^{P'}\varepsilon_1 - E_1^{CB}\varepsilon_1 + \frac{\nu}{\mu}\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right)}{(1+z)\left(k+\left(\frac{\nu}{\mu}\right)^2\right)}.$$

Recalling (12) and using (3), (6), and (9), we can now compute π_1, which is necessary to obtain the signal extracted by the central bank at time 2:

$$\begin{aligned}\pi_1 &= (1+\kappa)\big(E_1^P\varepsilon_2 - E_1^P E_2^{CB}\varepsilon_2\big) - E_1^P E_2^{CB}\varepsilon_2 - \kappa r_1 + \varepsilon_1\\ &= (1+\kappa)\big(\gamma_1^{op}-\gamma_2^{op}\big)\left[\varepsilon_{1,2}^P - \left(\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\big(E_1^{P'}\varepsilon_1 - E_1^{CB}\varepsilon_1\big)\right)\right]\\ &\quad - \kappa r_1 - \left[\gamma_2^{op}\varepsilon_{1,2}^P + \big(1-\gamma_2^{op}\big)\left(\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\big(E_1^{P'}\varepsilon_1 - E_1^{CB}\varepsilon_1\big)\right)\right] + \varepsilon_1.\end{aligned}$$

This expression can be rewritten as

$$\frac{\pi_1 + \kappa r_1 - \varepsilon_1}{(1+\kappa)\big(\gamma_1^{op}-\gamma_2^{op}\big)-\gamma_2^{op}} + \theta\varepsilon_{1,2}^{CB} = \varepsilon_{1,2}^P + \theta\frac{\mu}{\nu}\big(E_1^{P'}\varepsilon_1 - E_1^{CB}\varepsilon_1\big),$$

where we have introduced an auxiliary variable $\theta = 1 + \frac{1}{(1+\kappa)(\gamma_1^{op}-\gamma_2^{op})-\gamma_2^{op}}$. Now note that π_1 and ε_1 become known in period 2 (and r_1 is always known). It follows that the right-hand side in the previous expression is known to the central bank when period 2 starts and it can be used as a signal about ε_2. However, the central bank can improve this signal by replacing $E_1^{CB}\varepsilon_1$ with ε_1 so that the signal about ε_2 is now $\varepsilon_{1,2}^P + \theta\frac{\mu}{\nu}(E_1^{P'}\varepsilon_1 - \varepsilon_1)$, with variance $\frac{1}{\beta}\left(\frac{1}{k} + \theta^2\left(\frac{\mu}{\nu}\right)^2\right)$.

We next use Bayes's rule to find $E_1^P E_2^{CB}\varepsilon_2$. The relevant computation leads to

$$E_2^{CB}\varepsilon_2 = \frac{\left[\left(\frac{\mu}{\nu}\right)^2\theta^2 zk + z\right]\left[k\varepsilon_{1,2}^{CB} + (1-k)\varepsilon_{2,2}^{CB}\right] + k\left[\varepsilon_{1,2}^P + \frac{\mu}{\nu}\theta\big(E_1^{P'}\varepsilon_1 - \varepsilon_1\big)\right]}{\left(\frac{\mu}{\nu}\right)^2\theta^2 zk + z + k}$$

so that the compounded expectation is given by

$$E_1^P E_2^{CB}\varepsilon_2 = \frac{\left[\left(\frac{\mu}{\nu}\right)^2\theta^2 zk + z\right]\left[kE_1^P\varepsilon_{1,2}^{CB} + (1-k)E_1^P\varepsilon_{2,2}^{CB}\right]}{\left(\frac{\mu}{\nu}\right)^2\theta^2 zk + z + k}$$
$$+ k\frac{\varepsilon_{1,2}^P + \frac{\mu}{\nu}\theta(1-\gamma_1)\left[\left(E_1^{P'}\varepsilon_1 - \varepsilon_1^{CB}\right) + \left(\frac{\nu}{\mu}\right)\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right)\right]}{\left(\frac{\mu}{\nu}\right)^2\theta^2 zk + z + k}.$$

Using the expressions for the various private-sector expectations, we can deduce by identification

$$\gamma_2^{op} = \left[\left(\frac{\mu}{\nu}\right)^2\theta^2 zk + z\right]\frac{\frac{k^2}{k+\left(\frac{\nu}{\mu}\right)^2} + (1-k)\frac{k(z+1)+\left(\frac{\nu}{\mu}\right)^2}{k(z+1)+(z+1)\left(\frac{\nu}{\mu}\right)^2}}{\left(\frac{\mu}{\nu}\right)^2\theta^2 zk + z + k}$$
$$+ k\frac{1+\theta\left(\frac{x\left(\frac{\nu}{\mu}\right)^2}{k(z+1)+(z+1)\left(\frac{\nu}{\mu}\right)^2}\right)}{\left(\frac{\mu}{\nu}\right)^2\theta^2 zk + z + k}$$

from which we find (14).

Proof of Proposition 2

The parameters for r_1 are found by minimizing the unconditional loss function $E(\pi_1)^2 + E(\pi_2)^2$.[26] Using (14), the previous expression for π_1 can be rewritten as

$$\pi_1 = \frac{1}{\theta-1}\left(\varepsilon_{1,2}^P - \varepsilon_2\right) + \frac{\mu}{\nu}\frac{\theta}{\theta-1}\left(E_1^{P'}\varepsilon_1 - \varepsilon_1\right)$$
$$+ \left(\kappa\nu + \frac{\theta}{\theta-1}\right)\left[\left(\varepsilon_2 - \varepsilon_{1,2}^{CB}\right) + \frac{\mu}{\nu}\left(\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right]$$
$$+ (1-\kappa\mu)\varepsilon_1 - (1+\kappa\nu)\varepsilon_2,$$

[26] It is unconditional because, if it were conditional on central bank information, the coefficients μ and ν would be nonlinear functions of $E_1^{CB}\varepsilon_1$ and $E_1^{CB}\varepsilon_2$, so the rule would not be linear—and impossible to derive in closed form.

which implies that

$$E(\pi_1)^2 = (1-\kappa\mu)^2 E(\varepsilon_1)^2 + (1+\kappa\nu)^2 E(\varepsilon_2)^2 + \text{other terms},$$

where the other terms depend on k, $z = \frac{\alpha}{\beta}$, μ, and ν.

Similarly, note that $\pi_2 = \varepsilon_2 - E_2^{CB}\varepsilon_2$ and that $E_2^{CB}\varepsilon_2$ is optimally found by the central bank by using the signals $\varepsilon_{1,2}^{CB}$, $\varepsilon_{2,2}^{CB}$, and $\varepsilon_{1,2}^{P} + \theta\frac{\mu}{\nu}(E_1^{P'}\varepsilon_1 - E_1^{CB}\varepsilon_1)$ as indicated above, which gives

$$E_2^{CB}\varepsilon_2 = \left[\left(\frac{\mu}{\nu}\right)^2 \theta^2 zk + z\right] \frac{\left(\frac{\mu}{\nu}\right)^2 \theta^2 zk + z\left[k\varepsilon_{1,2}^{CB} + (1-k)\varepsilon_{2,2}^{CB}\right]}{\left(\frac{\mu}{\nu}\right)^2 \theta^2 zk + z + k}$$
$$+ k\frac{\varepsilon_{1,2}^{P} + \frac{\mu}{\nu}\theta\left(E_1^{P'}\varepsilon_1 - \varepsilon_1\right)}{\left(\frac{\mu}{\nu}\right)^2 \theta^2 zk + z + k}$$

so that

$$\pi_2 = \left[\left(\frac{\mu}{\nu}\right)^2 \theta^2 zk + z\right] \frac{k\left(\varepsilon_2 - \varepsilon_{1,2}^{CB}\right) + (1-k)\left(\varepsilon_2 - \varepsilon_{2,2}^{CB}\right)}{\left(\frac{\mu}{\nu}\right)^2 \theta^2 zk + z + k}$$
$$+ k\frac{\left(\varepsilon_2 - \varepsilon_{1,2}^{P}\right) - \dfrac{\mu}{\nu}\theta\left(E_1^{P'}\varepsilon_1 - \varepsilon_1\right)}{\left(\frac{\mu}{\nu}\right)^2 \theta^2 zk + z + k}$$

and $E(\pi_2)^2$ only includes terms in k, z, μ, and ν. It follows that the total unconditionally expected loss under opacity can be written as

$$L^{op} = (1-\kappa\mu)^2 E(\varepsilon_1)^2 + (1+\kappa\nu)^2 E(\varepsilon_2)^2 + \text{other terms}.$$

Since both ε_1 and ε_2 are assumed to be uniformly distributed, $E(\varepsilon_1)^2$ and $E(\varepsilon_2)^2$ are arbitrarily large relative to the other terms—in particular, the variances α^{-2} and β^{-2}. It follows that the rule that minimizes L^{op} sets these terms equal to zero. Using the expression for π_2, we find the unconditional expectation $E(\pi_2)^2$, which measures the second period loss:

$$E(\pi_2)^2 = \frac{1+\theta^2 k}{\theta^2 zk + z + k}.$$

Proof of Proposition 4

The study of (19) shows that $\beta\Delta L_1(\theta)$ reaches a minimum of $\frac{kz^2-(1+k)^2}{kz(1+k)(1+z)}$ when $\theta = -\frac{1+k+z}{kz}$. This minimum is positive when $z > \frac{1+k}{\sqrt{k}}$.

The Sign of $\gamma_1^{op} - \gamma_2^{op}$

Using the optimality condition $\frac{\mu}{\nu} = -1$, the parameters γ_2^{op} and θ are jointly determined by the two following equations:

$$\theta = 1 + \frac{1}{(1+\kappa)\big(\gamma_1^{op} - \gamma_2^{op}\big) - \gamma_2^{op}} = 1 + \frac{1}{(2+\kappa)\left(\gamma_1^{op} - \gamma_2^{op}\right) - \gamma_1^{op}}$$

$$\gamma_1^{op} - \gamma_2^{op} = \frac{z\theta k(\theta k - 1)}{(1+z)(1+k)(\theta^2 zk + z + kt)}.$$

Defining $x = (1+z)(1+k)(\gamma_1^{op} - \gamma_2^{op})$, we can rewrite these two equations as

$$\theta = 1 + \frac{1}{(2+\kappa)\dfrac{x}{(1+z)(1+k)} - \gamma_1^{op}} = \frac{((2+\kappa)x + z)}{(2+\kappa)x - [k(1+z)+1]}$$

$$x(\theta^2 zk + z + k) = z\theta k(\theta k - 1).$$

They can be combined to yield the following third-order equation in x:

$$A_3 x^3 + A_2 x^2 + A_1 x + A_0 = 0,$$

where

$$\begin{aligned}
A_3 &= kz(2+\kappa)^2 + (k+z)(2+\kappa)^2 \\
A_2 &= 2kz^2(2+\kappa) - 2(k+z)(2+\kappa)[1 + k(z+1)] \\
&\quad - kz(2+\kappa)[k(2+\kappa) - 2 - \kappa] \\
A_1 &= kz^3 + (k+z)(-k(z+1) - 1)^2 - z^2 k[k(2+\kappa) - 2 - \kappa] \\
&\quad - kz(2+\kappa)[zk + k(z+1) + 1] \\
A_0 &= -z^2 k(zk + k(z+1) + 1).
\end{aligned}$$

The graphical study of the solutions of this equation shows that, for κ not too large (the threshold exceeds any realistic value of κ), this equation admits a single noncomplex solution x, which is always positive.[27] This establishes our claim that $\gamma_1^{op} - \gamma_2^{op} > 0$.

For further reference, we have the following limit conditions:

- When $z \to 0$, $x \to 0$ (as $\frac{z^2}{(k+1)}$).
- When $z \to \infty$, $x \to 2\frac{k}{k+1}$ and $\gamma_1^{op} - \gamma_2^{op} \to 0$.
- When $k \to 0$, $x \to 0$ (as kz).
- When $\kappa \to \infty$, there are three possibilities, all of which imply that $\gamma_1^{op} - \gamma_2^{op} < 0$: $x \to 0$ (as $\frac{-z}{k}$), $x \to \frac{-zk(1-k)}{zk+z+k}$, or $x \to -\frac{k+2kz+1}{-2-\kappa+2k+k\kappa}$.

Proof of Proposition 5

Remember first that welfare in period 2 is always higher under transparency because it provides the central bank with more information and therefore a more precise estimate of the period shock. For opacity to welfare dominate transparency, therefore, it must reduce inflation volatility in period 1 by enough to offset the welfare loss of period 2. From (3) we know that period 1 inflation is driven by expected inflation and the extent to which the central bank fails to stabilize output in period 1. Using (10), (6), and (15), (3) can be rewritten as

$$\pi_1 = (1+\kappa)E_1^P \pi_2 - \kappa\big(r_1 + E_1^P r_2\big) + \varepsilon_1 = (2+\kappa)E_1^P \pi_2 - \psi. \quad (20)$$

The "policy miss" term $\psi = \kappa r_1 - (\varepsilon_1 - E_1^P \varepsilon_2)$ measures the private sector's perception of the extent to which the central bank fails to achieve its period 1 objective when it optimally chooses r_1. Without any information about the private signals, the central bank's best forecast of $E_1^P \pi_2$ is $E_1^{CB} E_1^P \pi_2 = 0$, which explains (10). The policy miss term can be rewritten as

$$\psi = \big(E_1^{CB}\varepsilon_1 - \varepsilon_1\big) + \big(E_1^P \varepsilon_2 - E_1^{CB}\varepsilon_2\big). \quad (21)$$

[27]When κ is above this threshold and z is not too large, we have three real solutions for x, two of which are negative. When z becomes large, again, there is a unique real solution, which is positive.

Let us now compare (20) under the two regimes. Under transparency, but not under opacity, $E_1^P\pi_2 = 0$, so opacity tends to add volatility to period 1 inflation, the more so the higher is κ. Furthermore, we can show that $Var^{op}(\psi) > Var^{tr}(\psi)$. This is quite intuitive: the first term in (21), the period 1 signal error, is regime invariant, while the second term reflects disagreements between the central bank and the private sector. When it announces $E_1^{CB}r_2$, the central bank fully reveals $E_1^{CB}\varepsilon_2$ and therefore moves $E_1^P\varepsilon_2$ toward $E_1^{CB}\varepsilon_2$.

It follows that opacity always raises period 1 inflation as well, unless $cov(E_1^P\pi_2, \psi)$ under opacity is positive and large enough to offset the other two effects. Thus $cov(E_1^P\pi_2, \psi) > 0$ is a necessary, but not sufficient, condition for opacity to raise welfare. Since $cov(E_1^P\pi_2, \psi) = (\gamma_1^{op} - \gamma_2^{op})\frac{k+1}{\alpha k} > 0$, as shown above, this condition is always satisfied. What is needed, therefore, is that $cov(E_1^P\pi_2, \psi) > 0$ be large enough. This is the case when z, k, and κ are small.

Proof of Proposition 6

Using the loss functions given in the text and using $\gamma_1^{op} = \frac{k(1+z)+1}{(1+z)(k+1)}$, we have

$$\beta\Delta L = \frac{1}{((z+1)(k+1))^2}\left(((2+\kappa)x - (k(z+1)+1))^2\frac{k+1+z}{kz} + ((2+\kappa)x+z)^2\right) - \frac{1+k(1+z)}{kz(1+z)} + \frac{k}{z(z+k)}\frac{((2+\kappa)x+z)}{(2k+k\kappa-2-\kappa)x+k+2zk+1}x,$$

where x, defined in this appendix (in the study of the sign of $\gamma_1^{op} - \gamma_2^{op}$), is the solution of a polynomial of degree 3. We can also write

$$\beta\Delta L = \frac{P(x)}{-(1-k)(2+\kappa)x+k+2zk+1},$$

with $P(x)$ of degree 2 where the coefficient of x^2 is positive $\forall$ k, z, κ. We do not specify the form of $P(x)$, since we will only need some of properties of this function, which we study graphically using Mapple. Note that x is a solution of the third-degree polynomial shown above in this appendix. For any such x, the sign of $-(1-k)(2+\kappa)x+k+2zk+1$ is always positive (the hypersurfaces defined by the third-degree equation for x and by $-(1-k)(2+\kappa)x+k+2zk+1=0$ do not intersect). As a consequence, the sign of ΔL reduces to the sign of the second-degree polynomial $P(x)$. Denoting $\Delta(k,z,\kappa)$ the discriminant of $P(x)$, we draw the following conclusions:

- If $\Delta < 0$, then $\Delta L > 0$ whatever x, the solution of the third-order equation, and transparency dominates.
- If $\Delta > 0$ and x, the solution of the third-order equation, lies outside the roots of P, then $\Delta L > 0$ and transparency dominates.
- If $\Delta > 0$ and x, the solution of the third-order equation, lies inside the roots of P, then $\Delta L < 0$ and opacity dominates.

The study of ΔL consists then in checking whether or not the roots of the third-order equation lie between the roots of $P(x)$. The results are presented graphically in the (k,z) plane in figure 2, which displays $\Delta(k,z,\kappa)=0$. Above this curve, $\Delta(k,z,\kappa) < 0$ and transparency dominates. The shape of the opacity zone below $\Delta(k,z,\kappa)=0$ has been determined from a graphical three-dimensional analysis using Mapple and is therefore not precisely known. The figures are also informed by the analytic study of the following limit cases:

- For $z \to 0$, $\Delta L < 0$.
- For $z \to \infty$, $\Delta L > 0$.
- For $k \to 0$, $\Delta L > 0$ when $z - 2\kappa - 4 > 0$, and $\Delta L < 0$ otherwise.

Proposition 6 states that, as κ increases, the curve $\Delta L = 0$ shifts up when k is small and down when k is large. This is not entirely accurate. The graphical analysis indicates that when κ is small, the curve shifts upward $\forall k$. For economically relevant values of κ, however, the statement in proposition 6 is accurate.

References

Archer, D. 2005. "Central Bank Communication and the Publication of Interest Rate Projections." Unpublished Paper, Bank for International Settlements.

Blinder, A. S., C. A. E. Goodhart, P. M. Hildebrand, D. Lipton, and C. Wyplosz. 2001. *How Do Central Banks Talk?* Geneva Report on the World Economy, Vol. 3. London: Centre for Economic Policy Research.

Clark, T. E., and M. W. McCracken. 2006a. "Averaging Forecasts from VARs with Uncertain Instabilities." Working Paper No. 06-12, Federal Reserve Bank of Kansas City.

———. 2006b. "Forecasting with Small Macroeconomic VARs in the Presence of Instabilities." Economic Research Paper No. 06-9, Federal Reserve Bank of Kansas City.

Cukierman, A., and A. H. Meltzer. 1986. "A Theory of Ambiguity, Credibility, and Inflation Under Discretion and Asymmetric Information." *Econometrica* 54 (5): 1099–1128.

Faust, J., and E. M. Leeper. 2005. "Forecasts and Inflation Reports: An Evaluation." Washington, DC: Board of Governors of the Federal Reserve System.

Ferrero, G., and A. Secchi. 2007. "The Announcement of Future Policy Intentions." Unpublished Paper, Bank of Italy.

Galí, J., and M. Gertler. 2007. "Macroeconomic Modeling for Monetary Policy Evaluation." *Journal of Economic Perspectives* 21 (4): 25–46.

Goodhart, C. A. E. 2005. "The Interest Rate Conditioning Assumption." Discussion Paper No. 547, Financial Markets Group, London School of Economics.

———. 2006. Letter to the Editor, *Financial Times*, June 29.

Gosselin, P., A. Lotz, and C. Wyplosz. 2008. "Interest Rate Signals and Central Bank Transparency." In *NBER International Seminar on Macroeconomics 2007*. Chicago: University of Chicago Press.

Hellwig, C. 2005. "Heterogeneous Information and the Welfare Effects of Public Information Disclosures." Unpublished Paper, University of California, Los Angeles.

Morris, S., and H. S. Shin. 2002. "Social Value of Public Information." *American Economic Review* 92 (5): 1521–34.

Qvigstad, J. F. 2005. "When Does an Interest Rate Path 'Look Good'? Criteria for an Appropriate Future Interest Rate Path—A Practician's Approach." Staff Memo No. 2005/6, Norges Bank.

Rudebusch, G. D., and J. C. Williams. 2006. "Revealing the Secrets of the Temple: The Value of Publishing Central Bank Interest Rate Projections." NBER Working Paper No. 12638.

Svensson, L. E. O. 2005a. "Social Value of Public Information: Morris and Shin (2002) Is Actually Pro Transparency, Not Con." NBER Working Paper No. 11537.

———. 2005b. "The Instrument-Rate Projection under Inflation Targeting: The Norwegian Example." Princeton University.

Walsh, C. E. 2007. "Optimal Economic Transparency." *International Journal of Central Banking* 3 (1): 5–36.

Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton, NJ: Princeton University Press.

———. 2005. "Central Bank Communication and Policy Effectiveness." In *The Greenspan Era: Lessons for the Future*, ed. Federal Reserve Bank of Kansas City. Kansas City: Federal Reserve Bank of Kansas City.

———. 2006. "Inflation-Forecast Targeting: A Monetary Standard for the Twenty-First Century?" Unpublished Paper, Columbia University.